SERIES ... ... ...

# Principles of Structural Engineering

206 ILLUSTRATIONS

Prepared Under Supervision of

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INTERNATIONAL CORRESPONDENCE SCHOOLS

GENERAL PROPERTIES OF MATERIALS
FOUNDATIONS
STATICS
FORCES ACTING ON BEAMS
STRESSES AND STRAINS
THEORY OF BEAMS

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# PREFACE

The volumes of the International Library of Technology are made up of Instruction Papers, or Sections, comprising the various courses of instruction for students of the International Correspondence Schools. The original manuscripts are prepared by persons thoroughly qualified both technically and by experience to write with authority, and in many cases they are regularly employed elsewhere in practical work as experts. The manuscripts are then carefully edited to make them suitable for correspondence instruction. The Instruction Papers are written clearly and in the simplest language possible, so as to make them readily understood by all students. Necessary technical expressions are clearly explained when introduced.

The great majority of our students wish to prepare themselves for advancement in their vocations or to qualify for more congenial occupations. Usually they are employed and able to devote only a few hours a day to study. Therefore every effort must be made to give them practical and accurate information in clear and concise form and to make this information include all of the essentials but none of the nonessentials. To make the text clear, illustrations are used freely. These illustrations are especially made by our own Illustrating Department in order to adapt them fully to the requirements of the text.

In the table of contents that immediately follows are given the titles of the Sections included in this volume, and under each title are listed the main topics discussed.

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# CONTENTS

## GENERAL PROPERTIES OF MATERIALS, §19, §20, §21

v

# GENERAL PROPERTIES OF MATERIALS
## (*Continued*)

# FOUNDATIONS, § 22

# STATICS, § 43

## FORCES ACTING ON BEAMS, §44

## STRESSES AND STRAINS, § 45

## THEORY OF BEAMS, § 46

# GENERAL PROPERTIES OF MATERIALS
## (PART 1)

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## ELEMENTS OF PRACTICAL PHYSICS

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### THEORY OF MATTER

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#### INTRODUCTION

**1.** Anything that occupies space may be defined as **matter.** Thus, substances such as wood, metal, steam, air, and gas are each considered as matter. It does not necessarily follow that the senses should be aware of the presence of matter, as the latter may exist without the senses being cognizant of it. Matter may be so finely divided as to be intangible, as well as invisible.

**2.** Whatever happens in nature is a **phenomenon.** The motions of the heavenly bodies, the rolling of a ball on the ground, the generation of steam from water, the formation of rust on the surface of iron, the circulation of the blood, the beating of the heart, thinking, speaking, walking—all these are phenomena. It will be seen from the definition and examples just given that the word phenomenon, as understood and used in science, does not mean something extraordinary or wonderful For science, there is nothing extraordinary, and a phenomenon is simply a fact—anything that happens

§ 19

**3.**    The invariable relation between a phenomenon and its cause is called a **law of nature.**    If a stone is let fall from a certain height above the ground several times, the interval of time required for it to reach the ground will invariably be the same.    The cause of the stone falling to the ground is attributed to the gravitation of the earth, and the particular law of nature governing the effect of gravitation is called the *law of gravitation*    Other laws of nature are those governing light, heat, electricity, motion, etc    A knowledge of the laws of nature enables the engineer to predict with certainty the effect of any proposed operation.

The sciences dealing with laws of nature are called the *physical sciences*, and it is these sciences that will be considered in this Section.    The physical sciences cover a very large field, but it is beyond the scope of this discussion to give more than is required to enable the engineer to understand the phenomena encountered in his every-day practice.

**4.    Physics and Chemistry.**—Two divisions of the physical sciences of special importance to the engineer are physics and chemistry.    These deal with the phenomena of matter.    When several samples of any kind of matter or material are subjected to various experimental tests, the material will suffer certain changes    Distinction is made between the nature of these changes, which are accordingly classified as *physical* or *chemical.*    A **physical change** is one that does not change the distinctive properties of the material, thus, a marble block split in two is still marble; the splitting of a block into smaller fragments of the same kind is therefore a physical change, and the process by which the block is split is a physical process    A **chemical change**, on the contrary, is one in which the distinctive properties are altogether lost. Thus, if a chip of marble is dropped into a tumblerful of acid, the marble will be dissolved in the acid, with a copious production of gas, and the solid marble is replaced by a gas and a liquid.    Since the material in this case has been entirely changed into something not even remotely similar to the original marble, the change is a chemical change

## CONSTITUTION OF MATTER

**5. Molecules and Atoms.**—It is a generally accepted theory that if a portion of matter is subjected to a process of subdivision, there will finally be obtained an extremely small particle that cannot be further subdivided by mechanical means  This minute particle is known as a **molecule**, and is supposed to be the smallest portion of matter that retains the properties of the original substance

If chemical methods are now employed, the subdivision may be carried still further, but in this process the original properties of the substances are destroyed and the molecules are resolved into smaller particles called **atoms.**   Applying these remarks to the subject considered in Art  4, it may be said that a physical change does not affect the molecule and that a chemical change is a change within the molecule

For example, if the process of subdividing a piece of marble by splitting it into two or more parts is imagined to be continued until the resulting particles become so minute that they cannot be divided further by any physical means, these minute particles are *molecules*.   A molecule of marble can be further resolved into smaller particles, if subjected to a chemical process, but then the resulting particles, or *atoms*, are no longer marble, as they have properties that are entirely different from those of marble.   Thus, if a molecule of marble is resolved chemically, there result 5 atoms, one of a solid substance known as calcium, one of a solid substance known as carbon, and 3 atoms of a gas known as oxygen   Atoms of calcium, carbon, and oxygen cannot be divided by any known process, for which reason such materials are called *elements*, as distinguished from such materials as marble, which are called *compounds*, since they consist of two, three, or more elements

**6. The Molecular Theory.**—The theory that all matter is composed of atoms arranged in groups or molecules is called the molecular theory   In it, all material things are conceived as being either *elements* or *compounds of elements*, both of

which are assumed to consist of the very small individual particles called atoms. As previously explained, an element is assumed to consist of only one kind of atoms, while a compound is assumed to consist of several kinds of atoms grouped in molecules. According to this theory, the molecules may be changed by loosening, exchanging, or adding atoms, whereas the individual atoms remain always unchangeable and unchanged; an atom of oxygen, for example, always remains an atom of oxygen, and all atoms of oxygen are exactly alike The difference between the elements arises from differences in their atoms, just as the difference between compounds arises from a difference in their molecules.

**7. Nature of Atoms and Molecules.**—Little is known about the nature and appearance of either atoms or molecules because they are so very minute that they cannot be made visible by any known means; in fact, the molecules are so small that it would take many thousands laid end to end to produce a speck large enough to be visible under the strongest known microscope, and they are assumed to be separated by distances many times greater than themselves. The atoms are of course much smaller than the molecules; the several atoms composing any one molecule are also supposed to be separated by distances many times greater than the size of the atoms themselves.

**8. The Corpuscular Theory.**—The molecular theory cannot be accepted as an established fact, but merely as a convenient working theory with which the engineer must be familiar in order to understand ordinary technical language. Investigations started in the last decade of the nineteenth century (by the discovery of *radium* and *radio-activity*, referred to hereafter) have led many prominent physicists to believe that the atoms are not unchangeable and indivisible, but that they consist, perhaps, of systems similar to the planetary system, in which many particles, separated by relatively very large distances, rotate about a common center  In ordinary elements, such as iron and the other metals, these particles (or *corpuscles*, as they are sometimes called) are bound to

regular paths within the boundary of each atom, but in certain elements, of which the metal radium is one, some of the atoms are in a state of disruption and the corpuscles are flying off in all directions with great energy. This phenomenon of disruption of the atoms is known as radio-activity, and the theory of corpuscles, advanced to account for radio-activity, is called the corpuscular theory. The corpuscular theory does not necessarily conflict with the molecular theory but is rather an elaboration thereof. The construction engineer must have some knowledge of the molecular theory, because he will occasionally find it referred to in his practice, but the corpuscular theory is to him of only theoretical interest and will therefore not be further explained here.

9. **Forms of Matter.**—All material objects exist in one of the following forms. *solid*, *liquid*, or *gaseous*. The manner in which temperature and pressure transform matter from one form into another may be explained by means of the kinetic molecular hypothesis, of which the main outline is as follows. The molecules that compose matter are not supposed to be at rest; on the contrary, various phenomena indicate that they are in constant motion. In a gas, which represents matter in its most rarified form, the molecules are widely separated from one another and are supposed to fly in straight lines through relatively long distances before they collide with one another or with the walls of the containing vessel and are deflected into other paths.

By subjecting a gas to the effect of heat, the rapidity of the molecular motion is increased  On the other hand, a decrease in temperature of the gas is followed by a slowing down of this motion and a crowding together of the molecules; a tendency to cohesion between the molecules then becomes more evident and the gas assumes eventually a liquid form.

A further reduction of temperature brings the molecules into closer proximity to one another and allows the force of cohesion to predominate, so that the molecules occupy fixed positions relative to one another; that is, the matter assumes

a solid form in which the molecules still retain a certain amount of individual mobility.

**10.** Water furnishes a familiar example of the various forms of matter. Under ordinary circumstances water is a liquid distinguished by its great mobility. By lowering its temperature, the water freezes and assumes a solid form, known as ice. If, on the other hand, the temperature of the water is raised, the motion of the molecules is increased to such an extent that the cohesive attraction is overcome and the water assumes a gaseous form and the molecules fly off into space. If they are confined in a closed vessel, this tendency of the molecules to fly off will cause an increase in the pressure exerted on the walls of the vessel, as in the case of a steam boiler.

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## SPECIFIC GRAVITY

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### PRELIMINARY CONSIDERATIONS

**11. Gravity.**—Experience teaches that, when a body is unsupported, it falls to the ground. This fact is ascribed to the attraction exerted by the earth on all bodies, and the cause of this attraction is known under the name of *gravity*, which will be explained more fully later on in this Section. The attraction of gravity on any particular body is called the **weight** of the body. Weights are measured in pounds, tons, grams, etc.

**12. Specific Gravity.**—In daily language, materials are spoken of as being light or heavy. For example, as a basketful of earth weighs more than a basketful of feathers, it is said that earth is *heavy* and feathers are *light*. To make the comparison more definite, the term *specific gravity* is introduced. The **specific gravity** of a body may be defined as *the ratio between its weight and the weight of an equal volume of water*. The value given for the specific gravity of a substance indicates how many times heavier a given volume of the substance is than the same volume of water.

**13.**  The specific gravity of water is equal to 1.  If the specific gravity of a certain substance is 2, it simply means that a cubic foot of that particular substance weighs twice as much as a cubic foot of water.  Since a cubic foot of water weighs 62.5 pounds, a liquid with a specific gravity of 2 weighs $62.5 \times 2$, or 125, pounds per cubic foot   The metal mercury is a very heavy liquid which has a specific gravity of 13.6; a cubic foot of mercury, therefore, weighs $62.5 \times 13.6 = 850$ pounds

**14.  Specific Gravity Formulas.**—If the specific gravity of a material is known, the weight of a cubic foot of the material can be easily computed by multiplying the specific gravity by 62.5 and, conversely, if the weight of a cubic foot of a material is known, its specific gravity may be found by dividing this weight by 62 5.

Therefore, if $W$ denotes the weight of a material in pounds per cubic foot and $S$ denotes its specific gravity,

$$W = 62 \ 5 \times S \qquad (1)$$

and
$$S = \frac{W}{62.5} \qquad (2)$$

EXAMPLE 1.—Turpentine has a specific gravity of .870.  Find its weight per cubic foot.

SOLUTION.—According to formula 1,
$$W = 62.5 \ S = 62.5 \times .870 = 54 \ 4 \text{ lb  per cu  ft.}  \text{Ans.}$$

EXAMPLE 2.—Alcohol weighs 50 pounds per cubic foot.  Find its specific gravity.

SOLUTION.—According to formula 2,
$$S = \frac{50}{62.5} = 80.  \text{Ans.}$$

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### DETERMINATION OF SPECIFIC GRAVITY

**15.  Practical Application of Specific Gravity.**—The engineer engaged in the design and erection of structures is constantly confronted with problems involving the weights of substances.  Thus, if a warehouse is to be constructed, the floors must be strong enough to carry the goods to be stored

# TABLE I
## APPROXIMATE SPECIFIC GRAVITIES OF VARIOUS SUBSTANCES

| Substance | Specific Gravity | Weight per Cubic Foot, in Pounds |
|---|---|---|
| *Metals:* | | |
| Platinum.. ... ..... .. . ... | 21.50 | 1,343.8 |
| Gold.. . ..... ..... ... . .. . . | 19.50 | 1,218.8 |
| Mercury....... ,.... ...... . | 13.60 | 850.0 |
| Lead (cast) . . . . . . . . . | 11.35 | 709 4 |
| Silver ....... . .... . .. | 10 50 | 656 3 |
| Copper (cast)...... ... ... ... | 8.79 | 549.4 |
| Brass...... .. . .. . . . .. .. | 8.38 | 523.8 |
| Wrought iron...... .. .. ..... .. . | 7.68 | 480.0 |
| Cast iron.. .. . .. . . | 7.21 | 450.0 |
| Steel... .... . ....... .. . .. | 7.84 | 490.0 |
| Tin (cast)........ . . . . . | 7.29 | 455.6 |
| Zinc (cast)..... .... . . . . | 6.86 | 428.8 |
| Antimony.. .. . . . . . . | 6.71 | 419 4 |
| Aluminum. ....... .. . . ... | 2.60 | 162.5 |
| *Woods:* | | |
| Ash.... .. . .... ..... .. . | .845 | 52.80 |
| Beech.... .. ... ..... ..... | .852 | 53.25 |
| Cedar..... ... . .. .. .. . | .561 | 35.06 |
| Cork...... ... . . .. ... .. ... | .240 | 15 00 |
| Ebony (American)..... .. .. . | 1 331 | 83.19 |
| Lignum vitæ.. . ..... .. ... | 1.333 | 83.30 |
| Maple........ .. .... ....... | .750 | 46 88 |
| Oak (old)... . . . . Oak (old)... | 1.170 | 73.10 |
| Spruce......... ... . .... .. | 500 | 31.25 |
| Pine (yellow)..... ... .. . ... | .660 | 41.20 |
| Pine (white)... .. ..... . ........ | .554 | 34.60 |
| Walnut..... .... . ... ... . | .671 | 41.90 |
| *Miscellaneous:* | | |
| Concrete....... . .. .. .. | 2.40 | 150 |
| Emery........ . . .. ... | 4.00 | 250 |
| Glass (average)... . . . . . . .. . | 2.80 | 175 |
| Chalk.... .. . . . . .... .. | 2.78 | 174 |
| Granite........... ... .... . | 2 65 | 166 |
| Marble .... .. .. . . .... .. | 2.70 | 169 |
| Stone (common)....... .. ... ... | 2.52 | 158 |
| Salt (common).. ..... .. ... | 2.13 | 133 |
| Soil (common)............. ... .. | 1.98 | 124 |
| Clay...... .......... .. ... . | 1.93 | 121 |
| Brick......... .. ..... .. | 1.90 | 118 |
| Plaster of Paris (average).... . ... | 2.00 | 125 |
| Sand... ... ... . . . .... . | 1.80 | 113 |

on the floors; a stronger floor is needed for machinery or heavy merchandise than for furniture or light merchandise.   The necessity for knowing the weight of substances arises also in many other cases and, to meet this need, printed tables are in existence, giving either the weight per cubic foot, or the specific gravity, or both.   As an example, the weights and specific gravities of various common metals, woods, and other solids

## TABLE II
### SPECIFIC GRAVITY OF VARIOUS LIQUIDS

| Substance | Specific Gravity | Weight per Cubic Foot, in Pounds |
|---|---|---|
| Acetic acid . . . . . . . . . . . . . . . | 1.062 | 66.4 |
| Nitric acid . . . . . . . . . . . . | 1.420 | 88.8 |
| Sulphuric acid . . . . . . . | 1.841 | 115.1 |
| Hydrochloric acid . . . . . . . . | 1.200 | 75.0 |
| Alcohol . . . . . . . . . . . . . . . . | .800 | 50.0 |
| Turpentine . . . . . . . . . . . . . . . | .870 | 54.4 |
| Sea-water (ordinary) . | 1 026 | 64.1 |
| Milk . . . . . . . . . . . . | 1.032 | 64.5 |

are given in Table I.   As stated, it makes no difference whether the weight per cubic foot or the specific gravity is given, and it might, therefore, seem unnecessary to consider specific gravity, since it would suffice to weigh a cubic foot of material on a pair of scales and note the result    However, this method meets with the practical difficulty of preparing a piece having exactly a given volume, such as 1 cubic foot, and in practice it is therefore frequently more convenient to determine the specific gravity according to the following briefly described methods.

**16.   Determination of Specific Gravity of Liquids.** To find the specific gravity of a liquid, it is only necessary to weigh a given volume of the liquid, weigh the same volume of water, and divide the weight of the liquid by that of the water.

A flask $a$, Fig. 1, with a long neck $b$, is most convenient for the purpose.    In the long neck of the bottle there is a narrow portion carrying a horizontal mark $c$, scratched in the glass with a diamond.    The flask is filled up to the mark $c$ with the liquid and weighed, and from this weight is subtracted the weight of the bottle, the remainder giving the weight of the liquid.    The same operation is then repeated with water, and the weight of the same volume of water is thus obtained.    The specific gravity is finally computed by dividing the weight of the liquid by the weight of the water.    Of course, if this experiment is to be scientifically accurate, many precautions must be taken; thus, the bottle must be perfectly clean and dry before either liquid or water is poured in, and a constant temperature must be maintained.    It will be noted that, by this method, the volume is not measured, this being unnecessary because the same volume of liquid and of water is used.    The specific gravities of various common liquids are given in Table II.

FIG 1

**17. Hydrometer.**—Where many determinations of specific gravity of nearly similar liquids are to be made, an instrument known as a *hydrometer* is found very convenient. There are several forms of hydrometers on the market; one extensively used is illustrated in Fig 2. It consists of a glass tube terminating at its lower end in a bulb loaded with any heavy substance. When placed in a jar containing the liquid of which the specific gravity is to be found, the instrument will float in an upright position.

FIG 2

The use of this instrument is based upon the fact that it will sink deeper into a liquid having a small specific gravity than into one having a greater specific gravity. Therefore, if some of the liquid to be examined is poured into the vessel $b$, and the hydrometer $a$ is placed in the liquid, it will sink

to a certain depth which can be read on a scale engraved on the narrow upper stem. In many instruments, figures indicating the specific gravity are printed on the scale, so that it is only necessary to read on the scale whatever figure comes level with the surface of the liquid in order to know the specific gravity.

**18. Baumé's Scale.**—The scale on the upper end of the tube in Fig 2 may, as stated, have figures giving directly the specific gravity, but some hydrometers have scales graduated according to other systems   One such system is known as *Baumé's*, so called because Baumé was the inventor of the hydrometer described   Since Baumé's scale is frequently referred to in engineering literature, it is sometimes convenient to be able to change the readings of the Baumé scale into specific gravity, this can be done by means of Table III.

<div align="center">

**TABLE III**

**SPECIFIC GRAVITY ACCORDING TO BAUMÉ'S SCALE**

</div>

| Degrees Baumé      . . . . | 0 | 10 | 20 | 30 | 40 | 50 | 60 |
|---|---|---|---|---|---|---|---|
| Liquid heavier than water... | 1 000 | 1 070 | 1 152 | 1.246 | 1.357 | 1 490 | 1.652 |
| Liquid lighter than water.. . . | | 1 000 | .936 | .880 | 830 | .785 | .745 |

In this table the first line gives various degrees Baumé, and the second and the third lines are the equivalent specific gravities for liquids heavier and lighter than water, respectively. Thus, if the liquid is heavier than water and the scale reads 30° Baumé, then the specific gravity is 1.246. If, however, the liquid is lighter than water, 30° Baumé corresponds to a specific gravity of .880

**19. Specific Gravity of Water.**—It has been stated before that the specific gravity of water is 1; that is, the specific gravity of water has been arbitrarily selected as the standard by which the specific gravity of all solids and liquids is measured   This standard must be qualified by a more pre-

cise definition of the term *water*, because the specific gravity of water is not always the same, but, on the contrary, it varies with the degree of purity and temperature of the water. In the language of the physical laboratory, water is understood to mean absolutely pure water at a temperature of 39.2° F. How to purify water by distillation, as well as methods of measuring temperature, will be described later. It is, however, necessary to state here that water has its greatest specific gravity at 39.2° F. If water is heated above this temperature it expands, so that the same quantity of water occupies more space, with the result that the specific gravity becomes less. In this respect, water behaves exactly like any other substance; it is a common rule for all substances to expand when heated. Conversely, when substances are cooled, they contract, but this general rule is not followed by water at 39.2° F.; at this temperature, water expands when cooled, increasing in volume until it freezes at 32° F.

**20. Determination of Specific Gravity of Solids.**
One of the common methods of determining the specific gravity of a solid is similar to that already explained for liquids. A flask with a long neck, but without the horizontal mark, is used in this case. The weight of the solid is found and noted. Then the bottle is filled to the brim with water and weighed. Finally, the solid is put into the bottle, causing an amount of the water to overflow equal to the volume of the solid introduced into the bottle. The bottle containing the solid and the remaining water is then weighed, and the specific gravity is found as in the following example illustrating the determination of the specific gravity of lead.

EXAMPLE.—A piece of lead weighs 2.68 ounces and a flask filled with water weighs 10.90 ounces. When the lead is put in the flask, the flask and contents are found to weigh 13.34 ounces. Find the specific gravity of lead.

SOLUTION.—The weight of the flask filled with water plus that of the lead is 10.90+2.68 = 13.58 oz. Hence, the displaced water weighs 13.58 −13.34 = .24 oz. The specific gravity of the lead is therefore $\dfrac{2.68}{.24}$ = 11.2. Ans.

It should be understood that the size of the piece of lead used in this example does not in any way influence the final result; the same specific gravity would result whether a piece twice as large or half as large were substituted.

**21. Buoyancy.**—The force that tends to make a body float in a liquid is called **buoyancy.** Bodies having a specific gravity greater than that of water, or unity, will sink in water, while those having a specific gravity less than unity will only



(a)                    (b)
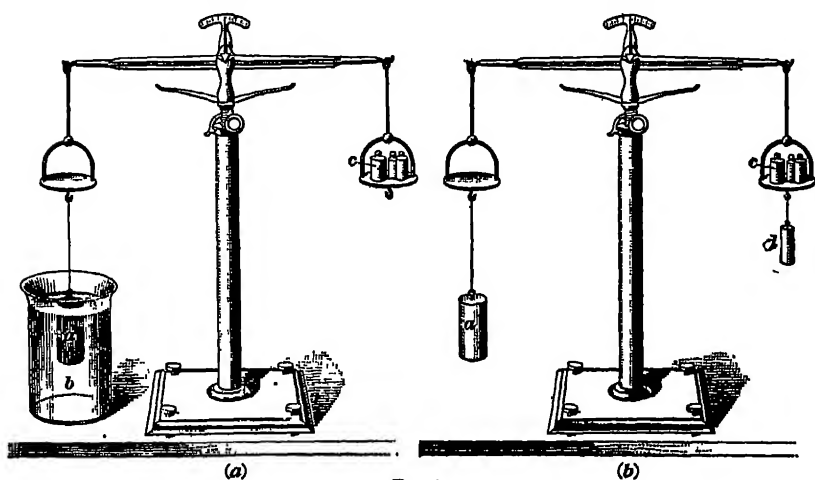
Fig 3

be partly submerged. As discovered by Archimedes and demonstrated by experiment, *when a body is immersed in a liquid, it weighs less than in air by an amount equal to the weight of the liquid displaced.*

In Fig. 3 (a), the body a, completely immersed in water b, is suspended from one end of a scale and balanced by weights c on the other side of the scale. If the vessel of water is removed, as in Fig. 3 (b), the scale will tip, showing that the body now weighs more than the weights c. The additional weight d, needed to balance the body a, is always found to be exactly equal to the weight of the volume of water displaced by a.

The loss in weight upon immersion is due to the buoyant force exerted on the body by the liquid, and the law stated

above is true for all bodies, no matter what the material, size, or shape may be or whether they are completely or only partly submerged.

The construction engineer must consider carefully the effect of buoyancy on the weight of any structure built in water. A cubic foot of concrete, weighing 150 pounds, exerts a downward force of only 150 − 62.5, or 87.5 pounds, when submerged in water. Bridge piers, dams, and retaining walls depend on their weight for stability and whenever water finds its way under the structure the reduction in weight due to buoyancy may lead to serious consequences.

**22. Buoyancy of Ice and Wood.**—Since ice floats on water, it must be lighter than water, and its formation must therefore be accompanied by an expansion of the water. In point of fact, ice has a specific gravity of .92, and it weighs, therefore, .92×62 5 = 57.5 pounds per cubic foot. Since the specific gravity of ice is about nine-tenths that of water, nine-tenths of a floating iceberg, for example, will be submerged and only one-tenth will project above the water.

The specific gravity of different kinds of pine varies between one-half and two-thirds that of water; hence, a pine log will float with one-half to two-thirds of its volume submerged. Some kinds of wood are lighter, others heavier, cork, having a specific gravity of only about one-fourth that of water, floats with three-fourths of its volume above water and only one-fourth submerged, while some woods are so heavy that they sink entirely.

**23. Specific Gravity of Gases.**—Although the properties of gases differ in many respects from those of liquids and, especially, from those of solids, weight is something that they all possess. It is quite as proper to speak of the weight of a cubic foot or of a gallon of air as it is to speak of the weight of a similar quantity of water, and so the ratio of the weight of a cubic foot of gas to that of a cubic foot of water could be considered as the specific gravity of the gas However, since the specific gravities of gases are so much less than that of water, it is preferable to use some other substance than water

as the basis of comparison.  In physics, the atmospheric air is used as the standard for gases and its specific gravity is taken as 1.  Table IV gives the specific gravities of a number of gases, of these, carbon monoxide and carbon dioxide, which are compounds of carbon and oxygen, will be referred to later; chlorine, oxygen, nitrogen, and hydrogen are elements  Hydrogen is the lightest known element, having a specific gravity of

### TABLE IV

#### SPECIFIC GRAVITY OF VARIOUS GASES

*At 32° F , and Under a Pressure of One Atmosphere*

| Substance | Specific Gravity | Weight per Cubic Foot, in Pounds |
|---|---|---|
| Atmospheric air. . . . . | 1.0000 | .08073 |
| Carbon dioxide  . . . .        . | 1.5290 | .12344 |
| Carbon monoxide . .        . | .9674 | .07810 |
| Chlorine . . . . . | 2.4910 | .20110 |
| Oxygen . .       . . . . | 1.1054 | .08924 |
| Nitrogen . . . . . .     . . . | .9674 | .07810 |
| Smoke (bituminous coal)     . . | .1020 | .00815 |
| Smoke (wood)      . .          . . | .0900 | .00727 |
| *Steam at 212° F.      . .          . | .4700 | .03790 |
| Hydrogen .       .  . | .0695 | .00561 |

*The specific gravity of steam at any temperature and pressure, compared with air at the same temperature and pressure, is .622.

only about $\frac{7}{100}$; that is, 100 cubic feet of hydrogen weighs exactly the same as 7 cubic feet of atmospheric air, or 1 cubic foot of hydrogen weighs only as much as $\frac{7}{100}$ cubic foot of air. Since light gases tend to float on heavy gases, a bag filled with hydrogen will rise in air.  This fact is utilized in the construction of certain types of balloons and dirigible airships. Hydrogen is, however, highly inflammable and very dangerous, and the gas known as helium, which is heavier than hydrogen but is not inflammable is sometimes used instead of hydrogen.

1.  Using Table I, find the weight of a concrete foundation 10 feet long, 4 feet wide, and 18 inches thick.                    Ans. 9,000 lb.

2.  A sample of stone having a volume of 3 cubic feet weighs 375 pounds. Find the specific gravity of the material.                    Ans. 2

3.  A piece of copper weighs 3.52 pounds; a flask full of water to be used as in the experiment of Art. 20 weighs 10 pounds.   When the copper is immersed in the water, the flask, copper, and water weigh 13.12 pounds. Find the specific gravity of the copper.                    Ans. 8.8

# ELEMENTS OF MECHANICS

## INTRODUCTION

### MOTION AND FORCE

**24.**  **Motion** is a change in the relative positions of two bodies, and is said to be possessed by either body with respect to the other.

**25.**  **Rest** is the condition of two bodies not in motion with respect to each other, and is said to belong to either body with respect to the other.   Each body is said to be fixed with respect to the other body.

**26.**  When the motion or rest of a body is referred to, without specifying any other body, it is generally understood that the other body is the earth or its surface   Thus, the motion of a train, of a steamer, of a horse, as usually spoken of, means the change of position of the train, steamer, or horse with respect to the ground—that is, to the surface of the earth, or to objects on that surface.

**27.**  A body may be in motion with respect to another body and at rest with respect to a third body.   For example, the smokestack of a locomotive is at rest with respect to the

boiler, since their relative positions do not change; yet, both may be moving with respect to objects on the ground.   Likewise, a man standing on the deck of a moving vessel is at rest with respect to the vessel, but in motion with respect to the water, the shore, etc.

**28.   Uniform Motion.**—A point is said to move with uniform motion when it passes over equal distances in any and every two equal intervals of time; otherwise, the motion is **variable**.   The fact, for instance, that a point moves over 10 feet during the first second of its motion, and over an equal space during the fiftieth second, is not enough to define the motion as uniform; that the motion may be uniform, the point must describe the same space (10 feet in this case) in every second, whatever instant is chosen in which to begin to count the time.   Thus, the space described by the point between the middle of the third second and the middle of the fourth must be the same as the space described in the twentieth second, or during the second in which the time of motion changes from 27.15 to 28.15 seconds, or, in short, during any and every interval of 1 second.

**29.   Definition of Force.**—Matter is always found either at rest or in motion   When a body is at rest it can be set in motion by the action of another body.   Thus, if a block of iron is lying on the ground, it can be set in motion by pulling it with a rope or by placing a strong magnet near it.   If the block is in motion, the action of the other body upon it may cause a change in the rate or direction of motion.   Furthermore, the block may be pushed by one person in one direction, while another person pushes it in the opposite direction.   In this case there may result no motion or change of motion on account of the neutralizing effects of the two actions.   We say, however, that there is a *tendency* to motion or to change of motion, because as soon as one of the opposing actions is removed, the other will cause the block to move, if it is at rest, or cause a change if it is in motion.

The action of one body on another producing, or tending to produce, motion or change of motion is called **force**.

**30. Inertia.**—If a body is not acted upon by a force, it will remain at rest or in uniform motion. That a body does not start to move by itself is familiar to everybody   That a body once set in motion does not come to rest unless acted upon by a force is not so obvious.   Moving bodies often come to rest apparently by themselves.   However, it is found that in such cases the motion is invariably resisted by some force acting in a direction opposite to the motion   For example, a rolling ball will gradually come to rest on a level surface, because the motion is resisted by the air and by the force of friction developed by the body on which it rolls.   Were it not for these opposing forces, the ball would roll at a uniform rate forever.

This property of matter by which it retains its state of rest or uniform motion, unless it is acted upon by a force, is called **inertia.**

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MEASUREMENT AND DELINEATION OF FORCES

**31. Measurement of Force.**—The attraction of the earth, known as gravity, is a force exerted by the earth on all bodies.   This force on any particular body is called its weight and is measured in pounds, kilograms, tons, etc   Forces otherwise exerted may be conveniently expressed in the same
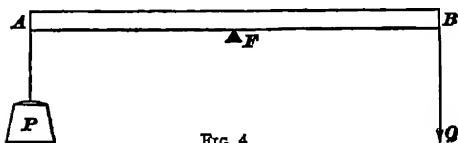


Fig. 4

units. The reason for this will be more apparent when it is considered that every force can be replaced by a weight. Thus, suppose a body $P$, Fig. 4, to be suspended from the extremity $A$ of a rod $AB$ resting at its center on a knife edge $F$.   If $P$ is to be prevented from falling, a weight equal to $P$ may be attached at $Q$, the string $BQ$ may be pulled downwards, or a piece of iron may be fastened at $Q$ with a magnet underneath it.   In all these cases, the weight of $P$ is balanced by, and is therefore equivalent to, the force at $Q$, by whatever means the latter may be produced.

As the two forces are equivalent, the one may be measured

by the other, and the force acting at $Q$ may be said to be 20 pounds, kilograms, tons, etc, according as $P$ weighs 20 pounds, kilograms, tons, etc   Suppose, for instance, that the weight of $P$ is 10 pounds, and that the pull at $Q$ is just sufficient to keep $P$ from falling.   Then, the force pulling at $Q$ is equal to 10 pounds.   If $P$ falls, the pull is less than 10 pounds; and if it rises, the pull is greater than 10 pounds.

**32.   Magnitude of a Force.**—By the magnitude of a force is meant the numerical value of the force, expressed in units of weight.   If, for example, a force is equivalent to a weight of 12 pounds, its magnitude is 12 pounds.

**33.   Direction of a Force.**—The direction of a force is the direction in which the force moves or tends to move the body on which it acts.   A *weight* always acts vertically downwards.

**34.   Point of Application of a Force.**—The point at which a force is considered to act on a body is called the point of application of the force   Thus, in Fig. 4, $A$ is the point of application of the weight $P$ and $B$ is the point of application of the force $Q$ acting on the rod $AB$.

**35.   Graphic Representation of Forces.**—As a great many problems relating to forces are solved by means of geometry, it is convenient to represent forces graphically, that is, by means of lines.   A force is represented graphically by drawing a line, called the **line of action**, in the direction in which the force acts and of a length equal to the magnitude of the force to a given scale   If, for instance, 1 inch of length



FIG. 5

is assumed to represent 1 pound, 2 pounds will be represented by 2 inches of length, and 6.5 inches will represent 6.5 pounds.

The direction of the force along its line of action is indicated by an arrow   Thus, in Fig. 5, the force $AO$ acting on the particle $O$ tends to move it from right to left, as indicated by the arrow

The decimally divided engineer's scale will be found of great assistance in representing forces graphically   On its

various edges this scale is divided into tenths, twentieths, thirtieths, fortieths, fiftieths, and sixtieths of an inch, the number of divisions in each case being a multiple of ten.    Thus, when forces are to be represented graphically to a scale such as 1 inch = 6,000 pounds, the scale of sixtieths, marked 60, will apply most conveniently.    Then each smallest division of the scale represents $\frac{6,000}{60} = 100$ pounds.

### EXAMPLES FOR PRACTICE

1.  A force of 50 pounds is to be represented to a scale of 1 inch = 20 pounds.  What will be the length of the line?        Ans.  2.5 in.

2.  A force has a length of 5.2 inches on a diagram drawn to a scale of 1 inch = 4 tons.  What is the magnitude of the force, in pounds?

Ans.  41,600 lb.

### BALANCING OF FORCES

**36.  Balanced and Unbalanced Forces.**—If two or more forces act on a body in such a manner that they cause no motion, owing to the neutralizing effects they produce, each force is said to be balanced by the combined action of the others, and is referred to as a balanced force.

When a force acts on a body, and there is no opposing force preventing the motion of the body, the force is called an unbalanced force.

These definitions apply to moving bodies as well as to bodies at rest.    If two or more forces are balanced when exerted on a body at rest, they are also balanced when the body is in motion; that is, they do not affect the motion of the body.

**37.  Equilibrium.**—A body is in equilibrium when it is under the action of balanced forces.    The balanced forces themselves are also said to be in equilibrium.

It should be noticed that equilibrium and rest are not equivalent terms.    A body may move uniformly while acted on by balanced forces, in which case it is in equilibrium, but not at rest.    In this case, however, the motion of the body is

not due to and is not affected by the balanced forces acting on it.

**38.  Action and Reaction.**—When a body is at rest, it is evident that the forces acting on it must balance among themselves.  Consider a body weighing 4 pounds lying on a flat surface   If the weight of the body were not balanced, the body would move; the surface, therefore, must exert an upward force equal to 4 pounds in order to keep the body from falling.   If a hand is pushed against a wall with a force of 10 pounds, the wall presses on the hand with the same force, but in an opposite direction   A weight of 20 pounds hanging from a string will evidently pull the string downwards with a force of 20 pounds, but the string must pull the weight upwards with the same force, as otherwise the weight would fall.   In general, whenever one body acts on another, the latter body acts on the former with the same force, but in the opposite direction

The force acting on a body is called the **action,** while the resistance to the force, exerted by the body, is called the **reaction.**   The relation, which is known as **Newton's law of action and reaction** and which applies to any system of bodies as well as to only two, may be briefly stated thus:   *To every action, there is always an equal and opposite reaction*

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### DIVISIONS OF MECHANICS

**39.  Mechanics** is the science of force and motion.   This science is divided into two general branches  (a) *dynamics* and (b) *kinematics.*

**40.  Dynamics** is the science of force and its effects.   In this science, the forces applied to bodies are given, and the resulting effects of these forces are determined; or the conditions of motion are given, and the forces necessary to produce that motion are determined.   Rest is considered a special case of motion in which the speed is zero

**41.  Kinematics** treats of motion alone, without reference to either force or the physical or mechanical properties of

bodies. Suppose, for instance, that a body is moving in a circle with uniform speed, and that it is required to determine what force is necessary to preserve this kind of motion; this is a problem in dynamics, for it relates to force   Suppose that it is required to determine what distance the body travels in a certain time; this is a problem in kinematics, for it can be solved without considering anything but the speed and path of the body.

**42.  Subdivisions of Dynamics.**—Dynamics is subdivided into the branches of (*a*) *kinetics* and (*b*) *statics*.

**Kinetics** treats of unbalanced forces and their effects

**Statics** treats of the equilibrium of forces   For the engineer engaged in building or bridge construction, the study of statics is of the utmost importance, because all such structures are subjected to the action of balanced forces.

NOTE.—The preceding divisions and definitions are of recent origin. Formerly, the term dynamics was used in the sense in which kinetics is now used, the latter term being then unknown.   Even today, the old meaning is frequently given to the term dynamics; the best modern writers, however, use it in the sense just defined—that is, to denote the science of force, in general, whether it produces motion or not.

### RESULTANT AND COMPONENTS OF FORCES

#### FORCES HAVING THE SAME LINE OF ACTION

**43.  Resultant of Forces.**—Assume that a body is acted upon by two or more forces at the same time.   Whatever the effect of the combined action of these forces may be, there usually may be found a single force which, if acting alone, will produce exactly the same effect   Such a
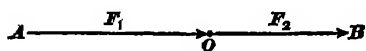


FIG 6

single force is called the **resultant** of the combined forces. With reference to the resultant, the forces combined are called **components.**

For example, if a force $F_1$ of 1,500 pounds acting along the line $AB$, Fig. 6, is pushing a body $O$ in the direction shown

by the arrow, and another force $F_2$ of 1,000 pounds acts along
the same line and in the same direction, the result will be
exactly the same as if the body were acted upon by a single
force equal to $1,500 + 1,000 = 2,500$ pounds in the direction
of the given forces.

If the force $F_2$ acts in a direction opposite to $F_1$, as in Fig. 7,
the result will be exactly the same as if the body $O$ were acted
upon by a single force equal to $1,500 - 1,000 = 500$ pounds in
the direction of the larger force.
In both cases, the forces $F_1$ and
$F_2$ are the components, but in



Fig. 7

the first case the resultant is 2,500 pounds and in the second
case it is 500 pounds.

It is evident that if, in Fig. 7, the forces $F_1$ and $F_2$ were
equal, the resultant would be equal to zero, which is equivalent
to saying that the two forces balance each other.
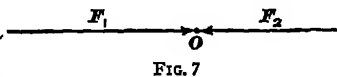
**44. Composition of Forces.**—When more than two
forces have the same line of action and they all have the same
direction, their resultant will be equal to the arithmetical sum
of the components.   If some of the forces act in opposite
directions, the forces may be arranged in two sets, each of which
contains forces acting in the same direction.   The sum of the
forces in each set constitutes a *partial resultant*.   The resultant
of the two partial resultants will be equal to their difference
This difference is the resultant of all the combined forces.

The process of finding the resultant when the components
are known is called **composition of forces.**   The following
examples will help to make the subject clearer

EXAMPLE 1.—A body is acted upon by five forces having the same line
of action, of which $F_1 = 75$, $F_2 = 100$, and $F_3 = 150$ pounds are acting in
the same direction.   The remaining forces $F_4 = 250$ and $F_5 = 125$ pounds
act in the opposite direction.   Find the resultant.

SOLUTION —The partial resultant of $F_1$, $F_2$, and $F_3$ is equal to $75 + 100$
$+ 150 = 325$ lb , and the partial resultant of $F_4$ and $F_5$ is equal to $250 + 125$
$= 375$ lb    The resultant of these partial resultants and, therefore, of all
the forces is $375 - 325 = 50$ lb. in the direction of $F_4$ and $F_5$.   Ans.

EXAMPLE 2.—If, in example 1, $F_4 = 200$ pounds while the other forces
retain their values, what will be the resultant of all the forces?

SOLUTION.—The partial resultant of $F_1$, $F_2$, and $F_3$ is the same as before and is equal to 325 lb. The partial resultant of $F_4$ and $F_5$ is 200+125 =325 lb. The resultant of all the forces is, therefore, equal to the difference between the two partial resultants, or 325−325=0 Since the value of the result is zero, the forces are balanced, or are in equilibrium.

**45. Resolution of Forces.**—In some cases, when a single force acts on a body, it is desirable to substitute for this force two or more components of which the given force is the resultant. The process of finding the components when the resultants are known is called **resolution of forces.**

For example, a force of 100 pounds may be resolved into two components having a common line of action and both acting in the same direction, as in Fig. 6. If one of these components is given a value of 60 pounds, the other must be equal to 40 pounds, because the sum of the components must be equal to 60+40=100 pounds. Or, the force of 100 pounds may be resolved into two components having the same line of action, but acting in opposite directions, as in Fig. 7. If one component is equal to 500 pounds in the direction of the given force, the other must be equal to 500−100=400 pounds in the opposite direction, because the difference between the values of the two components must be equal to 100, and 500 −400=100 pounds.

<div align="center">EXAMPLES FOR PRACTICE</div>

1. A body is acted upon by the following forces having a common line of action: $F_1$=300 pounds, $F_2$=150 pounds, and $F_3$=600 pounds are to the right; $F_4$=100 pounds and $F_5$=750 pounds are to the left. Find the value of the resultant.                    Ans. 200 lb. to the right

2. A force of 800 pounds upwards is to be resolved into three components having the same line of action. If two of these are, respectively, 800 pounds upwards and 400 pounds downwards, what is the value of the other component?                    Ans. 900 lb. upwards

<div align="center">FORCES HAVING DIFFERENT LINES OF ACTION</div>

**46.** When forces have different lines of action, either they may intersect each other or they may be parallel. In either case, the processes of composition and resolution are not quite so simple as in the case of forces having a common line of

action.   The cases to be considered here will include only those in which there are not more than two components   It will be seen that the method of finding the resultant of two forces may be extended to cases where there are several forces, by combining the forces in pairs and substituting for each pair a partial resultant.   Then the partial resultants may be treated in the same manner, until finally all the forces are combined into one resultant

**47.  Resultant of Two Intersecting Forces.**—In Fig. 8, let the lines $O\,A$ and $O\,B$ represent the lines of action of two forces acting on the point $O$, which is their point of intersection and therefore their common point of application   Selecting any convenient scale, the lines $O\,A$ and $O\,B$
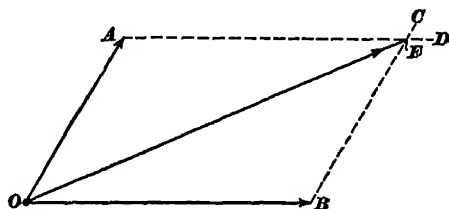


Fig. 8

are laid off to represent the magnitude of these forces and the arrowheads are added to indicate the direction of their action

To find their resultant, draw the lines $B\,C$ and $A\,D$ parallel, respectively, to $O\,A$ and $O\,B$, and intersecting at $E$   Then the line $O\,E$, joining the point of application $O$ and the point of intersection $E$, will be the line of action of the resultant, and $O\,E$ measured to the same scale as $O\,A$ and $O\,B$ will be the magnitude of the resultant

**48.  Parallelogram of Forces.**—It will be noticed that, in the construction of Fig. 8, the resulting figure $AOBE$ forms a parallelogram of which $OE$, the resultant, is the diagonal passing through $O$.   For this reason, the principle upon which this construction is based is called the **law of the parallelogram of forces.**

This law also has useful application for resolution of forces   Assume that, in Fig. 9, the resultant $OE$ is known and it is

required to find its components acting along the lines $O\,M$ and $O\,N$. From the point $E$ draw $EB$ parallel to $O\,M$ and $E\,A$ parallel to $O\,N$. These lines intersect $O\,N$ and $O\,M$
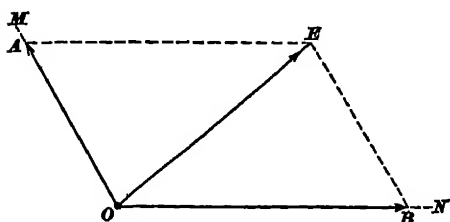
in $B$ and $A$, thus completing the parallelogram of forces, and $O\,A$ and $O\,B$ are the required components measured to the same scale as $O\,E$  Since the given force $O\,E$ acts away from $O$,

Fig 9

the directions of the components $O\,A$ and $O\,B$ are also away from $O$

In practice, it is often required to resolve a force into two components, one horizontal and one vertical  The method of solution of this problem does not differ from that used in the preceding case.  Let $O\,E$, Fig. 10, be the given force acting toward $E$.  Through $O$, draw the vertical line $O\,M$ and the horizontal line $O\,N$ and complete the parallelogram $OAEB$.  Then $A\,E$ and $B\,E$ are the required components, and act toward $E$.

**49.  Triangle of Forces.**—It is readily seen from Fig. 8 that $B\,E$ not only is parallel but is also equal to $O\,A$.  Therefore, point $E$ can be located more easily by simply laying off $O\,B$ equal and parallel to one of the given forces and then making $B\,E$ equal and parallel to the other force $O\,E$ is the resultant just as before. The same result is obtained if the forces are laid off from $O$ to $A$ and from $A$ to $E$.  In either case, the resulting triangle, $O\,B\,E$ or $O\,A\,E$, is called the triangle of forces.

**50.**  The principle of the preceding article can also be applied to the resolution of a force into two components in any directions.  The components of the force $O\,E$, Fig. 9, in

the directions $O M$ and $O N$ can be obtained by drawing from $O$ an indefinite line in the direction of $O N$ and from $E$, a line parallel to $O M$ which intersects the first at $B$.   The desired components are then $O B$ and $B E$.   Since the direction of the resultant is from $O$ to $E$ the directions of the components must be from $O$ to $B$ and from $B$ to $E$
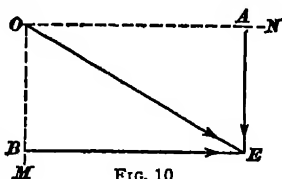
The same results are obtained if the lines are drawn from $O$ parallel to $O M$ and from $E$ parallel to $O N$.   The point of intersection is now at $A$ and the required components are $O A$ and $A E$, which are equal and parallel to $B E$ and $O B$, respectively.   The directions must be from $O$ to $A$ and from $A$ to $E$.

**51.**   The following applications will help make the preceding explanations clearer

EXAMPLE 1.—Fig 11 represents a boat $O$ towed through the lock of a canal by ropes from tractors on the shores.   If the pull in each rope is 1,000 pounds and the angle between the ropes is 60 degrees, find the resulting force pulling the boat
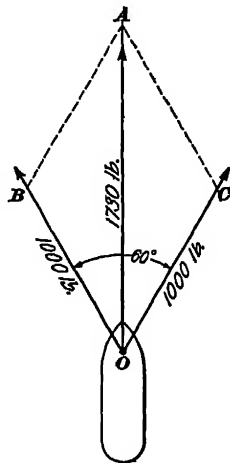
SOLUTION —Since the pulls in the ropes are the components of the resulting force, lay off $O B$ and $O C$ to scale, each representing 1,000 lb. Using a scale of 1 in $= 1,000$ lb , for which the scale marked **10** on the engineer's scale is most convenient, the length of each will be 1 in.   Complete the parallelogram of forces by drawing $B A$



FIG. 11

parallel to $O C$ and $C A$ parallel to $O B$    $O A$ then represents the resultant, and its length, measured to the same scale as $O B$ and $O C$, is the force pulling the boat, or 1,730 lb., obviously, it is in the direction bisecting the angle between the ropes.   Ans.

EXAMPLE 2 —The weight of 500 pounds in Fig 12 is suspended between two walls by ropes $O M$ and $O N$.   If the distance $O X = 15$ feet, $O Y = 5$ feet, and $X M = Y N = 10$ feet, find the pull in each rope.

SOLUTION.—Since the weight is kept from moving by the pulls in the ropes, the problem is to resolve the weight into its two components in the directions of the ropes.   The weight may be considered as pushing downward on point $O$ from above, as shown by the line $C O$    Lay off the length $C O$ to represent 500 lb , using a scale of 1 in. $= 200$ lb    This may be most

conveniently done by means of the scale marked **20** on the engineer's scale, where each inch is divided into twentieths; each division will then represent 10 lb.    Complete the triangle of forces by drawing $C A$ parallel to $O N$.    Since the pulls in the ropes are the components of the weight in the directions $O M$ and $O N$, their respective values are represented by lengths $O A$ and $A C$ to the same scale as was used in laying off $C O$. Since $O A$ scales 225 lb. and $A C$ scales 420 lb , the pull in rope $O M$ is 225 lb. and the pull in rope $O N$ is 420 lb.    Ans



Fig. 12
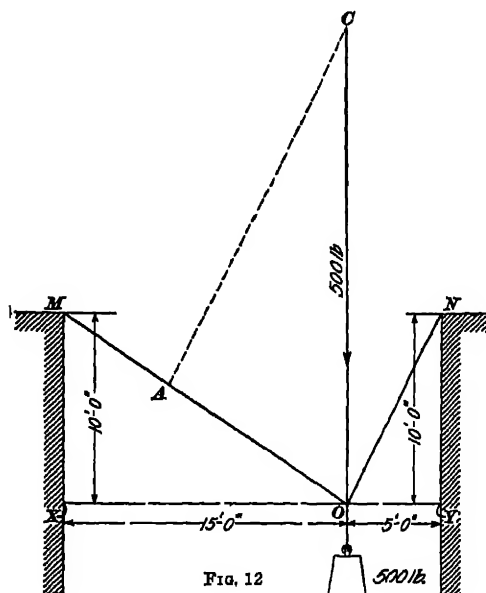
**52.    Resultant of Two Parallel Forces.**—If, in Fig. 13 (*a*), *A* and *B* are two parallel forces of the same magnitude and both point in the same direction, their resultant *R* is equal to the sum of the two forces, and the line of action will be situated midway between them.    If the two forces are not equal, the resultant is also equal to the sum of the two forces, but the line of action of the resultant is now situated, not midway between the two components, but nearer to the larger of the two, as indicated in Fig. 13 (*b*).    The rule applying in this case is that *the products of the forces and their respective distances to the resultant must be equal;* that is, the product of

the force $A$ and the distance $a$ must equal the product of the force $B$ and the distance $b$, or $A \times a = B \times b$. If, for instance, the force $B$ is twice as large as the force $A$, then the distance $a$ must be twice as large as the distance $b$. Thus, if $A$ is 10 pounds and $B$ is 20 pounds, and the distance between them is 6 feet, then this distance must be divided into two parts of which one is
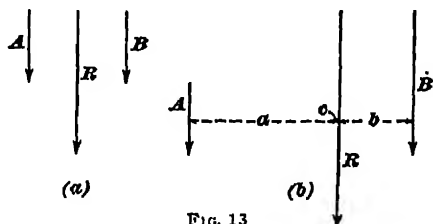


Fig. 13

twice as long as the other; therefore, $a$ is 4 feet and $b$ is 2 feet. The magnitude of the resultant is $10 + 20 = 30$ pounds.

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### EXAMPLES FOR PRACTICE

1. Two forces, having magnitudes of $F_1 = 20$ pounds and $F_2 = 30$ pounds, intersect at an angle of 45°. Find (a) the magnitude, and (b) the direction of the resultant by the graphical method    Ans. $\begin{cases} (a)\ 46.4\ \text{lb.} \\ (b)\ 27\frac{1}{4}°\ \text{with}\ F_1 \end{cases}$

2. A force of 25 pounds makes an angle of 60° with the horizontal. Find the magnitudes of the horizontal and vertical components by the triangle of forces.    Ans. $\begin{cases} \text{Horiz., } 12.5\ \text{lb.} \\ \text{Vert., } 21.7\ \text{lb.} \end{cases}$

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## MOMENTS AND THEIR RESULTANT

**53. Moment of a Force.**—A force may act on a body so that it tends to rotate the body about a fixed point. The tendency of a force to cause rotation about a point is called the **moment of the force about the point,** or **with respect to the point.** The value of the moment is the product of the magnitude of the force and the perpendicular distance from the point to the line of action of the force. In Fig. 13, the moment of $A$ about point $c$ is $A \times a$, and the moment of $B$ about $c$ is $B \times b$.

The point about which a moment is taken, as $c$ in Fig. 13, is called the **center of moments** or **origin of moments.**

The perpendicular distance, $a$ or $b$, from the origin to the line of action of the force is called the **arm** or **lever arm** of the force with respect to the origin.

If the magnitude of a force is in pounds, and the length of the arm is in inches, the product of these two quantities will give the moment expressed in a unit known as *inch-pounds*. If the arm had been measured in feet, the moment would have been obtained in a unit of measure known as *foot-pounds;* similarly, units such as *inch-tons* and *foot-tons* are sometimes used. Thus, if the force $A$, Fig. 14, is 3,000 pounds and the distance $a$ is 8 inches, the moment of $A$ about $p$ is $3,000 \times 8 = 24,000$ inch-pounds; if a force of 200 pounds has an arm of 6 feet, the moment is $200 \times 6 = 1,200$ foot-pounds.

**54. Positive and Negative Moments.**—The force $A$ in Fig. 13 ($b$) tends to cause rotation about $c$ in one direction, while force $B$ tends to produce rotation in the opposite direction. It is therefore important to consider the direction of a moment as well as its magnitude. When a force tends to produce rotation in the direction in which the hands of a clock move, the moment is usually assumed **positive** and is indicated plus ($+$) When the rotation is in a counter-clockwise direction, the moment is **negative** and is indicated minus ($-$). In Fig. 13 ($b$), the moment of $B$ about $c$ is positive and that of $A$ is negative.



FIG. 14
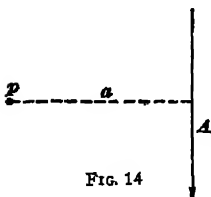
**55. Resultant Moment.**—If there are two or more forces acting on a body, the several moments can be replaced by a single moment which will produce the same rotation as the original system. This single moment is called the resultant moment.

If two forces act on a body so that they tend to rotate the body in the same direction, the resultant moment is equal to the sum of the individual moments. When the moment due to one force is opposed by the moment of another force, the resultant moment is equal to the difference between the two moments and acts in the direction of the greater.

When more than two forces tend to rotate a body in the same direction, the resultant moment is equal to the sum of the individual moments. When the rotation due to some of the forces is opposed by that due to the others, the moments may be combined into two *partial resultants*.
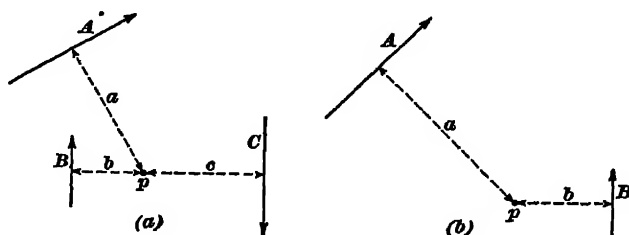


FIG 15

One of these partial resultants is equal to the sum of all the positive moments and the other is equal to the sum of all the negative moments. The final resultant is then equal to the difference between the partial resultants and acts in the direction of the greater.

In Fig. 15 (a), forces $A$, $B$, and $C$ are at distances $a$, $b$, and $c$, respectively, from the point $p$, and the moments of the forces about the point $p$ are $A \times a$, $B \times b$, and $C \times c$, respectively, as already explained. The resultant moment of the forces $A, B$, and $C$ is the sum of these moments, or $A a + B b + C c$, since it will be noted that, in this particular case, all the forces tend to produce rotation around the point $p$ in the positive direction However, there is
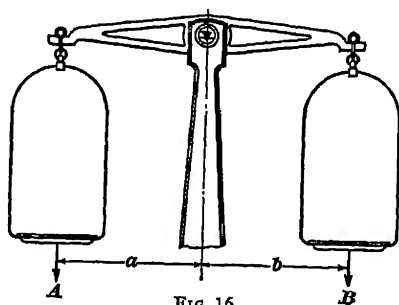


FIG 16

not always such coincidence of direction of rotation. Thus, in Fig. 15 (b), the moment $A \times a$ is positive and $B \times b$ is negative and the resultant moment is equal to the difference, or $A \times a - B \times b$. Equilibrium will exist when the resul-

tant moment is equal to zero; that is, when $A \times a$ equals $B \times b$.

**56.  Scale Beam and Lever.**—The ordinary *scale beam* is an application of this principle of equality of moments.  In Fig. 16, the two arms $a$ and $b$ are equal, and there can be equilibrium only when the weights $A$ and $B$ are equal.   In another form, Fig. 17, the arm
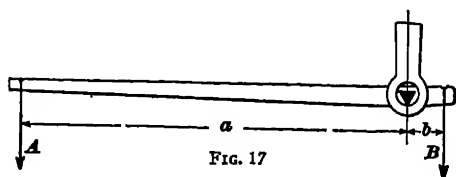


FIG. 17

$a$ is ten times as long as the arm $b$.   In this case, there can be equilibrium only when the weight $B$ is ten times as great as the weight $A$, as then the moments $A \times a$ and $B \times b$ are equal, and opposite in direction.

The *lever*, or crowbar, Fig. 18, used for shifting heavy loads, also acts upon this principle   A comparatively small pres-



FIG 18

sure $A$ on the long lever arm $a$ will lift the large burden $B$ on the short lever arm $b$.   Taking moments about point $c$ where the bar rests on a sharp edge, it is seen that $A \times a$
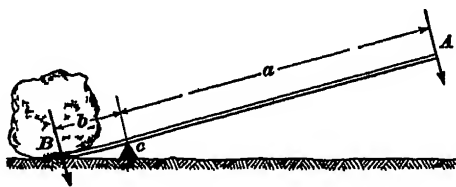


FIG. 19

$$= B \times b \text{ or } A = \frac{B \times b}{a}.$$
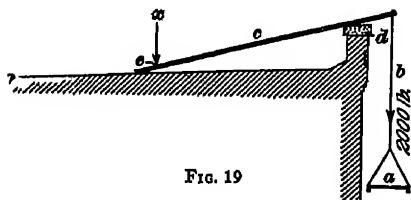
EXAMPLE.—In Fig. 19, $a$ is a swinging scaffold supported by ropes $b$ attached to the outer end of a pole $c$ riding on the coping $d$ of a building.   The scaffold, with the workmen, tools, etc., weighs 2,000 pounds; the pole overhangs the coping 2 feet, measured horizontally.   How much weight must be applied at $e$, at a horizontal distance of 8 feet from the coping, in order to hold up the scaffold?

SOLUTION.—To balance the men, there must be equality between the two moments that exist with reference to the coping, and if the unknown

weight is designated by the symbol $x$, $2,000 \times 2 = x \times 8$.   It follows that $4,000 = 8\,x$, and $x = \dfrac{4,000}{8} = 500$ lb.   Ans.

In practice, two to three times as much weight should be piled on, in order to be on the safe side

---

### EXAMPLES FOR PRACTICE

1.   A stone is to be moved by a crowbar as in Fig. 18   The bar $A\,B$ is 6 feet long and rests on a sharp-edged block $c$ which is 12 inches from the end $B$.   If the force exerted by the stone at $B$ is 150 pounds, what force is required at $A$ to move the stone?   Ans. 30 lb

2.   The weight at $B$ in Fig. 17 is 500 pounds and the distance $b$ is 6 inches.   If the bar itself is assumed to have no weight, at what distance $a$ must a weight $A$ of 100 pounds be placed to keep the bar level?   Ans. 30 in.

---

### CENTER OF GRAVITY

**57.   Definition.**—The weight of a body is the resultant of innumerable minute parallel forces of gravity acting on the particles of the body.   Since these forces are all vertical for any position of the body, the resultant is also vertical.
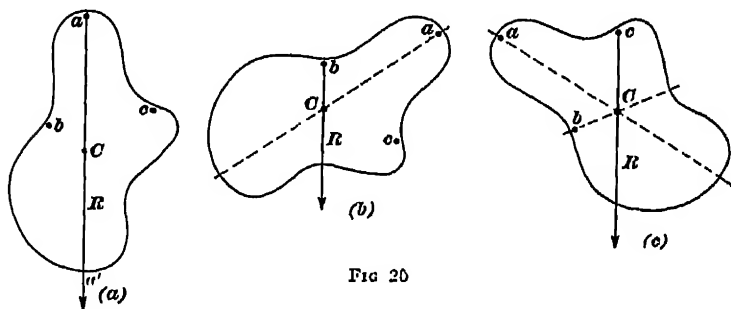


Fig 20

When the body is turned or rotated, the position of the resultant with regard to the body is changed, but the resultant always remains vertical and it always passes through a point which is called the *center of gravity*

If the body shown in Fig. 20 (a) is suspended by a cord fastened at point $a$, its weight will be balanced about the

point $a$ only when the weight of the body and the line of the extended cord are in one straight vertical line $a\,a'$ passing through the center of gravity of the body $C$   If the body is suspended from any other point, as $b$ or $c$ in Fig. 20 $(b)$ and $(c)$, the body will rotate until the point $C$ is vertically beneath the point of suspension so that the weight acts in the line of the extended cord.
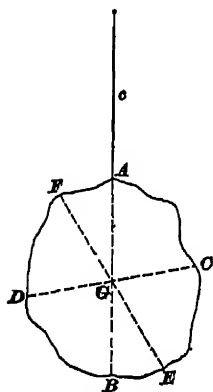
If the body is suspended from the center of gravity $C$, it will balance in any position.   The center of gravity may therefore be defined as that point of a body from which the body may be suspended so that it will remain in equilibrium in any position.



FIG. 21

**58.**  Based upon the principle explained in the preceding article, the center of gravity of a body may be determined by suspending the body by a cord from two or more points on the body, as $A$, $C$, and $E$ in Fig. 21.   $AB$, $CD$, and $EF$ are the imaginary extensions of the cord $c$ for the different positions of the body, and the center of gravity must be at their common point of intersection $G$.   However, this method is usually difficult to apply, because the center of gravity is generally within the body, but, with some modification, it can be employed for plane figures   The engineer seldom has to deal with the center of gravity of bodies but



FIG. 22

he frequently has to know the location of the center of gravity, or *centroid*, of plane figures.
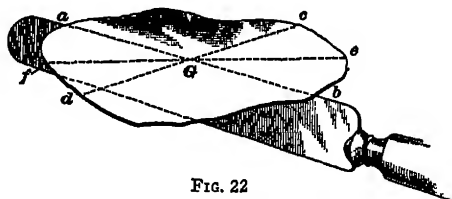
To find the center of gravity of the plane figure shown in Fig. 22, draw the outline of the figure, either full size or to some convenient scale, on a piece of heavy cardboard; cut

out the figure and place it on the edge of a knife as shown in the illustration, shifting the cardboard back and forth until a position is found where it balances; mark the line of the knife on the cardboard, as *a b*.   Then in a similar manner, find one or more other lines, as *c d* and *e f*, along which the cardboard is balanced.   All these lines should intersect in one point *G* which is the center of gravity, or centroid, of the figure.

**59.   Axis of Symmetry.**—Any figure that can be folded along a line in such a manner as to produce two parts whose outlines coincide throughout is said to be *symmetrical* with reference to the line, which is known as an axis of symmetry.
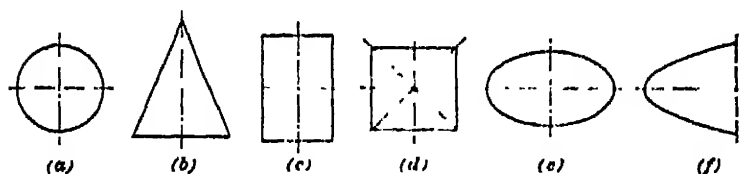


Fig. 23

Thus, in a circle, any diameter is an axis of symmetry because if a circle is folded on one of its diameters, the two semicircles will coincide, as seen from Fig. 23 (*a*).   If an isosceles triangle, Fig. 23 (*b*), is folded on the line passing through the vertex and perpendicular to the base, the two parts will coincide.   A rectangle may be folded, as shown at (*c*), on a line passing through the center and parallel to any one of the sides.

Figures such as the isosceles triangle in Fig. 23 (*b*) and the parabola in (*f*) have only one axis of symmetry   The rectangle (*c*) and ellipse (*e*) each has two axes of symmetry; the square has four axes of symmetry, as shown at (*d*), and the circle has an infinite number of axes of symmetry.

**60.   Center of Gravity of Symmetrical Plane Figures.**—If a figure is symmetrical about one axis, the center of gravity must be on that axis; and if symmetrical also about a second axis, the center of gravity is at the point of intersection of the two axes.   For example, a circle is

symmetrical about a diameter; hence, the point of intersection of two diameters, the center of the circle, is the center of gravity.

The center of gravity of any regular polygon is the center of the circumscribed circle.

**61.** The center of gravity of a triangle may be found in the following manner. In the triangle $ABC$, Fig. 24, let $AE$ be drawn from the vertex $A$ to the middle point $E$ of the opposite side $BC$. Supposing the triangle to be made up of little strips parallel to $BC$, the middle point of each strip must lie on the line $AE$; each strip will therefore balance on a knife edge placed below the line $AE$. Consequently, the triangle as a whole will balance on this edge and the center of gravity must, therefore, lie somewhere in $AE$. For similar reasons, the center of gravity must lie in the lines $CF$ and $BH$, drawn from the vertex $C$ to the middle point $F$ and from $B$ to the middle point $H$ of the sides $AB$ and $AC$, respectively. Hence, the center of gravity must lie at the point of intersection of $AE$, $BH$, and $CF$. Only two of these lines are required for the purpose, and the point of intersection thus obtained is at a distance from any base equal to one-third the length of the line to the vertex; that is, $FG = \frac{1}{3} FC$, $EG = \frac{1}{3} EA$, $HG = \frac{1}{3} HB$.
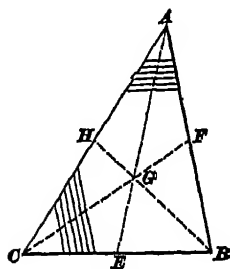


Fig. 24

**62.** The center of gravity of an irregular four-sided figure may be found by first dividing it, by a diagonal, into two triangles and joining the centers of gravity of the triangles by a straight line; then, by means of the other diagonal, divide the figure into two other triangles, and join their centers of gravity by another straight line; the center of gravity of the figure is at the intersection of the lines joining the centers of gravity of the two sets of triangles.

Another method by which to locate the center of gravity of an irregular four-sided figure is illustrated in Fig 25   Draw

the diagonals $a\,c$ and $b\,d$ and from their intersection $e$ measure the distance to any vertex, as $a\,e$.  From the opposite vertex $c$, lay off this distance, as $c\,f = a\,e$.  From $f$, draw a line to one of the other vertexes, as $f\,d$, and bisect this line as at $g$. Connect $g$ and $b$ and lay off one-third of its length from $g$ at the point $h$.  This point is the center of gravity of the figure.
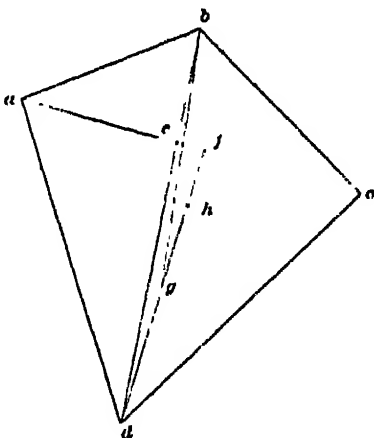
**63.  Center of Gravity of Solids.**—In the case of solids that are symmetrical about a plane, the center of gravity lies in the plane of symmetry.  When there are two planes of symmetry, the center of gravity lies on their line of intersection.  When there is a third plane of symmetry perpendicular to these two, as in the case of the sphere, cube, cylinder, and regular prism, the center of gravity is at the point of intersection of the three planes. Thus, for the sphere or cube, the center of gravity is at the center of the figure; for the cylinder or regular prism, the center of gravity is on the line joining the centers of the parallel sides and midway between these two points.
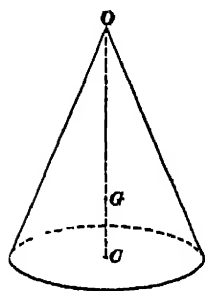
The center of gravity of a cone or pyramid lies on the line connecting the apex to the center of gravity of the base and at a distance from the base equal to one-fourth the length of the connecting line, that is, if $O$, Fig 26, is the apex and $C$ the center of gravity of the base, then $G$, the center of gravity of the cone or pyramid, lies on the line $O\,C$, and $C\,G = \frac{1}{4}\,C\,O$.



Fig. 25



Fig. 26

## STRENGTH OF MATERIALS

### FUNDAMENTAL PRINCIPLES

#### DEFORMATION AND STRESS

**64. Deformation and Elasticity.**—It is a familiar fact that a piece of rubber will stretch under a pull, and that the rubber, when released, will return to its original shape.   If the rubber is compressed instead of extended it will shorten, and when the pressure is removed the rubber will again return to its original shape.   In engineering language, the pull or pressure is called a *load* or a *force;* the amount of lengthening or shortening is called **deformation***; the property of the material that enables it to undergo deformation and again return to its original shape and size is called **elasticity**.   All materials are more or less elastic, whether solids, liquids, or gases, but since the construction engineer is concerned almost entirely with the elasticity of solids, the word elasticity as used in the following pages is applied exclusively to solid materials.

Some materials, such as putty and lead, are readily deformed by a force and do not return to their original shape when the force ceases to act; they are called **plastic materials**.   Other materials, such as glass and brick, can be deformed only slightly without causing rupture, and are known as **brittle materials**.

**65. Unit Deformation.**—The lengthening or shortening of a piece of rubber or any other material is the difference between the original length and the new length, measured

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*The term *strain* is sometimes used to indicate deformation, but since strain is often confused with the term *stress*, to be defined later, it will not be employed in this Section.

in inches or more usually in fractions of an inch, and is called the **total deformation.** However, it is often more convenient to consider the deformation that takes place in any unit length of the body. This quantity, which is called the **unit deformation** and is designated by the symbol *e*, is equal to the total deformation in inches divided by the original length in inches, or the total deformation in feet divided by the original length in feet. Thus, if a rubber band 10 inches long is stretched to a length of 11 inches, the total deformation is $11-10=1$ inch, and the unit deformation is $\frac{1}{10}$ inch per inch of original length. Generally, if the original length of a bar is denoted by *l* and the total deformation by *d*,

$$e = \frac{d}{l}$$

**66. Stress.**—An interesting and important problem arises when the attempt is made to ascertain the effects produced on a body by its deformation. It is obvious that the deformation itself, whether it is a lengthening or shortening of the body, is merely an outward manifestation of something that takes place within the body. It is reasonable to assume that a lengthening is the result of a general lengthening of the distances between the molecules and that a shortening is the result of a general shortening of the distances between the molecules. The changes in the molecular structure of the body are resisted by certain forces acting between molecule and molecule within the body, these forces are called **stresses.** It follows that there are two kinds of forces, namely, *external forces*, that is, forces acting from the outside upon bodies, and *internal forces*, or *stresses*, acting within the body. Since the stresses, as considered by the engineer, usually occur only when external forces are acting upon a body, it is customary to say that the stresses are *caused* by the external forces One of the most important problems in engineering is to determine the kind and magnitude of the stress produced in a given body acted upon by external forces

The load or stress acting upon a unit area of a body is called a **unit load** or **unit stress.** Thus, if a load of 20,000 pounds

is supported by a rod having an area of 2 square inches, the unit load, or unit stress, is $\dfrac{20,000}{2} = 10,000$ pounds per square inch.    If $P$ represents the total load on a body, and $A$ the
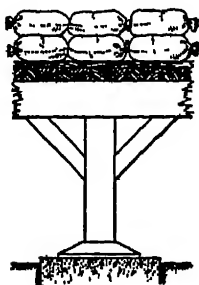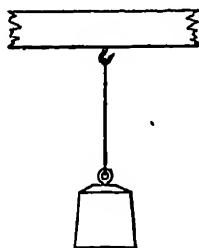


FIG. 27                                FIG. 28

area of cross-section of the body, the unit stress $s$ can be found by the formula

$$s = \frac{P}{A}$$

**67.  Classification of Stresses.**—Three distinct kinds of stresses are recognized; namely, (a) *compressive stress,* (b) *tensile stress,* and (c) *shearing stress.*

A **compressive stress,** or *compression*, is caused in a body when it is subjected to forces which tend to shorten it.    For example, compression is produced in the post in Fig. 27, which carries a load at its upper end.

A **tensile stress,** or *tension*, is produced in a body when it is subjected to forces which tend to stretch it.    Thus, tension exists in the rope in Fig. 28, which supports a weight suspended from its lower end.
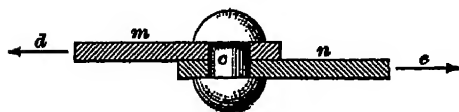


FIG. 29 .

A **shearing stress,** or *shear*, in a body tends to cause some parts to slide past adjacent parts, as in the rivet $c$ uniting the two steel bars $m$ and $n$ shown in the sectional view,

Fig 29. If these bars are pulled by the forces *d* and *e* in oppo-
site directions, the bars *m* and *n* will tend to slide along each
other; but this sliding motion is resisted by the rivet *c*   If
the forces are greater
than the strength of
the rivet, the rivet
will be severed or cut
on the line *a b*, Fig. 30.
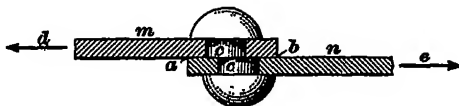


FIG 30

**68.** Although all three kinds of stresses described are
constantly present in all structures, they are not easily
observed, because the deformations produced by them are
too small to be noticed.   However, in the case of a piece of
rubber, its extensibility is easily observed by merely pulling
on the ends   Similarly, compression can be observed if a
block of lead is placed on top of a cube of rubber   The latter
will decrease in height and increase in width   Shear also can
be illustrated by punching a hole in a piece of cardboard

Tension, compression, and shear are called **simple stresses**
because the stress in any structure may be reduced to com-
binations of these three.

In a member subjected only to a simple stress, the deforma-
tion is produced in the direction of the applied force or load.
Thus, in Fig. 27, the post is shortened by the applied load
and the deformation is in the direction in which the load acts.
Also, in Fig 28, the rope is stretched in the direction in which
the suspended weight acts.   In the case of shear, too, the
sliding tends to occur in
the direction of the ap-
plied force.
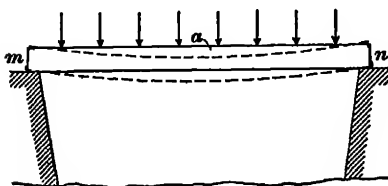


FIG 31

Another manner of pro-
ducing deformation is illus-
trated in Fig 31, where
a plank *a* supports a body
which is being moved
across a ditch   The weight of the body acts in the direction
indicated by the small arrows and the plank is deflected as
shown by the dotted lines.   When the plank bends, there is

a shortening in the material near the top of the plank and a
lengthening in the material near the bottom of the plank
Although the plank deflects downward, the deformations
occur in the direction *m n* and therefore not in the direction
of the weight on the plank.   When the deformation takes
place in a direction other than that of the external load, the
member bends and the stresses produced are called **bending**
**stresses.**   Bending stresses can be reduced to combinations
of the simple stresses, and will be discussed more fully in
another part of this Section

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**ELASTICITY AND RESISTANCE OF MATERIALS**

**69.**   The various materials employed by the construction
engineer differ as regards their ability to resist the three kinds
of stresses referred to.   Some display a great resistance to



tensile forces; others show great
compressive strength but a very
low resistance to tension, and so
forth.   It is therefore of impor-
tance that the engineer should
choose the material best suited
for the purpose in view
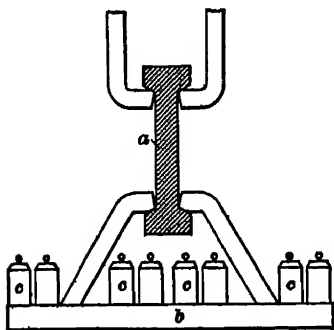
Besides employing the right
kind of material, it is also
economical to use the quantity
which is just enough to make
the structure safe.   The stresses
to which a material may be sub-

Fig. 32

jected without danger of failure are determined by actual
measurements in experimental tests.   The properties which
a material exhibits in resisting stresses are called **elastic**
**properties.**

**70.   Tension Test.**—In order to investigate its elastic
properties, the material under consideration is shaped into
pieces called *test specimens,* which are extended or compressed
in devices known as testing machines.   These machines are

complicated in design, but the principle underlying the test is very simple and may be conveniently illustrated by Fig. 32, where $a$ is a specimen of the material to be tested, in the shape of a bar or rod of uniform cross-section.   This rod is suspended from the ceiling and carries at its lower end a platform $b$ on which weights $c$ are placed

71. **Modulus of Elasticity.**—As the rod $a$ is subjected to a pulling force, equal to the total weight on the platform, it stretches and the deformation increases as more weights are added   If measurements are taken of the elongations corresponding to the different loads on the platform, it will be found that, up to a certain limit, the two values are proportional   Thus, if a pulling load of 10,000 pounds stretches the rod  01 inch, a load twice as great, or 20,000 pounds, will produce an increase in length of $2 \times 01 = .02$ inch, and the deformation under a load of 40,000 pounds will be $4 \times .01 = 04$ inch

It will be noticed that if each load is divided by the corresponding elongation, the quotient will be the same in each case.   Thus, for the first load, $\dfrac{10,000}{.01} = 1,000,000$; for the second, $\dfrac{20,000}{02} = 1,000,000$; and likewise $\dfrac{40,000}{04} = 1,000,000$.

When specimens of the same material and length but of different cross-sectional areas are tested, it is found that for the same total loads the elongations are greater for the specimens of smaller cross-sectional area, but that for the same unit loads the elongations are practically the same.   Furthermore, when specimens of the same material but of different lengths are tested, it is found that for the same unit loads the total elongations are greater for the longer specimens, but that the unit elongations are practically the same.   Therefore, in order to eliminate the effect of the variation of the dimensions of the test specimens on the results, it is necessary to consider in each case the relation between the unit loads and unit elongations.   For most structural materials, it is found that up to a certain limit of loading the ratio of the unit load to

the unit elongation is practically constant in all specimens of the same material.

Since the unit load and the unit stress are numerically equal, it may be stated for most structural materials that *up to a certain limit of loading the ratio between the unit stress and unit deformation has a constant value.* This ratio is called the **modulus of elasticity,** and the relation is known as **Hooke's law.**

Each kind of structural material has a characteristic value of the modulus of elasticity, usually expressed in pounds per square inch, which serves as an indicator of the elastic properties of the material. Values of the modulus of elasticity of materials most frequently used in construction work are given in Table V.

If the modulus of elasticity is denoted by $E$, the unit load or unit stress by $s$, and the unit deformation by $e$, Hooke's law may be stated by the formula,

$$E = \frac{s}{e} \qquad (1)$$

The formula for finding $e$ when $E$ and $s$ are known is

$$e = \frac{s}{E} \qquad (2)$$

For determining $s$ when $E$ and $e$ are given, the formula is

$$s = E\,e \qquad (3)$$

EXAMPLE 1.—A steel rod, 10 feet long and 2 square inches in cross-section, is stretched .12 inch by a weight of 54,000 pounds  What is the modulus of elasticity of the material?

SOLUTION —The unit load, or unit stress, $s = \dfrac{54,000}{2} = 27,000$ lb. per sq. in.  The unit deformation $e = \dfrac{.12}{12 \times 10} = .001$ in. per in. length

Hence,  $E = \dfrac{s}{e} = \dfrac{27,000}{001} = 27,000,000$ lb. per sq. in.   Ans

EXAMPLE 2.—If the modulus of elasticity is 28,000,000 pounds per square inch, what elongation will be produced by a load of 40 tons on a bar 20 feet long and $4\frac{1}{2}$ square inches in area?

SOLUTION.—The unit stress $s = \dfrac{40 \times 2,000}{4.25} = 18,820$ lb. per sq. in.   Unit

deformation $e = \dfrac{s}{E} = \dfrac{18,820}{28,000,000} = .000672$ in. per in. length.

Total elongation $= 20 \times 12 \times .000672 = .161$ in.   Ans.

**72.   Elastic Limit.**—It must be carefully borne in mind that Hooke's law holds good only up to a certain limit of loading per square inch of the material and that when this limit is exceeded the deformation of the rod increases at a greater rate than the load.   This limit of loading on a unit area up to which the ratio between the unit stress and unit deformation is constant is called the **elastic limit** of the material.

**73.   Permanent Set.**—Another important phenomenon takes place when a material is loaded beyond the elastic limit. For loads within the elastic limit, it is found that on the removal of the load the rod returns to its original length, and retains its original strength; but when the load exceeds the elastic limit of the material, the deformation not only increases more rapidly in proportion to the load but on the removal of the load the rod does not fully recover its original dimensions. There then remains a permanent deformation, called **permanent set,** which is the greater the more the load exceeds the elastic limit of the material   The material is also considerably weakened and this process of weakening is more intense when the material is subjected repeatedly to loads exceeding the elastic limit   For example, if a piece of rubber is stretched many times beyond its elastic limit its permanent elongation increases more and more and finally it breaks   It is therefore important in engineering work that no material should be stressed beyond its elastic limit, and the construction engineer should familiarize himself with the values of the elastic limit of materials used in construction to support loads.   Table V gives these values for materials most frequently used in construction work.

**74.   Ultimate Strength.**—By increasing the load on the platform in Fig 32 the rod can finally be broken   The total unit elongation at rupture is called the **ultimate deforma-**

tion, and the total load on the platform at the moment of breaking is the **ultimate load**, or *breaking load*.

The breaking load depends upon the material of the rod and upon its area of cross-section. It is obvious that the total load required to pull apart a rod 2 inches square is greater than that for a rod 1 inch square. The larger rod, which has an area of 4 square inches, can carry four times the load of the rod 1 square inch in area. In other words, if it takes 60,000 pounds to break the 1-inch rod, it requires $60,000 \times \frac{4}{1}$ $=240,000$ pounds to break the 2-inch rod.

Dividing the ultimate load in pounds by the area of cross-section in square inches, the quotient is found to be nearly constant for any material and is a unit stress, in pounds per square inch, called the **ultimate strength**. Thus, in the preceding example, $\frac{60,000}{1 \times 1} = 60,000$ pounds per square inch, and also $\frac{240,000}{2 \times 2} = 60,000$ pounds per square inch. It is to be noted that the ultimate strength is practically independent of the length or shape of the piece.

EXAMPLE —Find the ultimate strength of a piece of steel $\frac{1}{2}$ inch in diameter, which breaks under a load of 12,000 pounds

SOLUTION —Area of   cross-section $= .7854 \ d^2 = .7854 \times (\frac{1}{2})^2 = .196$ sq. in.
Ultimate strength $= \dfrac{\text{total load}}{\text{area}} = \dfrac{12,000}{.196} = 61,200$ lb  per sq. in.    Ans.

**75.** Table V gives the average values of the properties of various materials explained in the preceding articles. The omission of some of the values is due to the fact that they are not definite   The elastic limits and moduli of elasticity are for tension and compression only.

**76.  Resistance of Materials to Tension and Compression.**—For all materials, the principles explained for tension apply similarly for compression except that in the latter case the deformation is a shortening instead of an elongation. The values of the elastic limit, ultimate strength,

## TABLE V
### STRENGTH AND PROPERTIES OF MATERIALS

| Material | Ultimate Strength Pounds per Square Inch | | | Modulus of Elasticity Pounds per Square Inch | Elastic Limit Pounds per Square Inch |
|---|---|---|---|---|---|
| | Tension | Compression | Shear | | |
| Steel .... .    ... | 60,000 | 60,000 | 50,000 | 30,000,000 | 35,000 |
| Wrought iron   . | 50,000 | 50,000 | 40,000 | 27,000,000 | 30,000 |
| Cast iron.. .. | 20,000 | 80,000 | | 15,000,000 | |
| Concrete (1 · 2 : 4). | 150 | 2,000 | 1,300 | 2,500,000 | |
| Stone...    . | | 6,000 | 1,500 | 5,000,000 | |
| Brick..    . | | 2,500 | | | |
| Timber (with grain). | 10,000 | 8,000 | 500 | 1,500,000 | 3,000 |

and modulus of elasticity for steel and wrought iron have been found very nearly equal for both tension and compression Cast iron, natural stone, and concrete have a much higher elastic limit and ultimate strength in compression than in tension, while timber along the grain is stronger in tension It is thus seen that a tensile test for steel and wrought iron is sufficient to determine the quality and properties of these materials.   Due to the unreliability and low resistance of cast iron, stone, and concrete when subjected to tension, they are not used for this purpose if it can be avoided

**77. Hardness.**—It is a common mistake to confuse the words *strength* and *hardness*.  By **hardness** is understood the quality of a material by which it resists abrasion or scratching of its surface; thus, aluminum will scratch tin and aluminum is therefore *hard* compared with tin, but aluminum is *soft* compared with gold because gold will scratch aluminum. There is no direct way of measuring hardness; hardness cannot be expressed in either pounds or inches or any combination of these units.   In the **mineralogical scale,** the hardness is determined by comparison with the following ten minerals, named in the order of increasing hardness.   (1) talc, (2) rock salt, (3) calcspar, (4) fluorspar, (5) apatite, (6) feldspar, (7) quartz, (8) topaz, (9) corundum, (10) diamond.

Thus, if a certain material is scratched by calcspar and scratches rock salt, its hardness is between 2 and 3, and if it seems to be about midway between these, its hardness number is taken as 2.5. Diamond is the hardest of all materials, since it scratches all and is itself scratched by none

Hard materials are likely to be *brittle*, or easily broken into fragments, but this general rule is not without important exceptions

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### EXAMPLES FOR PRACTICE

1. A block 9 inches long and 8 square inches in cross-section is compressed $\frac{1}{16}$ inch by a force of 60 tons. What is the compression modulus of elasticity for the material?          Ans. 2,160,000 lb. per sq. in.

2. The tension modulus of elasticity of a rod 15 feet long and 1 5 square inches in cross-section being 24,000,000 pounds per square inch, determine: (*a*) the elongation caused by a force of 30,000 pounds; (*b*) the force necessary to cause an elongation of $\frac{1}{16}$ inch.          Ans. $\begin{cases} (a) & .15 \text{ in.} \\ (b) & 12,500 \text{ lb.} \end{cases}$

3. Using Table V, find the total pull required to break a piece of steel having a diameter of $1\frac{1}{2}$ inches          Ans. 106,000 lb.

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### WORKING STRESSES

**78. Kinds of Loads.**—The method of applying a force or load has an important effect on the stress produced by the load  Loads may be classified as (1) *dead* and (2) *live*  The weight of the structure itself, which is always constant both in direction and value, is **dead load.** The material which is stored on the floor and is probably changing continually in position and weight is called **live load,** although it is not in motion. People, furnishings, trains on bridges, are other examples of live load. The dead load is always present, while there may or may not be a live load

Live loads may be either *static* or *dynamic*. **Static loads** mean loads applied without a great shock and include the ordinary live loads in buildings  Dynamic loads may mean either swiftly moving loads which are suddenly applied to the structure, or stationary loads which cause vibration; an example of the first is a train moving over a bridge, while

an example of the second is machinery on the floor of a building.

When a dynamic load is applied to a structure, the structure receives a shock and the effect is similar to that which occurs when a moving body collides with another body. This collision between two bodies is known as **impact.** Due to the effect of impact, a dynamic load produces a much greater stress than a static load of equal magnitude.

The weight of a person, or of something which takes up very little area on the loaded surface is called a **concentrated load,** while the weight of stored material taking up considerable area on the structure is a **distributed load.** The force of the wind and the weight of snow are distributed live loads.

**79. Factor of Safety.**—As explained in Art. **73,** if a material is subjected to a load greater than its elastic limit there will remain a permanent deformation after the load is removed. If this load is applied frequently it is liable to cause complete failure even though it is much less than the

### TABLE VI
#### FACTORS OF SAFETY

| Material | Static Loads in Buildings | Dynamic Loads | |
|---|---|---|---|
| | | Varying Stress in Bridges | Shocks and Vibration |
| Timber...... . | 8 | 10 | 15 |
| Brick and stone .. | 15 | 25 | 30 |
| Cast iron...... | 6 | 15 | 20 |
| Wrought iron .. | 4 | 6 | 10 |
| Steel..... . | 4 | 7 | 15 |

ultimate strength of the material   To insure safety, structures are therefore designed so that the unit stress is considerably below the elastic limit. The safe unit stress which is employed in designing structures is known as the **working unit stress,** and the number by which the ultimate strength

of the material is divided in order to give the working unit stress is called the **factor of safety.** The values of the factors of safety used in common practice for different materials are given in Table VI.

In modern practice, the factor of safety is not usually employed, since the safe unit stress or working unit stress should be based upon the elastic limit rather than upon the ultimate strength. Besides, the working unit stresses are usually specified by the building laws of the locality. However, the factor of safety may be useful in some cases.

EXAMPLE 1.—How many tons of steady load can be safely laid on a timber post 8 inches square if the ultimate compressive strength of the material is 8,000 pounds per square inch?

SOLUTION.—From Table VI, the factor of safety is 8   The working unit stress is equal to the ultimate strength divided by the factor of safety, or $\frac{8,000}{8} = 1,000$ lb. per sq. in.   Since the area of the post is $8 \times 8 = 64$ sq. in., the total safe load is $64 \times 1,000 = 64,000$ lb.   Ans.

EXAMPLE 2.—A wrought-iron rod is subjected to a varying load.   If the ultimate strength of the wrought iron is 50,000 pounds per square inch, what is the allowable unit stress in the rod, if it is to carry the load safely?

SOLUTION.—From Table VI, the factor of safety is 6   As the ultimate strength is 50,000 lb. per sq. in., the maximum safe unit stress must not exceed $\frac{50,000}{6} = 8,330$ lb. per sq. in.   Ans.

**80.   Direct and Transverse Loading.**—A body of uniform section is subjected to direct loading when the line of action of the external load coincides with the axis of the body, that is, when it passes through the center of gravity and the point of support of the body.   The external force is then said to be **concentric.**   When the line of action of a load is parallel to the axis of the body but does not pass through its center of gravity, the load is **eccentric.**

A concentric load in most cases produces only simple stress, while an eccentric load always produces bending stress in addition to the simple stress.   Most bodies under a concentric load will change in length but remain straight, while an eccentric load not only changes the length of the member but

also tends to bend it.    Bending under a concentric load occurs only in the case where a long slender member is subjected to compression.

All forces do not necessarily act on a body in the direction of its axis.    Take, for example, the case of the load on the plank *a* laid across the ditch in Fig 31.    This is **transverse loading** and the plank tends to bend into the form shown by the dotted lines

### EXAMPLES FOR PRACTICE

1.   What unit stress is allowable in a square wooden post to safely support a steady load if the ultimate compressive strength of the wood is 6,000 pounds per square inch?                    Ans. 750 lb. per sq. in.

2.   A steel member in a building supports vibrating machinery and is subject to the shocks of suddenly applied loads.    What is the allowable unit stress in tension to be used in designing the member?

Ans. 4,000 lb. per sq. in.

## SIMPLE STRESSES IN STRUCTURES

### FUNDAMENTAL RELATIONS

**81.   Formulas for Simple Stress.**—It has been explained that when the line of action of a load coincides with the axis of the member upon which it acts, simple stress is produced    In this case, the distribution of the stress over the cross-sectional area of the member is uniform and the relations between the total external load $P$, the area of cross-section $A$, and the unit stress $s$ are given by the following formulas·

$$P = A s \qquad (1)$$

$$A = \frac{P}{s} \qquad (2)$$

$$s = \frac{P}{A} \qquad (3)$$

These formulas apply for all cases of tension and shear, but they can be used for compression only when the length of the

member does not exceed ten times its least width.    Longer compression members bend even under concentric loads and will be considered later in this Section

The maximum direct load will usually be known and the safe unit stress or working stress will be specified    For safety the structure must be designed so that the unit stress produced in no place exceeds this value.    The required area of material in a cross-section can be found from formula 2 by substituting for $P$ the maximum load and for $s$ the working unit stress.

EXAMPLE 1.—What will be the diameter $d$ of a steel rod required to safely resist a pull of 30,000 pounds?    Assume the working stress as 16,000 pounds per square inch.

SOLUTION.—

The required area of cross-section $A = \dfrac{P}{s} = \dfrac{30,000}{16,000} = 1.875$ sq. in

Since the area of a rod having a diameter $d$ is .7854 $d^2$,
$$1.875 = .7854\ d^2$$

Hence,    $d^2 = \dfrac{1.875}{.7854}$ and $d = 1.55$ in. $= 1\tfrac{9}{16}$ in.    Ans

EXAMPLE 2.—A cast-iron post, having a length eight times its least dimension and an area of 30 square inches, supports a concentric load of 200 tons at its top.    If the allowable compressive strength of the material is 15,000 pounds per square inch, is the post safe?

SOLUTION.—Since the length is less than ten times the least dimension, formula 3 applies and the actual unit stress in the material is

$$s = \frac{P}{A} = \frac{200 \times 2,000}{30} = 13,300 \text{ lb. per sq in}$$

Since this is less than the working value of 15,000 lb. per sq in., the post is safe

**82.   Gross and Net Areas of Sections.**—A member often consists of several pieces of metal connected together. These connections are usually made by means of *rivets*.    In Fig. 33 is shown an elevation at ($a$) and a cross-section at ($b$) of a member composed of two steel shapes, commonly known as *angles*, which are riveted together.    To make the connection, holes $\frac{1}{16}$ inch larger in diameter than the rivets are first punched or drilled in the legs of the angles and the angles are then placed together so that the holes are opposite    A

heated rivet consisting of a cylindrical rod with a formed head at one end, as shown in Fig 33 (c), is then inserted in each hole, and by the aid of special tools the protruding end of the

rivet is hammered or pressed until the rivet completely fills the hole and the second head is formed, thus making a secure connection.
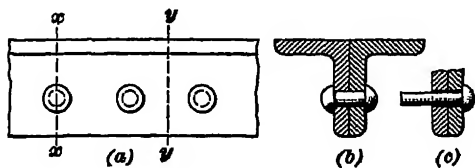


FIG. 33

Rivets are also used for connecting several members of a structure, as in Fig. 34, where the two members *a* and *b* are connected by means of the plate *c* and the rivets.   Sometimes connections are made by means of *bolts*.   A bolt, shown in Fig. 35, is a cylindrical rod which has a formed head *h* at one end and is threaded at the other end   After the holes are made in the pieces of metal to be connected, the bolts are inserted in the holes and the nuts *n* are screwed on the threaded ends to hold them in place.
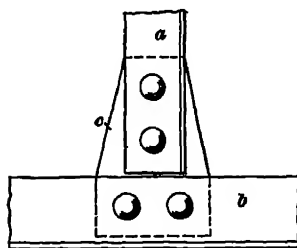


FIG. 34

The holes made in members to receive rivets or bolts reduce the material available at that point   Thus, in Fig. 33 there is less material available at section $x\,x$ than at section $y\,y$, and if the member were subjected to a tensile stress there would be more tendency for it to fail at section $x\,x$ than at section $y\,y$   Therefore, in designing tension members, where the material is pulled apart, only the area remaining in the cross-section through the rivet holes is considered effective in resisting the tensile stress   In designing compression members, however, it is assumed that when



FIG. 35

the member is subjected to a load the metal around the hole presses against the rivet or bolt, and therefore the member is considered just as capable of resisting com-

pressive stress at a section through the rivets or bolts as elsewhere.
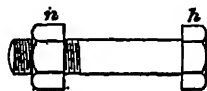
The total cross-sectional area of a member is known as its **gross area**, and the area left in a cross-section after allowance has been made for the holes in it is called the **net area**.
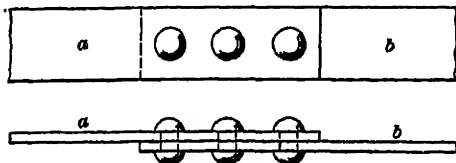


Since in punching or drilling holes the material around the hole is somewhat injured, in deducting for holes from the gross area of a member to obtain the

Fic. 36

net area, the diameter of each hole is considered $\frac{1}{8}$ inch larger than that of the rivet or bolt.

In designing tension members, the net area is substituted for $A$ in the formulas of the preceding article, while in designing compression members the gross area is substituted for $A$, as illustrated in the following examples:

EXAMPLE 1.—Bars $a$ and $b$ in Fig. 36 are each $\frac{3}{8}$ inch thick and are connected with $\frac{3}{4}$-inch rivets. If the width of the bars is 3 inches and the safe unit stress for the material is 16,000 pounds per square inch, what pull can be safely resisted?

SOLUTION.—The gross area of one bar is $3 \times \frac{3}{8} = 1.125$ sq. in. The width to be deducted for each rivet hole is $\frac{3}{4} + \frac{1}{8} = \frac{7}{8}$ in., and the area to be deducted from the gross area is that of one hole, or $\frac{7}{8} \times \frac{3}{8} = .328$ sq. in. The net area of each bar is therefore $1.125 - .328 = .797$ sq. in.

The total safe pull is $P = s A = 16,000 \times .797 = 12,800$ lb., nearly. Ans.
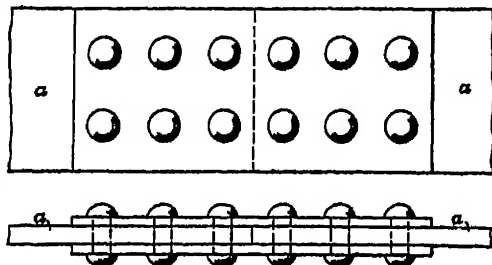


Fic. 37

EXAMPLE 2.—The central plates $a$ in Fig. 37 are $\frac{3}{4}$ inch thick and are connected to the outside plates with $\frac{1}{2}$-inch rivets. If the safe unit stress of the material is 16,000 pounds per square inch, find the width required to resist a pull of 60,000 pounds.

SOLUTION —The required net area $A = \dfrac{P}{s} = \dfrac{60,000}{16,000} = 3\ 75$ sq  in.   To
obtain the gross area, the allowance for rivet holes must be added to the
net area   Since the plate is $\frac{3}{4}$ in  thick, the area to be allowed for each
rivet hole is $\frac{3}{4} \times (\frac{3}{4} + \frac{1}{8}) = .656$ sq  in , and since there are two rivet holes
in a cross-section, the area to be allowed is $2 \times .656 = 1.31$ sq. in.   The
required gross area of the plate will  therefore be $3.75 + 1.31 = 5.06$ sq. in.
and the necessary width is $\dfrac{5\ 06}{.75} = 6\ 75$ in $= 6\frac{3}{4}$ in.   Ans.

EXAMPLE 3.—A compression member of a truss, having an area of
8 square inches and a length nine times its width, carries a concentric load
of 84,000 pounds   What is the unit stress in the member?

SOLUTION.—Since the length of the member does not exceed ten times
its width, the formula for direct stress can be applied.   Also, the gross
area is used for compression members and no allowance is made for rivet
holes.   Hence,

$$s = \frac{P}{A} = \frac{84,000}{8} = 10,500 \text{ lb. per sq in.   Ans.}$$

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### EXAMPLES FOR PRACTICE

1.   Steel bars $a$ and $b$, Fig. 36, are to transmit a pull of 21,500 pounds.
If each bar has a thickness of $\frac{1}{2}$ inch and the bars are connected with $\frac{7}{8}$-inch
rivets, find the required width.    The safe stress of the material may be
taken at 16,000 pounds per square inch.                     Ans.  4 in., nearly

2.   What is the total safe concentric load on a concrete post 6 inches
in diameter and 4 feet 6 inches long if the safe compressive strength of the
material is 300 pounds per square inch?
                                Ans. 8,480 lb.

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### RIVETED JOINTS

**83.   Lap  and  Butt  Joints.**
An important application of the
formulas for simple stress is their
use in investigating the strength of
riveted joints    Riveted joints are



FIG. 38

found in practically every steel structure and the number of
rivets to be used in any case is a very important problem
     When two pieces of material are joined as shown in Fig. 38,
it is known as a **lap joint.**   A more efficient joint and the
one commonly used is the **butt joint,** either with a single
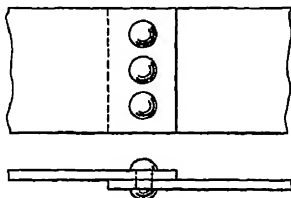
cover-plate or with double cover-plates, as shown in Fig. 39. The following discussion applies to both types, but the lap joint is used for illustration because of its simplicity.



With Single Cover Plate          With Double Cover Plate
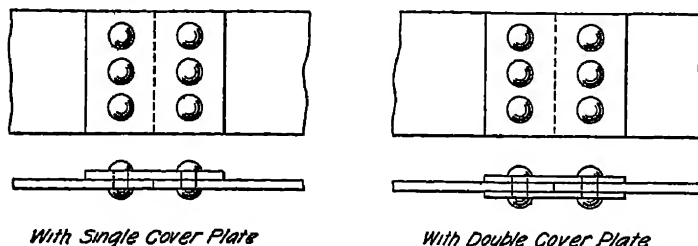
FIG. 39

**84.  Failure of Riveted Joints.**—The joint in Fig 40 (*a*) may fail in four ways when there is a stress in the plates:

1.    One of the plates may tear as in Fig  40 (*b*).

2.    The rivet may be sheared as explained in Art. **67** and as shown in Fig 40 (*c*).

3.    The plate may fail in direct compression, as shown in Fig. 40 (*d*).

4.    The plate may be sheared out by the rivet, as shown in Fig. 40 (*e*).



(a)          (b)          (c)          (d)          (e)
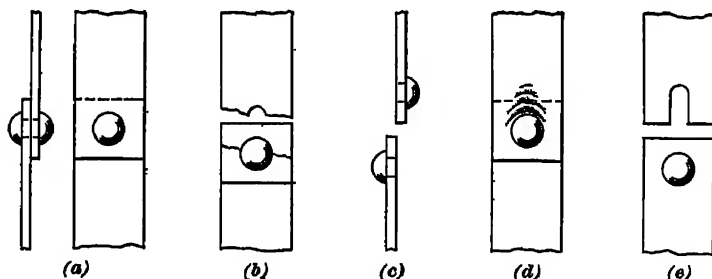
FIG. 40

If the member is properly designed, as explained in Arts  **81** and **82,** the first method of failure cannot occur   The fourth condition can only be caused by having the rivet too near the edge or side of the plate.   The second and third causes

of failure are the ones to be considered as affecting the number and size of the rivets, and the following articles will be devoted to their study.

**85. Shear on Rivets.**—When rivets connect but two plates, the tendency to shear occurs only at the plane or section between the plates, as *a b* of Fig. 41 (*a*), and the rivet is then said to be in *single shear*. Fig 41 (*b*) shows a rivet connecting three plates; the plates in this case tend to shear the rivet not only at plane *a b* but also at *c d*, and the rivet is therefore said to be in *double shear*. It is evident that a rivet in double shear is twice as strong as a rivet in single shear since there is twice as much resisting area. The total safe stress which a rivet can transmit without shearing is called its **shearing value.** The shearing value of a rivet in



Fig. 41

single shear is equal to the area of the cross-section of the rivet multiplied by the safe unit shearing stress. For double shear, the shearing value is twice as great as for single shear.
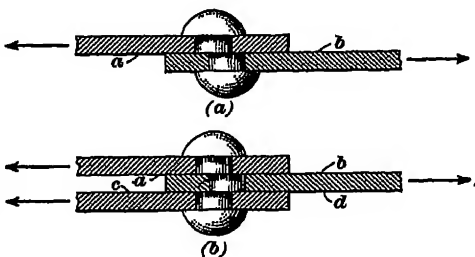
**86. Bearing on Plates.**—The third type of failure stated in Art 84 occurs when the diameter of the rivet is large in comparison with the thickness of the plate. The material of the plate pressing against the rivet acts like a short compression member and the unit bearing stress on the material equals the total load transmitted by a rivet in shear divided by the area of contact perpendicular to the direction of this load. The area of contact, in square inches, thus defined equals *t d*, where *t* is the thickness of the plate in inches and *d* is the diameter of the rivet (not the rivet hole), also in inches. The total safe bearing strength of the plate on one rivet is its **bearing value.**

**87.   Allowable Unit Stress.**—Due to the added resistance of the material around the hole, the unit stress allowed in bearing on a rivet is much greater than the safe compressive strength in a short column.   Also, higher unit stresses are allowed on rivets driven in the shop than on field-driven rivets.

The values of the allowable unit stresses for different materials will be given in a later Section.   It is sufficient to say here that for structures subjected to static loads, the shearing strength may be taken as 12,000 pounds per square inch for shop rivets and 10,000 pounds per square inch for field rivets. The bearing strength on a rivet is usually twice the shearing strength, or 24,000 and 20,000 pounds per square inch, for shop and field rivets, respectively.   In any particular case the safe stresses depend upon the character of the work and are usually specified.

**88.   Critical Value of a Rivet.**—Let $s_b$ be the allowable unit bearing stress on a rivet in pounds per square inch and let $s_s$ be the allowable unit shearing stress in pounds per square inch.   If $d$ is the diameter of the rivet in inches and $t$ the thickness of the plate in inches, the shearing values of one rivet are $.7854\,d^2 s_s$ in single shear and $1.57\,d^2 s_s$ in double shear, while the value of the plate in bearing is $t\,d\,s_b$.   After deciding whether the rivet is in single or double shear, the value in bearing or shear which gives the least strength of a rivet for the particular case is called the **critical**
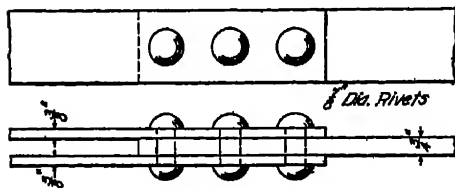


FIG. 42

value of the rivet and is the one to be used in calculating the number of rivets required.

**89.**   The following examples illustrate the above principles and also explain more fully the action of the rivets.

EXAMPLE 1.—In the tension joint of Fig. 42, the central plate is $\frac{3}{4}$ inch thick, the two outside plates are each $\frac{3}{8}$ inch thick, and there are three $\frac{7}{8}$-inch shop rivets.   Find the critical value of each rivet.

SOLUTION.—The area of cross-section of a $\frac{7}{8}$-in. rivet is $.7854 \times (\frac{7}{8})^2$ $= .601$ sq in and the bearing areas on one rivet are $\frac{7}{8} \times \frac{3}{8} = .328$ sq. in. for each outside plate and $\frac{7}{8} \times \frac{3}{4} = .656$ sq in. for the central plate. Each rivet is in double shear and the shearing strength of one rivet is then $2 \times .601$ $\times 12,000 = 14,420$ lb. The bearing strength of the two outside plates is $2 \times .328 \times 24,000 = 15,740$ lb , which is also equal to the bearing value on the central plate. The safe strength of one rivet, or its critical value, is therefore $14,420$ lb. Ans.

EXAMPLE 2.— The total stress to be transmitted from plate $c$ to plate $f$ in Fig. 43 is 44,000 pounds. If plate $c$ is $\frac{9}{16}$ inch thick, $f$ is a $\frac{3}{4}$-inch plate, and connection angles $d$ are each $\frac{1}{2}$ inch thick, find (a) the required number of $\frac{3}{4}$-inch shop rivets $a$, and (b) the number of $\frac{3}{4}$-inch field rivets $b$.
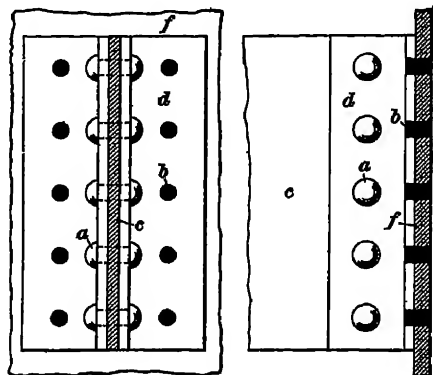
SOLUTION —(a) The shearing value of one rivet $a$ in double shear is $2 \times .7854$ $\times (\frac{3}{4})^2 \times 12,000 = 10,600$ lb. The bearing value on the $\frac{1}{2}$-in. angles is $2 \times \frac{1}{2} \times \frac{3}{4} \times 24,000 = 18,000$ lb. The bearing value on the $\frac{9}{16}$-in. plate is $\frac{9}{16} \times \frac{3}{4} \times 24,000$ $= 10,130$ lb. The last is the critical value, and the number of shop rivets required is

$$\frac{44,000}{10,130} = 4.34, \text{ say } 5. \text{ Ans.}$$

FIG 43

(b)   The value of one rivet $b$ in single shear is $.7854 \times (\frac{3}{4})^2 \times 10,000$ $= 4,420$ lb    The bearing value on the $\frac{1}{2}$-in angle is $\frac{1}{2} \times \frac{3}{4} \times 20,000 = 7,500$ lb    The bearing on the $\frac{3}{4}$-in plate is obviously greater and is not considered   The shearing value is the critical one, and the number of field rivets is

$$\frac{44,000}{4,420} = 10, \text{ or } 5 \text{ in each angle. Ans}$$

EXAMPLE 3.—If in Fig. 43, plate $c$ is $\frac{5}{8}$ inch thick, plate $f$ is $\frac{3}{4}$ inch thick, angles $d$ are $\frac{1}{2}$ inch thick, and there are four $\frac{7}{8}$-inch shop rivets $a$ and eight $\frac{7}{8}$-inch field rivets $b$, find the safe strength of the joint.

SOLUTION —The shearing value of one shop rivet $a$ in double shear is $2 \times .7854 \times (\frac{7}{8})^2 \times 12,000 = 14,430$ lb    The bearing value on the $\frac{1}{2}$-in. angles is $2 \times \frac{1}{2} \times \frac{7}{8} \times 24,000 = 21,000$ lb    The bearing value on the $\frac{5}{8}$-in plate is $\frac{5}{8} \times \frac{7}{8} \times 24,000 = 13,130$ lb    The last is the critical value of one rivet, and therefore the strength of the four rivets $a$ is $4 \times 13,130 = 52,520$ lb.

The shearing value of one field rivet $b$ in single shear is $.7854 \times (\frac{7}{8})^2$ $\times 10,000 = 6,010$ lb.    The bearing value on the $\frac{1}{2}$-in. angle is $\frac{1}{2} \times \frac{7}{8} \times 20,000$ $= 8,750$ lb.    The bearing value on the $\frac{3}{4}$-in plate does not have to be found, since it is obviously greater.    The critical value of each rivet is therefore 6,010 lb. and the strength of the eight rivets $b$ is $8 \times 6,010$ $= 48,080$ lb.    This is less than the strength of the rivets $a$ and therefore the safe strength of the joint is 48,080 lb    Ans.

1    Find the critical value ($a$) of a $\frac{7}{8}$-inch field rivet in double shear and in bearing on an $1\frac{1}{8}$-inch plate, ($b$) of a $\frac{3}{4}$-inch shop rivet in single shear and in bearing on a $\frac{5}{16}$-inch plate.    Ans. $\begin{cases} (a) & 12,020 \text{ lb.} \\ (b) & 5,300 \text{ lb.} \end{cases}$

2    Using the allowable stresses given in Art. 87, determine the safe strength of the joint in Fig. 43 if plate $c$ is $\frac{3}{8}$ inch thick, and there are four $\frac{3}{4}$-inch shop rivets $a$ and eight $\frac{3}{4}$-inch field rivets $b$.    The plate $f$ is $\frac{3}{4}$ inch thick and angles $d$ are $\frac{1}{2}$ inch thick.    Ans. 27,000 lb.

3.    Find the number of $\frac{7}{8}$-inch rivets in the connection shown in Fig 43 to transmit a total stress of 44,000 pounds.    Plate $c$ is $\frac{9}{16}$ inch thick, plate $f$ is $\frac{5}{8}$ inch thick, and angles $d$ are $\frac{7}{16}$ inch thick.    Rivets $a$ are shop driven, and rivets $b$ are field driven.    Ans. $\begin{cases} 4 \text{ rivets } a \\ 8 \text{ rivets } b \end{cases}$

## BENDING STRESSES

### BEAMS

**90.    Definitions.**—Any horizontal member supported at one or more points and subjected to transverse loads is called a **beam.**

When a beam rests freely on two supports near its ends and spans an opening between the supports it is a **simply supported beam** or **simple beam.**    Thus, the plank in Fig 31 is a simple beam

When one end of a beam is rigidly held in place and the other end is unsupported, the beam is known as a **cantilever.** A flagpole extending horizontally from the face of a building and having one end fastened inside the building is a cantilever.

Beams resting on more than two supports are called con-
tinuous.

In this Section, only simple beams will be considered.

**91.  Stresses in a Beam.**—When a simple beam bends,
or *deflects*, as it is usually termed, it takes the form shown in
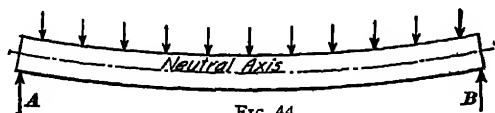


Fig 44

Fig. 44, in which the depth of the beam and the distance which
the unsupported part deflects below the original horizontal posi-
tion are shown exaggerated.   The material at the bottom of
the beam is elongated while that at the top is shortened.
Somewhere near the center the material is not deformed at all.
This surface, in which the material keeps its original length, is
called the **neutral surface** and the intersection of this sur-
face with any cross-section gives the **neutral axis** of the
cross-section.   Since it requires a stress to produce a deforma-
tion, the portion of the beam which elongates is in tension
and the part which is shortened is in compression, whereas
the material in the neutral surface is not stressed at all.   Thus,
the neutral axis of a section divides the part in tension
from the part in compression and is a
single line which usually passes through
the center of gravity of the section

The cross-section of a rectangular
beam is shown in Fig. 45, where *a b* is
the neutral axis.   Part *a b d c* will be in
compression and part *a b f e* will be in
tension.
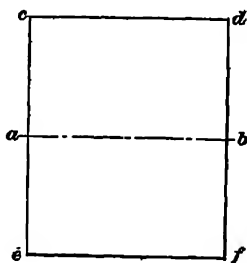
**92.  Shear in a Beam.**—Shear has
already been defined as the tendency for
one part of a member to slide past the adjacent part    The
consideration of shear is very important in the design of beams

Assume the beam of Fig 46 to be composed of a series of
separate blocks placed side by side, similar to a stack of



Fig. 45

pennies held horizontally between the fingers. If a small weight is placed on the pennies near the center of the stack, the pennies will tend to slide vertically past each other, but they may be prevented from falling by the horizontal pressure of the fingers. There is the same tendency for the blocks in Fig 46 to slide under the load $W$, or for any two adjacent parts of a solid beam to slide past each other when the beam carries a load. In the case of a solid beam, however, the sliding or shear is resisted by the cohesion of the particles of the material.

**93. Reactions.**—It has been previously shown that every action on a body requires an equal and opposite reaction
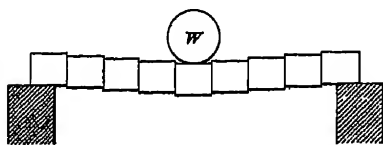


Fig. 46

to keep the body from moving. The necessary resisting forces on a beam are supplied by the supports. The opposing thrusts exerted by the supports are the **reactions** of the beam. A reaction is represented graphically as an external force acting on the beam at the supports. For a simple beam carrying vertical loads, the reactions are always vertically upwards, as $A$ and $B$ of Fig. 44.

**94. Modulus of Rupture.**—When a simple beam breaks due to a transverse load, it is found that the maximum unit stress developed in the material does not agree with the ultimate strength either in tension or compression. The ultimate unit stress under transverse load is called the **modulus of rupture** and its value is obtained by actual tests. Just as in the case of direct stress, a factor of safety must be applied to the modulus of rupture to give the safe unit bending stress.

### COLUMNS

**95.** A column is a structural member which is subjected to a compressive stress by a load placed upon its upper end If the ratio between the length and least width of a column

is less than 10, the column is called *short*.   When the length exceeds ten times the least width, the column is *long*.

A short column will be slightly flattened by a concentric load but will remain straight.   In this case the column is subjected to a simple stress only, and, as previously explained, the formulas of Art. 81 can be applied in investigating its strength.

A long column, even under a concentric load, will bend sideways before failure, as shown by Fig. 47 (*a*).   Due to this bending, the column is subjected to a bending stress in addition to the simple stress, and is not so strong as the short column of equal area in Fig. 47 (*b*).   This bending in long columns requires the use of complicated formulas for their design, which will not be discussed here.   For the case of long wooden columns, however, the simple method given in the next article may be applied.
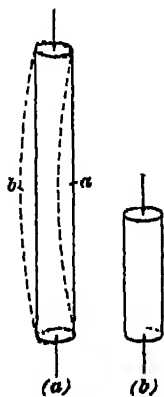


(*a*)   (*b*)
Fig. 47

**96.   Formulas for Long Wooden Columns.**—A short wooden column may be designed, or the strength of a given short column may be determined, by the formulas for simple stress in short columns given in Art. 81.   In the case of long columns, however, the value of *s* must be found from the following formula, which has been adopted by the American Railway Engineering Association for wooden columns, whether round, square, or rectangular in section:

$$ s = s' \left( 1 - \frac{1}{60} \frac{l}{d} \right) $$

where $s$ = working stress, in pounds per square inch;
$\quad s'$ = safe compressive strength for a short column of the same material; its values for various kinds of woods are given in Table VII;
$\quad l$ = length of column, in inches,
$\quad d$ = least dimension of cross-section of column, in inches.

## TABLE VII
### VALUES OF *s'* FOR WOODEN COLUMNS

| Kind of Wood | $s'$ Pounds per Square Inch |
|---|---|
| Longleaf yellow pine, white oak. . . | 1,300 |
| Douglas fir, hemlock...... ... . | 1,200 |
| Shortleaf yellow pine, spruce, cypress | 1,100 |
| White pine, tamarack.... . .. . | 1,000 |
| Red cedar, redwood. .. .. . .. . | 900 |
| Norway pine..... . .... .. . | 800 |

EXAMPLE 1.—What is the allowable concentric load on an $8'' \times 10'$ column of longleaf yellow pine 12 feet long?

SOLUTION.—Since the length $l$ of the column is $12 \times 12 = 144$ in., and the least width $d$ is 8 in., the ratio of length to width, $\dfrac{l}{d} = \dfrac{144}{8} = 16$, and the column is long   To apply the formula of Art. 81 to long columns, $s$ must first be found.  From Table VII, $s' = 1,300$.

Hence, $s = s'\left(1 - \dfrac{1}{60}\dfrac{l}{d}\right) = 1,300\left(1 - \dfrac{1}{60} \times 18\right) = 910$ lb  per sq  in.

Then, from formula 1 of Art 81,
$$P = A\,s = 8 \times 10 \times 910 = 72,800 \text{ lb.}  \text{Ans.}$$

EXAMPLE 2.—The weight of concrete and forms to be supported by $4'' \times 4''$ spruce posts 9 feet long is 800 pounds per foot.  If the posts are spaced 8 feet apart, will the forms be secure?

SOLUTION.—Here $l = 9 \times 12 = 108$ in.; $d = 4$ in ; $\dfrac{l}{d} = \dfrac{108}{4} = 27$, and from Table VII, $s' = 1,100$.  Then, $s = s'\left(1 - \dfrac{1}{60}\dfrac{l}{d}\right) = 1,100\left(1 - \dfrac{1}{60} \times 27\right) = 605$ lb. per sq. in.  Then from formula 1 of Art. 81, the total load which each post can safely support is $P = A\,s = 4 \times 4 \times 605 = 9,680$ lb.

The load which one post will have to carry is $800 \times 8 = 6,400$ lb.  Since the allowable load exceeds the actual, the posts are ample strong.

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### EXAMPLES FOR PRACTICE

1.  What is the total safe load which can be carried by a $6'' \times 8''$ shortleaf yellow pine column 10 feet long?               Ans  35,200 lb.

2.  What total load can be safely supported by a 6-inch square hemlock column 15 feet long?               Ans. 21,600 lb.

# GENERAL PROPERTIES OF MATERIALS

## (PART 2)

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## PRESSURES IN LIQUIDS AND GASES

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### PRESSURE IN LIQUIDS

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#### FUNDAMENTAL PRINCIPLES

**1. Vertical Pressure in Liquids.**—If some cement is carried in a pail, the bottom of the pail must necessarily support the weight of the cement. Similarly, if water is poured into a cylindrical glass, the bottom of the glass must support the weight of the water. The weight carried by the entire bottom is equal to the total weight of water, and each square inch of the bottom carries its proportionate share of the total. Therefore, each square inch of the bottom surface carries the weight of the column of water directly above it. Let Fig. 1 represent a vessel filled with liquid to the brim, and let $a\,b\,d\,c$ be a small area of the bottom; then the area $a\,b\,d\,c$ will obviously carry the weight of the column of .liquid resting upon it, or, in other words, the pressure on $a\,b\,d\,c$ is equal to the weight of the volume of liquid $a\,b\,d\,c\,e\,f\,g\,h$.

If, as an example, the area $a\,b\,d\,c$ is exactly 1 square inch, and the height of the vessel is 8 inches, then the volume $a\,b\,d\,c\,e\,f\,g\,h$ is $8\times1=8$ cubic inches. A cubic foot contains $12\times12\times12=1,728$ cubic inches and 1 cubic foot of

water weighs 62.5 pounds.   Therefore, 1 cubic inch of water

weighs $\dfrac{62.5}{1,728}$ = 03617 pound, and 8 cubic inches of water

weigh .03617×8 = 289 pound.
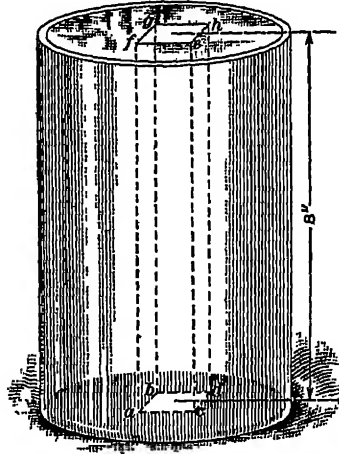


FIG. 1

The pressure on each square inch of the bottom is, therefore, 289 pound, that is, the pressure on the bottom is .289 pound per square inch    If the bottom is 10 square inches in area, the total weight of the water in the vessel would be   .289×10 = 2 89   pounds. If another liquid is substituted having a specific gravity of 2, the pressure becomes twice as great, or   289×2 = 578 pound per square inch, and the total pressure = .578×10 = 5 78 pounds

EXAMPLE.—If a submarine descends to a depth of 100 feet, what would be the pressure on each square foot of its deck?

SOLUTION —Let the area $a\ b\ c\ d$, Fig. 2, be 1 ft. square.   Then it must carry the weight of a column of water 1 ft square and 100 ft. high, having therefore a volume of 1×100 = 100 cu ft., and weighing 100×62 5 = 6,250 lb.   The pressure on the deck of the submarine is therefore 6,250 lb. per sq. ft.   Ans.

**2. Horizontal Pressure in Liquids.**—If, in Fig. 3, the vessel $a$ is provided with tubes $f$, carrying gauges for measuring the horizontal pressure, at depths



FIG. 2

of 1 foot, 2 feet, 3 feet, and so forth, it will be found that the gauge $b$, situated 1 foot below the surface of the water, indicates
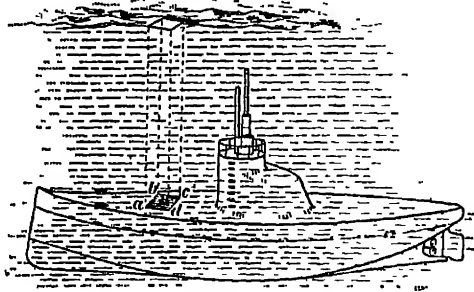
a pressure of 62.5 pounds per square foot.    The gauge $c$ is at a depth of 2 feet and the pressure is $2 \times 62\ 5 = 125$ pounds per square foot.    The gauge $d$ is at a depth of 3 feet and the pressure is $3 \times 62.5 = 187.5$ pounds per square foot    In other words, the pressure exerted sidewise against the vertical walls of the vessel is exactly as great as that exerted vertically against any horizontal surface at the same depth.    At the very bottom, the horizontal pressure on the sides of the vessel is just as great as the vertical pressure on the bottom.
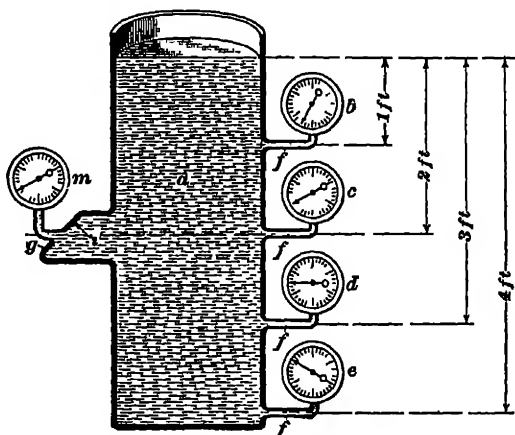


FIG. 3

**3.    Normal Pressure.**—If the pressure on a sloping surface, as at $g$, Fig. 3, is investigated, it will be found that the pressure indicated on the gauge $m$, situated at the center of the surface is exactly the same as the previously determined vertical and horizontal pressures existing at the same depth, corresponding in this case with that shown by the gauge $c$. The direction of a sloping surface does not in any way influence the magnitude of the pressure exerted against it by a liquid. Therefore, the pressure $p$ against any surface $g$, 1 square foot in extent, is equal to the product of $h$, the depth below the surface of the liquid in feet, and $w$, the weight of a cubic foot of the liquid.    Stated as a formula,

$$p = w\,h \qquad \textbf{(1)}$$

For water, $w = 62.5$, and therefore

$$p = 62.5\,h \qquad (2)$$

It is important to remember that the pressure of a liquid is always *normal*, that is, exerted in a direction perpendicular to the surface as indicated by the arrow $i$, Fig. 3, provided both the surface and the liquid are at rest    That the liquid is motionless is a tacit understanding und .rlying all the foregoing statements in the subject of liquid pressure.

**4.  Intensity of Pressure.**—In the preceding article there has been introduced a unit of measure very frequently employed in engineering computations, namely, the *unit*
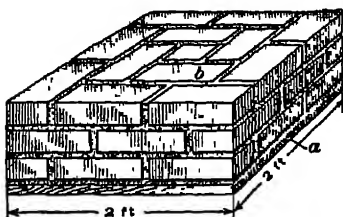


FIG. 4

*pressure*, expressed in pounds per square inch or in pounds per square foot.  In order to explain the nature of this unit of measure, Fig. 4 shows a platform $a$ 2 feet square and carrying a pile $b$ of material weighing 1,152 pounds.  Since each side of the platform is 2 feet long, the surface of the platform has an area of $2 \times 2 = 4$ square feet, and therefore each square foot of the platform carries one-fourth of the total weight, or $\dfrac{1,152}{4} = 288$ pounds.  In engineering language it is said that the load on the platform is 288 pounds per square foot.  Since 1 square foot contains 144 square inches, 1 square inch of the surface carries $\frac{1}{144}$ of 288, or 2 pounds, and the platform is said to carry a load of 2 pounds per square inch.  Thus, the pressure can be stated either as 2 pounds per square inch or as 288 pounds per square foot, since the two expressions are equivalent.

**5.**  Where, as in Fig  4, the material is piled to a uniform height over the entire area, the load is said to be *uniformly distributed*, because any two equal areas, no matter how small and no matter where taken, carry the same load    For example, in Fig. 4 an area of $\frac{1}{100}$ of a square inch taken at any point,

whether at the middle or near the edge, carries $\frac{1}{100} \times 2 = .02$ pound.

If, however, the load is piled in a heap, as in Fig. 5, a small area $a$ situated under the middle of the pile carries more weight than an equal area $b$ near the edge, as shown by the difference in the heights of the small prisms $a\ a'$ and $b\ b'$.  A load so arranged is said to be *unevenly distributed* or is referred to as a *non-uniform* load   When the load is unevenly distributed, the rate of pressure, or, as it is often called, the *intensity* of pressure, varies from point to point.   However, the pressure at a given point is also a unit pressure expressed in pounds per unit area, meaning the pressure that a unit area would have if loaded uniformly at the same rate as the very small area at the given point is loaded.
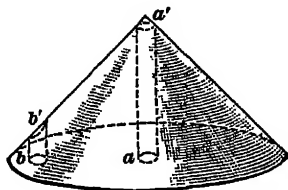
As an illustration, consider the



FIG 5

pressure gauges $b, c, d,$ and $e$ in Fig 3, each of these gives the pressure, in pounds per square foot, existing at the openings of the pipes $f$.   This, however, must not be understood to mean that there is a pressure of 62.5 pounds per square foot at a depth of 1 foot, and that the pressure then suddenly jumps to $62.5 \times 2$ $= 125$ pounds per square foot at a depth of 2 feet.   On the contrary, the pressure increases uniformly downward without any sudden changes   The small connecting tubes $f$ do not have to be 1 square foot or 1 square inch in cross-sectional area, since the pressure measured is merely pressure existing at the center of the tube, and means the pressure a square inch or square foot would have if the entire area had the same intensity as at the given point.   It must be noted, however, that in the case of an unevenly distributed load the expression pressure per unit area is often used to indicate the average unit pressure on a given area, which is the total pressure on this area divided by the area

EXAMPLE —Sand is piled on a platform in the form of a cone, as in Fig 5.   The diameter of the base is 10 feet and the height at the center is 3 feet.   If the weight of a cubic foot of sand is 90 pounds, find (*a*) the

total weight on the platform; (b) the intensity of pressure per square inch directly under the center of the pile; (c) the average pressure on a square inch of the platform.

SOLUTION.—(a) The area of the base is $A = .7854 \times 10^2 = 78.54$ sq. ft. The volume of the sand is $V = \frac{1}{3} \times 78.54 \times 3 = 78.54$ cu. ft. The weight of the sand is $W = 78\,54 \times 90 = 7,069$ lb. Ans.

(b) The height of the sand pile on a very small area at the center is practically 3 ft , and the volume of a small prism having an area of 1 sq. in. and a height of 3 ft. is $V = \frac{1}{144} \times 3 = \frac{1}{48}$ cu. ft. and the weight of this prism is $W = \frac{1}{48} \times 90 = 1.88$ lb. The intensity of pressure at this point is therefore 1.88 lb per sq. in. Ans.

(c) The total area of the base is $78.54 \times 144 = 11,310$ sq. in , nearly. The average weight on 1 sq. in. of the platform is $\frac{W}{A} = \frac{7,069}{11,310} = .63$ lb. The average pressure is then .63 lb. per sq. in. Ans.

**6.** Let $h$ be the height of the water in the tank shown in Fig. 6 (a). The pressure on the bottom $c$ is uniformly dis-



FIG. 6

tributed and its value on any square foot of area is $p_a = 62\,5\ h$. The pressure on the wall $b$ due to the water is zero at the point $e$ in the surface and increases uniformly to the value $p_g = 62\,5\ h$ at $g$, the intersection of $b$ and $c$. The variation of pressure at different points may be represented by the triangle $x\,y\,z$ in Fig 6 (b), where $x\,y$ is the total depth $h$ and the base $y\,z$, laid off to any convenient scale, is the unit pressure at $g$ or $p_g$ The length of any other line $m\,n$, parallel to $y\,z$ and measured to the same scale as $y\,z$, is the unit pressure at $f$ at depth $e\,f$ below the surface. The average unit pressure on the wall $b$ is equal to the average width of the triangle $x\,y\,z$, or one-half of $y\,z$. This average unit pressure
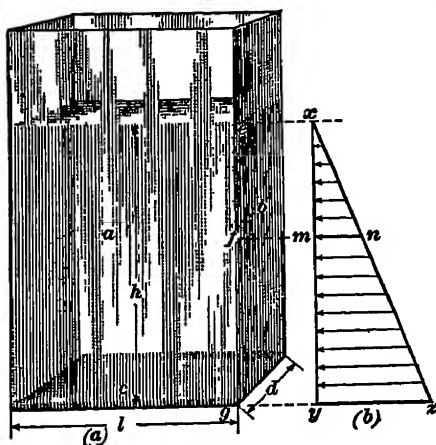
is at a point whose depth below the top of the water is one-half of $xy$, or $\dfrac{h}{2}$. Hence, the average unit pressure on the wall $b$ is equal to the unit pressure at its mid-point or $p_b = 62.5 \times \dfrac{h}{2} = 31\ 25\ h$. The area of wall $b$ is equal to $h \times d$ and the total pressure on $b$ is $P_b = 31\ 25\ d\ h^2$. Similarly the average unit pressure on wall $a$ is at its mid-point and its value $p_a = 31\ 25\ h$. The total pressure on wall $a$ is then $P_a = 31\ 25\ l\ h^2$.

In Fig. 7, $a\,b\,c\,d$ is the cross-section of a dam which holds back water of depth $h$. The water presses against the surface $xy$ with an intensity which varies from zero at $x$ to a value of $62\ 5\ h$ at $y$. Just as in the case of a vertical surface, the variation of unit pressure can also here be represented by a triangle, as $x\,y\,z$, Fig. 7, and the unit pressure at any point $m$ can be represented by the line $m\,n$ parallel to $y\,z$ and measured to the same scale as $y\,z$. Here, however, the direction of the pressure



FIG. 7

is normal to the surface $xy$ instead of horizontal. The average normal pressure is equal to one-half the base of triangle $x\,y\,z$, or $p = \dfrac{yz}{2} = \dfrac{62\ 5\ h}{2} = 31\ 25\ h$. This average pressure is at the mid-point of surface $xy$. It must be remembered that although the direction of the pressure is normal to the back of the dam, the height $h$ to be used in calculating the unit pressure is always vertical and is not measured along $xy$.
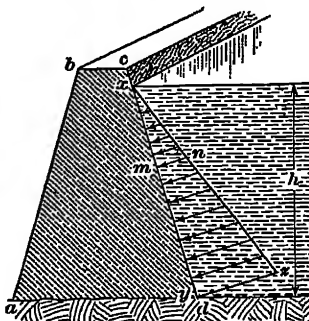
Let the length $xy$ be represented by $l$ and consider a section of the dam 1 foot in thickness. The area of the surface of pressure is then $l \times 1 = l$ square feet, and the total pressure normal to this surface, $P = 31.25\ h\ l$.

EXAMPLE 1.—The tank shown in Fig. 6 ($a$) is 10 feet long and 8 feet wide and the depth of the water is 12 feet. Find the total pressures on the bottom and walls.

SOLUTION.—The unit pressure on the bottom $c$ is $p_o = 62.5\,h = 62.5 \times 12$ $= 750$ lb. per sq. ft.

The average unit pressure on the surface $b$ is the same as that at its mid-point, which is at a distance of 6 ft. below the surface of the liquid. Hence, the average unit pressure on $b$ is $6 \times 62.5 = 375$ lb. per sq. ft.

The middle point of the surface $a$ is also 6 ft. below the surface of the liquid, hence, the average unit pressure on this surface is likewise 375 lb. per sq. ft.

The total pressure on the various surfaces may now be found:

> For surface $a$, $10 \times 12 \times 375 = 45,000$ lb.
> For surface $b$, $\;\;8 \times 12 \times 375 = 36,000$ lb.
> For surface $c$, $10 \times \;\;8 \times 750 = 60,000$ lb.   Ans.

EXAMPLE 2.—In Fig. 7, $h = 30$ feet and the back of the dam is sloped 1 foot horizontal in 6 feet vertical. Find the total normal pressure on 1 foot of the dam.

SOLUTION.—The average unit pressure on the dam is that at the mid-point of the surface $x\,y$, or $p = 31.25\,h = 31.25 \times 30 = 937.5$ lb. per sq. ft.

The vertical projection of length $x\,y$ is 30 ft. and the horizontal projection is $30 \times \frac{1}{6} = 5$ ft.   Length $x\,y = \sqrt{30^2 + 5^2} = 30.4$ ft.

The total normal pressure $P = 937.5 \times 1 \times 30.4 = 28,500$ lb.   Ans.

**7. Effect of Form and Size of Vessel.**—It has been stated that the intensity of pressure at any point in a liquid depends upon the depth of the point below the surface and the weight of a cubic foot of the liquid. Thus, the pressure at a point, 10 feet below the surface of the water, is the same whether the point is situated in a pipe 1 inch in diameter, in
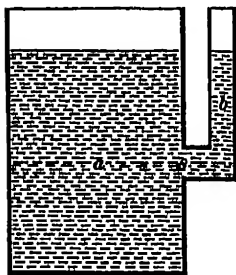
a well 5 feet in diameter, or in a lake 1 mile in width. In the Atlantic Ocean, some 3,000 miles in width, the pressure at a depth of 10 feet would also be the same but for the fact that salt water has a slightly higher specific gravity than fresh water.

Fig. 8 shows a large open vessel $a$ with a smaller tube $b$ projecting from it. It is a known fact that a liquid will rise to the same height in both the tube and the vessel. Since any unit of section $s$ at the mouth of the tube is in equilibrium, the unit pressure due to the weight of the small column in $b$ must be equal to that from the large volume above $s$ in $a$.

Therefore, the size of the vessel, as regards width and breadth, has no influence upon the unit pressure; nor is the pressure affected by the form of the vessel.

8.    **Center of Pressure.**—The total pressure on any surface loaded with a distributed load is the resultant of innumerable small forces each acting on a very small area    The point of application of this resultant, or its intersection with the loaded surface, is called the *center of pressure*.   This point should not be confused with the center of gravity of the loaded surface, as the center of gravity is a fixed point depending only upon the shape of the surface, whereas the location of the center of pressure depends upon the distribution



FIG 9

of the load and coincides with the center of gravity only when the load is uniformly distributed.
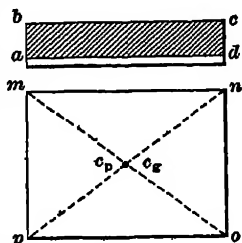
To illustrate the principles of the foregoing paragraph, two typical cases of loading will be considered    In Fig. 9, *a b c d* represents a cross-section of a uniformly distributed load over the rectangular area *m n o p*.   In this case the center of pressure $c_p$ coincides with the center of gravity $c_g$ of the rectangle, which is at the intersection of the diagonals *m o* and *p n*
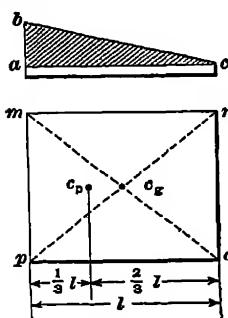


FIG 10

In Fig 10, *a b c* represents a cross-section of a triangular load distributed over the same area *m n o p* in such a manner that the intensity increases uniformly from zero at *c* to a pressure represented by the line *a b*.   In this case the center of pressure $c_p$ is at the center of gravity of the triangle or at a distance $\frac{1}{3} l$ from *m p*, or $\frac{1}{6} l$ from $c_g$.

As a further illustration of these principles reference is made to Fig. 6. Since the bottom of the vessel is loaded uniformly, the center of pressure of *c* coincides with its center of gravity. The portion *b*, however, is a rectangle whose upper edge

is at the surface of the water, and the values of pressures at different points are represented by the angle $x\,y\,z$ of Fig. 6 (b). The center of pressure of are is midway between the vertical edges at the distance $\frac{12}{3}$ inches from the base, and the total pressure on the sid may be considered as acting at this point. Similarly, center of pressure for side $a$ is 4 inches from the bottom the vessel. Also, in the case of the dam shown in Fig. 7, center of pressure is at a distance $\frac{1}{3}x\,y$ from the bottom $\frac{2}{3}x\,y$ from the surface. If $x\,y=30\ 4$ feet, as in example 2 Art. 6, the center of pressure is at a distance $\frac{2}{3}\times30.4=20\ 3$ below the surface.

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### EXAMPLES FOR PRACTICE

1. What is the pressure on a square yard of surface at the bottom tank which contains water to a height of 24 feet?    Ans. 13,50

2. A pressure gauge which reads in pounds per square inch is pl. $4\frac{1}{2}$ feet below the surface of a body of water. What reading will it rec
Ans. 1.95, ne

3. In the tank shown in Fig. 6, $l=12$ feet, $d=9$ feet, and $h=16$ Find the total pressure (a) on the bottom; (b) on wall $a$; (c) on wall $b$
Ans. $\begin{cases} (a) & 108,00( \\ (b) & 96,000 \\ (c) & 72,000 \end{cases}$

4. The height of the water behind the dam shown in Fig. 7 is 40 and the back of the dam slopes 1 foot horizontal in 8 feet vert (a) Considering a section 1 foot wide, find the total normal pressure $l$ the back of the dam. (b) At what distance below the water surl measured along the back of the dam, will the pressure $P$ be applied?
Ans. $\begin{cases} (a) & 50,400 \text{ lb., ne} \\ (b) & 26.9 \text{ ft.} \end{cases}$

---

### CAPILLARITY

**9. Cohesion and Adhesion.**—Forces of attraction e not only between molecules of the same kind, but also betw the molecules of unlike substances, as can be exemplified the adherence of cement mortar to stone or brick, or glu wood.

The attractive forces between molecules of the same kind are called **cohesive forces,** while those between molecules of unlike substances are referred to as **adhesive forces.**

**10. Capillary Phenomena.**—An interesting phenomenon occurs when molecules of a liquid and of a solid come in contact under certain conditions. When a glass plate $a$ is partly immersed in water, the surface of the water will take the form shown in Fig. 11 $(a)$. When the glass plate $a$ is immersed in mercury instead of in water, or when, instead of



(a)

$a$

(b)
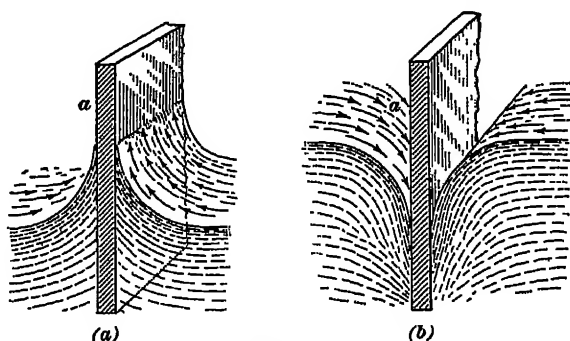
FIG. 11

the glass plate, a paper sheet covered with paraffin is immersed in water, the surface of the liquid near the solid will assume the shape shown in $(b)$. In the case shown in $(a)$ the adhesive forces between the molecules of water and of glass are greater than the cohesive forces between the molecules of water; for this reason the water near the glass rises above the level in the container. In the case shown in $(b)$, the attraction of cohesion exerted upon the particles in the surface of the liquid by the surrounding liquid is greater than the adhesive force tending to hold the liquid against the solid.

**11. Capillary Tubes.**—A tube of very small diameter is called a **capillary tube.** If instead of a plate a capillary tube is immersed in a large vessel of water, the adhesive force of water to glass will cause the water to rise in the tube against the force of gravity. The difference in level between the water inside the tube and that outside will depend on the diameter

of the tube.    In tubes of small diameter the water will rise to
greater heights above the outside level than in larger tubes,
as shown in Fig. 12 (*a*).    The pores of a sponge or of blotting
paper act just like small tubes and absorb a liquid by reason
of their capillary action.

If the glass tube is replaced by one made of paper covered
with paraffin wax, the conditions will correspond to those rep-
resented in Fig. 12
(*b*) and the water
within the tube will
occupy a level lower
than that of the sur-
rounding water    The
difference between the
two cases is some-
times explained by
saying that water
moistens (adheres to)
glass, but does not
moisten paraffin.
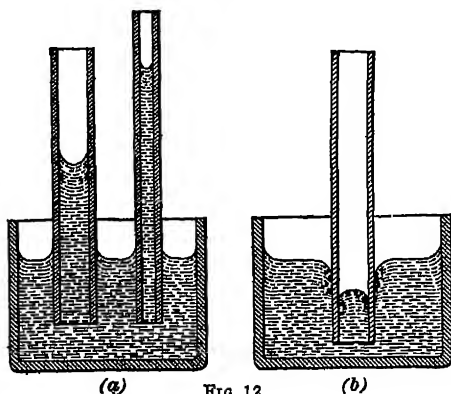


(*a*)      Fig. 12      (*b*)

**12.    Capillarity of Masonry.**—Capillary attraction is of
particular importance to the construction engineer because
any water in contact with masonry foundations will ascend
in the minute pores of the masonry.    Most masonry walls,
whether of stone, brick, or concrete, are somewhat porous,
and how to prevent water from rising into the building on
account of capillary attraction is a serious problem    Several
methods are used to exclude the water; one of these depends
on the principle that capillary tubes of paraffin wax and similar
substances actually expel water by their peculiar capillary
action.    The substance intended as a waterproofing agent is
mixed, either as a fine powder or as a liquid, with the mortar
to be waterproofed and, when the mortar subsequently hard-
ens, there is formed a very thin film of water-repellent mate-
rial on the walls of the capillary pores of the mortar of the
masonry.    As long as this film remains intact, it effectively
prevents water from entering the joints of the masonry.

## PRESSURE OF GASES

**13. Atmospheric Pressure.**—An examination of the nature of gases will show their great similarity to very light or thin liquids. Thus, the atmosphere is a gas that envelopes the earth, and man may therefore be supposed to live on the bottom of a vast ocean of gas many miles deep. Just as the objects at the bottom of the ocean are subjected to a pressure due to the weight of the water above them, all objects on the surface of the earth are under a pressure due to the weight of the air above them. This pressure is known as atmospheric pressure.

The atmospheric pressure decreases from the surface of the earth upward, because the height of overlying air decreases; it is less on the mountain top than in the valley, and greatest at the bottom of the deepest mine. Since the atmospheric pressure is dependent upon the altitude, it is necessary to fix one particular pressure as the standard by which all other pressures should be measured; this standard pressure is that existing at the level of the sea. Even this pressure varies somewhat from day to day and from hour to hour, but its average value is taken as 14.7 pounds per square inch. The manner in which this value may be determined will be explained in a subsequent article. Every square inch of the human body (or any other object upon



FIG. 13

the earth) is subjected to atmospheric pressure; that it is not felt is due to the fact that the same pressure exists both inside and outside of bodies open to the air.

To demonstrate that the atmosphere exerts a pressure, the apparatus of Fig. 13 may be used. As the air is exhausted through a tube *a* from a stout cylinder *b* covered with a bladder *c*, tied on so as to be practically air-tight, the pressure of the atmosphere will depress the bladder until finally it bursts with a loud report. To measure the magnitude of the atmospheric pressure a *barometer*, to be described later, is used.
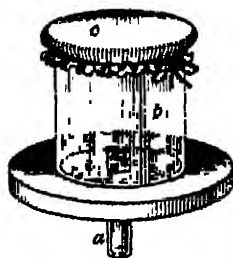
**14. Vacuum.**—Fig. 14 (*a*) shows a closed vessel filled with a liquid, such as water. If one-half of the volume of water is removed, the remaining liquid will fill only one-half of the vessel, as at (*b*); but if this experiment is repeated, as at (*c*), with a visible gas such as smoke, it will be found that if half of the gas is removed, the remaining half will continue to fill the entire vessel, as at (*d*). The density of the gas is, however, only half of what it was before; that is, the mean distance between the molecules is now twice as great   One important difference between liquids and gases is, therefore, that a gas will expand to fill spaces to which it has access, while a liquid will not so expand.

. By means of an instrument called an *air pump* it is possible to remove gradually the air from a closed vessel. While the air is being removed a pressure gauge attached to the vessel will show a gradually decreasing pressure. When the air is entirely removed the pressure gauge will indicate zero; the space within the vessel is then unoccupied by matter, and there is



(*a*)    (*b*)

(*c*)    (*d*)
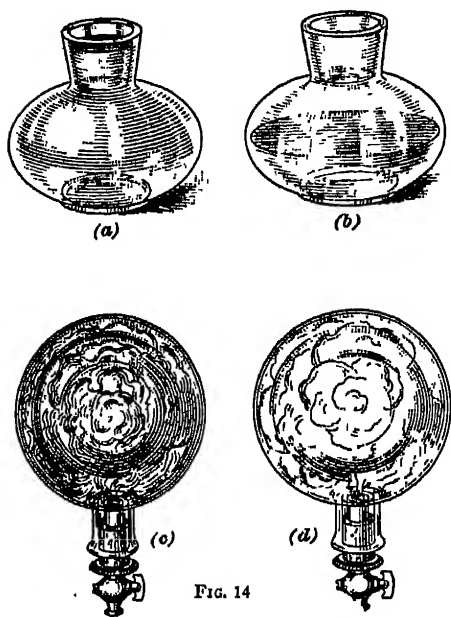
Fig. 14

said to be a *vacuum* within the vessel. When the air is not entirely removed there is said to be a *partial vacuum* within the vessel   In practice, it is not possible to create a perfect vacuum in the sense that there is absolutely nothing within the vessel, although it is possible to approach more or less closely to this state.

**15.  Mercury Barometer.**—The glass tube $a$, Fig. 15 $(a)$ is about 1 yard long and is closed at its lower end $b$.  The tube is completely filled with mercury, and while the end $c$ is temporarily closed, the tube is inverted and the end $c$ is entirely submerged in the cup $d$, which is partly filled with mercury.  If then the obstruction at $c$ is removed, the upper surface of the mercury in the tube sinks and after a few oscillations comes to rest at a level $e$ about 30 inches above the surface of the mercury in the cup $d$.  The apparatus thus made is a **barometer,** and is shown in Fig. 15 $(b)$.
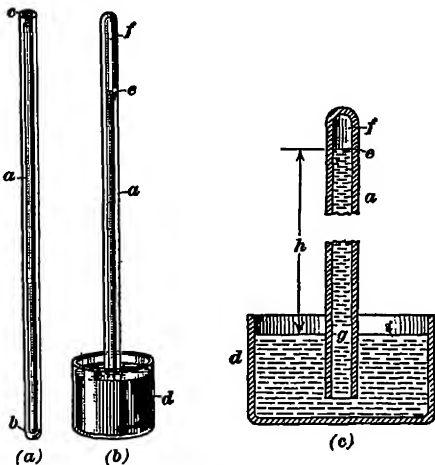


FIG. 15

Fig 15 $(c)$ shows an enlarged partial view of $(b)$ with the central portion of the tube broken away.  In the best barometers the space $f$ above the top of the mercury is for all practical purposes a perfect vacuum, and above the surface $e$ there is no pressure.  At a unit of surface $g$, level with the surface $i$ of the mercury in the cup $d$, there is a pressure equal to the weight of a column of mercury of unit area, having the height $h$ of about 30 inches if the barometer is at sea level   The unit pressure on the surface $g$ must be equal to the unit pressure on the surface $i$, because if these pressures are not equal, there would be a motion toward the surface having the smaller pressure   The pressure on the surface $i$ is the atmospheric pressure, and thus the atmospheric pressure on any area is equal to the weight of a column of mercury of equal area about 30 inches high.  Assuming that the area of the tube is 1 square inch, this column contains $1 \times 30 = 30$ cubic inches of mercury, or $\dfrac{30}{12 \times 12 \times 12}$ cubic feet.  If this column con-

sisted of water, it would weigh $\dfrac{30}{12\times12\times12}\times62.5$ pounds, but, since the liquid is mercury, having a specific gravity of 13.6, the weight is $\dfrac{30}{12\times12\times12}\times62\ 5\times13.6=14\ 7$ pounds    It follows that the pressure of the atmosphere per square inch is 14.7 pounds

**16.    Aneroid Barometer.**—If the experiment illustrated in Fig. 15 is repeated on top of a mountain, the height of the mercury column will be less than 30 inches because the atmospheric pressure is less at the higher elevation, as already stated.    Thus, at a point 5,000 feet above the sea, the height of the mercury column is about 24.7 inches; at 10,000 feet it is 20 5 inches; at 15,000 feet it is 16 9 inches; at 3 miles it is 16.4 inches, and at 6 miles it is 8.9 inches.    Therefore, if it is known how much lower the barometer stands on a certain mountain top than in the valley adjoining, it is possible to compute the difference in elevation between the two stations Explorers and engineers working in unknown, rough country often employ this method to determine the approximate elevation, but the barometer used is a different and more convenient type known as an *aneroid barometer*.    In this type a cylindrical box is provided with a thin, elastic, metallic cover. The air is exhausted from the box, and the cover thereby becomes very sensitive to changes in the atmospheric pressure, bending inwardly to an extent depending on the pressure.    This minute movement, many times multiplied by a suitable lever system, is indicated by a pointer.    The external shape of the apparatus is not unlike that of the ordinary alarm clock, except that it has but one hand, and the face has two scales, one of which is graduated to show inches of barometric pressure and the other to indicate the height above sea level, in feet.

**17.    Water Barometer.**—Any liquid may be used instead of mercury to counterbalance the atmospheric pressure, but, since all others are lighter than mercury, a greater height of column is required.    Thus, since water is 13.6 times lighter

than mercury, it requires a water column $13.6 \times 30 = 408$ inches $= \dfrac{408}{12} = 34$ feet high to balance the atmospheric pressure at sea level.   Therefore, in order to construct a water barometer, the tube would have to contain a column of water 34 feet high, and such an instrument is impractical.

**18.  Compressibility of Gases.**—It has already been stated that all materials are compressed when a load is placed upon them.   There is, however, a great difference in the amount of compressibility between different materials; this difference is especially striking when solids and liquids, taken
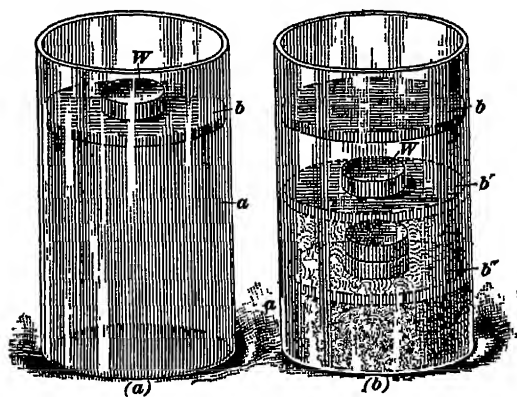


Fig. 16

as one group, are compared with another group comprising the gases.   The reduction in length, or volume, under pressure is relatively insignificant in solids and liquids, whereas it is very great in gases.

In Fig. 16 (a), the vessel a contains water, and a metal plate b which rests on the water fits inside the vessel tightly so that no water can escape between the edges of the plate and the walls of the vessel.   It will be found that even if a great weight W is placed on top of the plate, it will be impossible to lower the plate any appreciable amount.   Now imagine that the vessel contains a colored gas such as smoke, and the plate rests at a certain level as shown by the full lines in

Fig. 16 (b).  A relatively small weight $W$ placed on top of the plate will lower it in the vessel a certain noticeable distance, and the plate will take the position $b'$.  It will also be noticed that the smoke becomes more dense.  If the weight placed on the plate is increased, the plate will sink farther; when the weight is twice its first value, the plate will be in position $b''$ and the total movement from $b$ to $b''$ will be twice that from $b$ to $b'$; a weight three times as great will move the plate three times as far, etc.  In general, it can be stated that the distance moved by the plate will be proportional to the weight placed on it.  To show that the same quantity of smoke is still under the plate, on removal of the weight the plate will return to its original position.  The conclusion can therefore be drawn that gases are very compressible and that they have the power of expanding to their original volume.  This fact is utilized in many ways, compressed air and steam being frequently employed for driving hammers, drills, locomotives, and other appliances used in construction work.
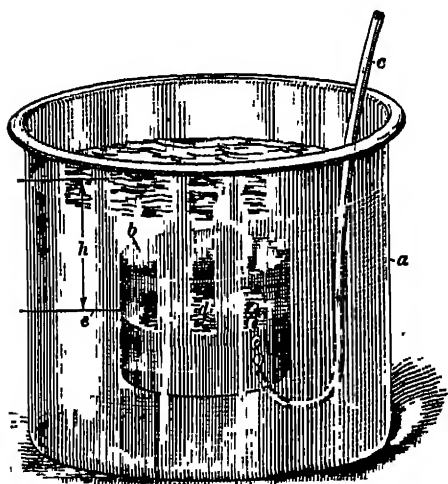


Fig. 17

**19.  Diving Bell.**—Compressed air is of special value for work in deep water, such as in the construction of bridge foundations in rivers, or in tunneling in very wet ground, where it is utilized to drive out the water.  In practice the details of application vary somewhat, but the principle is well illustrated by the so-called diving bell, which is a huge airtight box, open at the bottom, and into which compressed air is pumped through a pipe.  Fig. 17 shows the essential parts

of this apparatus on a scale so small that it can easily be duplicated; *a* is a vessel containing water and representing a river, lake, or ocean in which the work is to be performed; *b* is a cylinder, open at its lower end, representing the diving bell; and *c* is a tube through which compressed air is blown into the bell.

**20.**  If the bell *b*, Fig. 17, is full of water, a current of compressed air introduced through the tube *c* will lower the level of the water within it.  When the water in the bell stands at the level *d*, the pressure per square inch on the air within the bell is equal to the pressure per unit area in the vessel *a* at a corresponding level, such as at the point *e*, situated at the same depth, *h*   The unit pressure at *e* is the sum of two pressures; namely, the weight of the water column of unit cross-sectional area situated above the point *e* and the super-imposed atmospheric pressure.  It follows that air at atmospheric pressure cannot flow through the tube *c* into the bell, but that this pressure must be increased by an amount equal to that exerted by a column of water of unit section and of a height *h*.

**21.**  In the experimental tank the water pressure is, necessarily, small, but in actual construction work great pressures are encountered.  Thus, at a depth of 36 feet below the surface of water, the pressure due to the water is $36 \times 62\ 5$ pounds per square foot, or $\dfrac{36 \times 62.5}{144} = 15.6$ pounds per square inch, and the total pressure is $15.6 + 14\ 7 = 30.3$ pounds per square inch.  Since the pressure increases with the depth, there must be a depth beyond which it is impossible for a diver to work, because of injurious effects on the human body   Ordinarily, the pressures met with at less than 60 feet cause no particular difficulties, but it requires a man of strength and experience to work at a depth of 100 feet

Considerable quantities of air being required for filling the bell, as well as for supplying fresh air for breathing purposes, it is necessary to have a machine for compressing the air, as well as a reservoir with compressed air ready for instant

use.   This reservoir is usually a cylindrical, riveted steel drum into which the air is pumped by a machine known as an *air compressor* and acting on the same principle as a pump.

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### EXAMPLES FOR PRACTICE

1. Find the pressure, in pounds per square inch, corresponding to 24 inches of mercury.                                    Ans. 11.8

2. If the height of a column of mercury in a barometer is 29 inches, what will be the maximum height of a column of water which can be supported by the atmospheric pressure?                    Ans. 32.9 ft.

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## PUMPS

**22.**  The difference in the behavior of liquids and gases when subjected to changes in pressure is utilized in raising water or other liquids to a higher level by means of *pumps*. Three common types are the *suction pump*, the *force pump*, and the *centrifugal pump*.   A brief description of the operation of each follows.

**23.  The Suction Pump.**—A section of an ordinary suction pump is shown in Fig. 18.   Suppose the piston to be at the bottom of the cylinder and to be just on the point of moving upwards in the direction of the arrow.   As the piston rises, it leaves a vacuum behind it.   The air in $P$ then raises the valve $V$, and expands in the cylinder $B$, whereby its pressure is diminished below that of the atmosphere.   The atmospheric pressure on the surface of the water in the well causes the water to rise in the pipe $P$.   When the piston descends, the valve $V$ closes, and the air in $B$ escapes through the valves $u$.   After a few strokes, the water fills completely the space under the piston in cylinder $B$, so that, when the piston reaches the end of its stroke, the water entirely fills the space between the bottom of the piston and the bottom of the cylinder and also the pipe $P$.   The instant that the piston begins its down stroke, the water in the chamber $B$ tends to fall back into the well, and its weight forces the valve $V$ to its seat, thus preventing any downward flow of

the water.   The piston now tends to compress the water in the chamber *B*, but this is prevented through the opening of the valves *u* in the piston    When the piston has reached the end of its downward stroke, the weight of the water above closes the valves *u*.   On the upward stroke of the piston all the water resting on top of it is lifted and discharged through the spout *A*, the valve *V* again opening, and the water filling the space below the piston as before.

It is evident that the distance between the valve *V* and the surface of the water in the well must not exceed 34 feet, the highest column of water that the pressure of the atmosphere will sustain, since otherwise the water in the pipe would not reach to the height of the valve *V*. In practice, this dis-



Fig. 18

tance should not exceed 28 feet    This is due to the fact that there is a little air left between the bottom of the piston and the bottom of the cylinder, a little air leaks through the valves, which are not perfectly air-tight, and a pressure is needed to raise the valve against its weight, which, of course, acts downwards    There are many varieties of the suction pump, differing principally in the valves and piston, but the principle is the same in all.
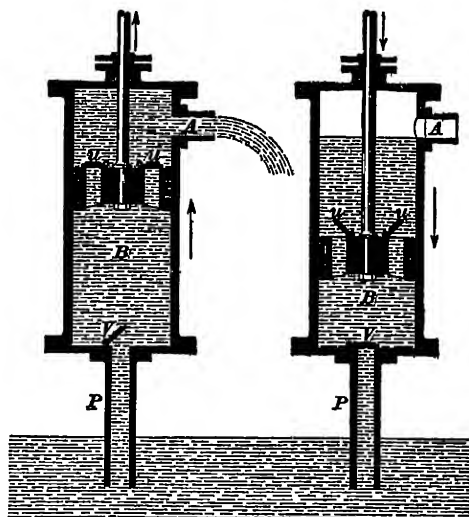
**24.  Force Pump.**—The force pump differs from the suction pump in several important particulars, but chiefly in the fact that the piston is solid; that is, it has no valves    A section of a *suction and force pump* is shown in Fig. 19.   The

water is drawn up the suction pipe, as before, when the piston rises; but when the piston reverses, the pressure on the water caused by the descent of the piston closes the valve $V$, opens the valve $V'$, and forces the water up the delivery pipe $P'$. When the piston again begins its upward movement, the valve $V'$ is closed by the pressure of the water above it, and the valve $V$ is opened by the pressure of the atmosphere on the water below it, as in the previous case. The force pump can raise water to much greater heights than are possible with the suction pump.
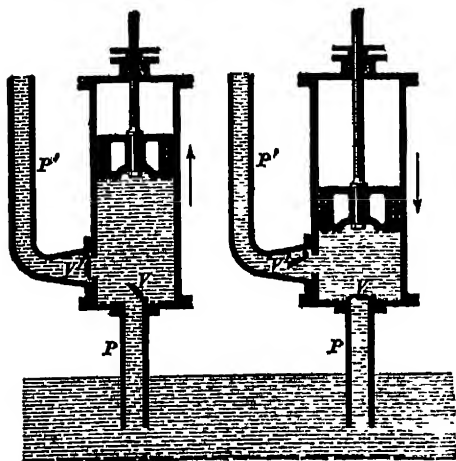


Fig. 19

**25. Centrifugal Pump.**—The centrifugal pump is a very valuable instrument for raising water to great heights. Fig. 20 represents a centrifugal pump with half of the casing removed. The hub $e$ is hollow, and is connected directly to the suction pipe $a$. The curved arms $v$, called *vanes* or *wings*, are revolved with a high speed in the direction of the arrow, and the air enclosed between them is driven out through the discharge passage $d$ and delivery pipe $b$. This creates a partial vacuum in the casing and suction pipe, and causes the water to flow in through $c$. This
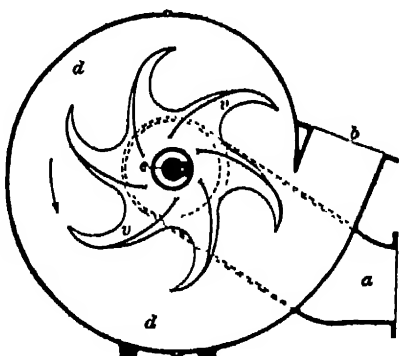


Fig. 20

water is also made to revolve with the vanes and, of course, with the same speed. Due to this speed of rotation, the water is forced outwards toward the ends of the vanes and finally leaves the pump by means of the discharge passage and delivery pipe.

Centrifugal pumps have no valves or restricted passages to hinder the flow of a liquid through them, and are therefore serviceable for pumping water containing large quantities of mud, sand, gravel, or anything that can be carried through the pump and pipes by a current of water.

## EFFECTS OF ENERGY ON MATTER

### ENERGY AND WORK

#### DEFINITIONS

**26. Work and Unit of Work.**—A body at rest will not move of its own accord or without an impulse from outside, and the influence that causes a body at rest to move is called a *force*. If, while under the influence of a force, a body moves through a certain distance, the force is said to have performed the *work* of moving the body. For example, it requires work to lift a bag of cement from a wagon up to a platform. This work may be done by a laborer carrying the bag on his back, or by a horse pulling on a rope, or by a hoisting engine driven by steam or electricity, and thus work may be done in many different ways.

In referring to the *amount* of work done, it makes no difference whether it is performed by men, horses, or mechanical means, in any event the work performed is the lifting of a weight through a certain distance. The amount of work done depends upon only two factors, in this case the weight of the bag of cement and the vertical distance through which it is lifted  The actual amount of work performed is measured by the product of the weight of the bag, in pounds, and the distance, in feet, through which it is lifted.

The amount of work performed is therefore defined by the product of *force times distance*.    Expressed as a formula,

$$W = F\,s$$

where
$\quad\quad\quad W = \text{work, in foot-pounds;}$
$\quad\quad\quad F = \text{force, in pounds;}$
$\quad\quad\quad s = \text{distance, in feet}$

The **foot-pound** is the unit used in measuring work and represents the amount of work required to lift a weight of 1 pound through a vertical distance of 1 foot.   In general, when any force of 1 pound acts through a distance of 1 foot in any direction, the amount of work done is 1 foot-pound. If a bag of cement weighing 94 pounds is lifted to a height of 10 feet, the force exerted is 94 pounds and the distance through which it acts is 10 feet   Hence, the work done is $W = F\,s = 94 \times 10 = 940$ foot-pounds.

**27.   Power and Unit of Power.**—The *amount* of work performed is of less interest to the engineer than the *time* required for performing the work.   For example, 40 bags of cement, each weighing 94 pounds, may be lifted by a motor in 10 minutes to a platform 12 feet above the ground. The total amount of work done is $94 \times 12 \times 40 = 45,120$ foot-pounds.   Supposing that it should require 1 hour for men to hoist the bags, the amount of work performed is the same in both cases, but the *rate* at which it is performed, that is, the work performed in a given unit of time, is different.

The rate at which work can be performed by a man, machine, or other source is called the **power** of the particular source of work.   Power is measured by the amount of work done in a given time.   In the example of the preceding paragraph, the motor does 45,120 foot-pounds of work in 10 minutes or $\dfrac{45,120}{10} = 4,512$ foot-pounds in 1 minute.   Thus, the power of the motor is 4,512 foot-pounds per minute.   The power of the men is $\dfrac{45,120}{60} = 752$ foot-pounds per minute.

If $P$ represents the power of the source of work, and $W$ is the total amount of work done in time $t$,

$$P = \frac{W}{t}$$

The unit of power used in the United States and England is the *horsepower*, abbreviated H. P., which is equal to 33,000 foot-pounds per minute, or 550 foot-pounds per second, since $33,000 \div 60 = 550$    If 1 horsepower is available for doing work, it is possible to lift 33,000 pounds through a height of 1 foot in 1 minute or 1,000 pounds through 33 feet in the same time, 1 minute, or any other weight through such a height in 1 minute that the product of weight and height is equal to 33,000.    To find the power of a machine, the following rule may be applied ·

**Rule.—I.**    *Divide the work, in foot-pounds, performed by the machine, by the number of seconds required to do the work, and the quotient represents the power, in foot-pounds per second.*

**II.**    *Divide the power thus obtained by 550; the result is the number of horsepower developed by the machine.*

EXAMPLE 1.—A crane lifts 16,000 pounds to a height of 32 feet in 8 seconds.    What is the horsepower developed?

SOLUTION.—Applying the formula, $W = F s$, the total work performed is $16,000 \times 32 = 512,000$ ft -lb.    The power $P = \dfrac{W}{t} = \dfrac{512,000}{8} = 64,000$ ft.-lb. per sec    Then $64,000 \div 550 = 116$ H. P. developed by the crane.    Ans.

EXAMPLE 2.—How many foot-pounds of work can be done in 10 minutes by a machine which develops 5 H. P.?

SOLUTION.—    1 H. P. is 33,000 ft -lb. of work done in 1 min.    Therefore, 5 H  P. $= 5 \times 33,000 = 165,000$ ft.-lb. in 1 min.    In 10 min., the work done is $10 \times 165,000 = 1,650,000$ ft.-lb.    Ans.

**28.    Energy.**—*The energy of a body is defined as its capacity for doing work.*    In a body possessing energy the molecules have the property which gives the body the ability to exert a force.    When a body moves under the action of a force, it performs work    Energy is therefore the ability to perform work.

**29. Kinds of Energy.**—Suppose that a body weighing 500 pounds is raised through a height of 5 feet    In thus raising the body the work performed is $500 \times 5 = 2,500$ foot-pounds, independent of the source that supplied the energy. If the weight is allowed to descend, it is now the acting force, and in moving through the same distance it does work equal in amount to that expended in raising it.    The body in a high position has stored energy and can do work equal to its weight multiplied by the distance through which it is lowered.

If a spring is extended or compressed, work is done equal to the amount  of the force applied multiplied by the change in length of the spring.    When released the spring will return to its original length and in so doing can perform the same amount of work that was spent on it.    Thus, in the extended or compressed state there is a certain amount of energy stored in the spring which it possesses because of its stretched or compressed condition.    Similarly compressed air possesses energy merely because it is compressed, and can do work in returning to its original state.

The energy which a body possesses by reason of its position, state, or condition is called **potential energy.**    Other examples of potential energy are the heat stored in a steam boiler, the chemical energy stored in coal, and the electrical energy stored in a storage battery.

An example of energy in a different form is that possessed by a body in motion.    For example, a moving car possesses energy because of its motion; this energy shows itself when the car collides with bodies at rest and causes them to move, thus performing work.    The energy which a body possesses because of its state of motion is called **kinetic energy.**

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**EXAMPLES FOR PRACTICE**

1.    A load of 3 tons is raised to a height of 6 feet.    How many foot-pounds of work are done?                                    Ans. 36,000 ft.-lb.

2.    An elevator carrying a load of 1,650 pounds rises 200 feet in 1 minute.    What horsepower is thus developed?                    Ans. 10 H. P.

### TRANSFORMATION OF ENERGY

**30.   Conservation of Energy.**—The law of conservation of energy asserts that energy cannot be destroyed. When energy apparently disappears, it is found that an equal amount appears somewhere, although perhaps in another form. Frequently, potential energy changes to kinetic energy, or vice versa. To illustrate, take the case of a steam hammer or a pile driver. The ram is raised to a certain height above the pile and on being released strikes the head of the pile   In its highest position the ram had the potential energy equal to its weight multiplied by its distance above the pile. When it reaches the pile just before striking, the ram has lost its potential energy due to position, but during the motion it has gained kinetic energy equal in amount to the potential energy which it had before starting to fall   After the blow the ram comes to rest and has neither potential nor kinetic energy, but the kinetic energy has for the most part been expended in doing the work of driving the pile a certain distance into the earth. A small part has been expended in heating the ram and head of the pile and another part in producing sound

**31.   Heat** is a form of energy due to the rapid vibration of the molecules of bodies. In many cases where energy apparently disappears, it reappears in the form of heat   Thus, the work done against frictional resistance appears as heat, which is usually dissipated into the air as fast as produced.

Conversely, there are examples of heat energy transformed into other forms   A pound of coal has potential energy, which, when the coal is burned, is liberated and changed into heat energy. The heat applied to water produces steam, and the steam has potential energy. Finally, the steam in giving up its energy does work in a steam engine, and this work is expended in overcoming the friction of shafts, belts, and machine parts, and ultimately reappears in the form of heat   However, in all these transformations, the total amount of energy—the sum of the kinetic and potential energy—remains always the same. Such is the law of the conservation of energy.

**32. Efficiency.**—Although energy that is spent in doing any kind of work is never lost in nature, from the human point of view a distinction is made between useful work and wasted work. For example, a load $W$, Fig 21, is lifted by means of the force $F$ applied to the rope $R$ passing over the pulley $P$. The useful work done is equal to the weight $W$ multiplied by the distance $h$. But actually more work must be done by the force $F$ to raise the weight because part of the total work is spent in overcoming the friction between the rope and the pulley. The energy spent in overcoming the friction is not absolutely lost because it is turned into heat, but it is not available for the purpose of lifting the load.
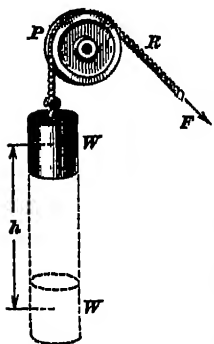


Fig. 21

In designing machinery, effort is therefore made to have the useful work as large a percentage of the total work expended as possible. The ratio of the useful work done to the total work spent by a machine is called the efficiency of the machine. Thus, if a motor utilizes 80 per cent. of the total energy spent, it is said to have an efficiency of 80 per cent.

**33.** The steam engine is considered an efficient machine because it utilizes a large proportion of the energy in the steam; an efficiency of from 75 to 92 per cent. is often reached. The steam boiler, on the other hand, is very wasteful, since only about 10 per cent. of the heat developed by the burning of the coal on the grate becomes available in the form of steam under pressure. The combined efficiency of a boiler having an individual efficiency of 10 per cent. and an engine having an individual efficiency of 80 per cent. is, theoretically, $\frac{80}{100} \times \frac{10}{100} = \frac{8}{100}$, or 8 per cent. In reality, it is usually much less on account of loss of heat from pipes and other surfaces, as well as losses by friction and otherwise.

What is here stated of the steam engine is generally true of any mechanical contrivance; a relatively large proportion of the original energy is lost at every step, and the amount of energy available at the working end of the machine is often only a small fraction of the energy originally available   Thus, if in a concrete-products plant it is desired to install four machines each requiring $1\frac{1}{2}$ horsepower for its operation, a total of $4 \times 1\frac{1}{2} = 6$ horsepower will obviously be consumed if the four machines are operated simultaneously.   It would, however, not do to install a 6-horsepower motor to furnish the power, because a great deal of energy is diverted on the way from motor to machine; in practice, a motor capable of developing 8 or 10 horsepower would probably be found necessary.

**34.  Suddenly Applied Loads.**—It is of great importance in structural work to take into consideration the kinetic energy of loads suddenly applied to any part of a structure   In general, if a beam is slowly and gradually loaded, it will be found that the corresponding deflection increases in direct proportion to the load.   Thus, if a certain beam deflects 1 inch under a load of 1 ton, it will deflect 2 inches under a load of 2 tons.   If, however, the load of 1 ton is applied suddenly, it is found that the deflection is now not 1 inch, but nearly 2 inches.   It is therefore to be seen that a sudden application of a load is much more dangerous than a gradual application.   In the engineering sciences a distinction is therefore made between the static effect of a load and the dynamic effect of a load.   The *static effect* is the effect produced by a stationary load and the *dynamic effect* is the effect produced by a quickly applied load   The dynamic effect is somewhat like that of a blow or shock; in engineering, a shock is often called *impact*.   Impact is dangerous to engineering structures because the structure must not only carry the static load but also absorb the kinetic energy of the impact.

Impact may be caused not only by a heavy load suddenly applied to a structure but also by a lighter load dropped from a height; the final kinetic energy may in this case be much

greater and will have to be counteracted by the resistance of the structure.

**35. Impact Usefully Employed.**—The fact that the kinetic energy in a moving body may be converted into other forms of energy by means of impact applied to a body at rest is utilized in various ways in engineering and in the industries. Examples are found in machines for riveting and punching, in pile drivers, steam hammers, ore-crushing machines, etc. Consider, for instance, the case of a hammer when used for driving a nail. The weight of the hammer head resting on the nail is unable to drive the nail into a block of wood. But, when the nail is struck by the moving hammer, the motion gives the hammer an amount of kinetic energy sufficient to drive the nail.

Small weights, moving quickly and striking with great frequency, are often used instead of large masses striking at relatively long intervals and with less speed. Examples of this kind are found in the pneumatic riveting machine and in the steam pile driver.

The kinetic energy expended in the impact of two bodies is not lost, but appears in other forms, such as setting another body in motion, as momentary deformation of material, as heat, and so forth.

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## HEAT

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### EXPANSION BY HEAT

**36. Heat a Form of Energy.**—It has been indicated in the preceding articles that heat is a form of energy. According to the modern conception of heat, when a force is applied to a body and heat results, the energy expended is transformed into energy within the body causing an increase in the speed of motion of the molecules and also a change in the relative distances of the molecules from one another

**37. Expansion and Contraction.**—It is assumed that by reason of the intensified molecular motion in a heated body,

whether solid, liquid, or gaseous, the distances between the molecules increase; therefore an increase in heat should expand a body, or increase its volume. Conversely, cooling the body should cause contraction, or a decrease in volume. Such is found to be the case with nearly all substances, the one notable exception being water under certain conditions; at temperatures near that of freezing, water contracts when heated, but it, too, follows the general rule of expansion and contraction for higher temperatures. This expansion or contraction is proportional to the change in temperature to which the body is subjected.

**38. Effects of Expansion and Contraction.**—That bodies expand when heated and contract when cooled is of great importance to the engineer, because the elongation or contraction of materials, such as steel and concrete, takes place with great force although the amount of the linear expansion is but a small fraction of the original length. Thus, a bar of steel 14 feet long may expand only $\frac{1}{16}$ inch if its temperature is raised a certain amount, but if, for example, the cross-section of the bar is 2 inches square, and the bar is fitted tightly between two supports, it can be shown that this increase in temperature will cause the bar to exert a pressure against the supports of about 60,000 pounds    For this reason, railroad rails are so laid that there is a small space between successive lengths of rail in order that the rails may expand freely in the summer, as they might otherwise buckle.

Concrete expands practically the same amount as steel when heated to the same extent, but, since concrete is weaker than steel, the effect on the concrete is even greater than the effect on steel. This is especially true when the concrete is cooled, since tension stresses will occur in the concrete during its shrinkage; concrete is comparatively weak in tension and, the shortening of concrete in cool weather is likely to cause it to crack. The concrete engineer will therefore always take great care to eliminate as far as possible the so-called *temperature stresses* in concrete work, by so arranging the details that the concrete can expand and contract freely. For this

reason long stretches of concrete pavement require so-called
*expansion joints*, which are open spaces filled with compres-
sible material, such as asphalt, and which permit the free
expansion and contraction of the concrete during changes
in the temperature.   This prevents the pavement
from buckling when expanding, or tearing apart
when contracting.
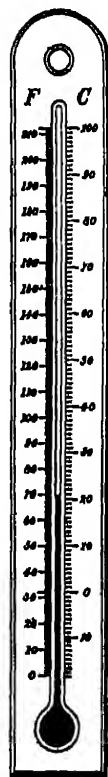
**39.  Mercury Thermometer.**—The fact that
liquids expand when heated and contract when
cooled is made use of in measuring temperatures
by means of instruments known as thermometers.
A thermometer consists of a glass tube of very
narrow bore, terminating at its lower end in a bulb
or enlargement, as shown in Fig. 22.   The bulb
and a portion of the tube are filled with any suit-
able liquid, the liquid metal *mercury* being most
commonly used for the purpose.   When the mer-
cury in the bulb is heated it expands, causing the
level of the mercury to rise.   When the tempera-
ture falls, the mercury level falls also, because of
the contraction of mercury when cooled.   The
position of the mercury level, and, therefore, the
temperature, is read on a scale printed upon the
mounting used for supporting the tube or on a
separate strip of metal attached to it.

**40.  Systems of Measuring Temperature.**
On the thermometer shown in Fig. 22 there are two
scales; the *Fahrenheit* scale, so named after its

FIG. 22

inventor, is marked *F* and the *centigrade* scale is
marked *C*, since these are the symbols commonly used to
indicate the respective scales.   The Fahrenheit system is
generally used in the United States and England; the centi-
grade system is used throughout the world in scientific work
and also by the general public in most European countries.

In both systems the unit of measurement is called the
degree, denoted by the sign °; thus, 5° F. is read five degrees
Fahrenheit and 15° C. is read fifteen degrees centigrade.   On

the centigrade scale the zero point was arbitrarily taken at the mercury level corresponding to the temperature of melting ice; that is, the temperature of melting ice was called 0° C. The mercury level at the temperature of boiling water was called 100° C. By dividing the interval between these two points into 100 equal parts, the distance corresponding to 1° was obtained. The scale was then graduated at convenient intermediate points and the divisions were also extended above 100° and below 0°. Temperatures below zero increase downwards and are written with a minus sign (−). Thus, a temperature of 10° below zero is written −10° and is higher than −40°.

On the Fahrenheit scale, the temperature of melting ice was called 32° and that corresponding to boiling water was taken at 212°. This interval was divided into 180 equal parts, each of which represents 1°. Thus, 180 Fahrenheit degrees are equivalent in size to 100 centigrade degrees. The graduations on the Fahrenheit scale were carried above 212° and below 32°, the size of a degree being the same in all parts of the scale. As in the centigrade system, temperatures below zero are called minus

**41. Conductivity.**—If one end of an iron bar is thrust into an open fireplace in which a brisk fire is burning, it will not be long before the other end of the bar becomes too hot to be held in the hand; iron is therefore said to *conduct* heat. All bodies conduct heat to a certain extent, but there is a great difference between their conductive powers; thus, if the preceding experiment is repeated with a stick of wood, the stick can be held in the hand until it is nearly consumed by fire. It is therefore obvious that wood does not conduct heat as well as iron. Materials are accordingly classified as *good conductors* and *poor conductors*.

The metals are good conductors; liquids and gases are poor conductors. Organic substances, such as wood, cotton, wool, straw, bran, etc , are poor conductors, for which reason a covering with straw is an effective means of preventing freezing in the winter. As a general rule, materials having an open

structure, such as the organic materials just mentioned, are poor conductors because the air enclosed in the many small air spaces contained within them is a poor conductor; thus, dense bricks conduct heat better than porous bricks. Similarly, solid rock is a better conductor than broken stone, although solid rock is a poor conductor compared with metals. Strictly speaking, it is not correct to say that any given material is either a good or a poor conductor unless it be also stated with what other material it is compared; conductivity can be measured only by comparison. Materials having a comparatively low conductivity are called *heat insulators;* insulators are often used in building construction to maintain a more constant temperature, by excluding the heat in the summer and retaining the artificial heat during the winter.

### CHANGES OF STATE

**42.** If water is heated beyond 212° F , it boils and becomes steam. If it is cooled below 32° F., it freezes and becomes ice. The passing of a material from a liquid to a gas or to a solid, or either of the reverse processes, is called a **change of state.** Temperatures at which a material changes state are called **critical temperatures.**

**43. Fusion.**—Solids *melt* or *fuse*, that is, become liquid, at a certain temperature called the **melting point** or **temperature of fusion.** In most solids there is a very definite distinction between the liquid and the solid state, but not in all. Thus, there is a very marked difference between water and ice, while iron, clay, and glass pass from the solid state into the liquid state by a gradual softening. The hotter the material becomes, the softer it gets, until finally it becomes liquid. A material heated to a state of softness without being liquefied is said to be heated to *incipient fusion*, or to be in a state of *vitreous fusion*.

Table I gives the temperatures of fusion of a number of substances. Not all substances can be fused, because some are decomposed before they reach a sufficiently high tem-

perature.    There are no substances in nature which are abso-
lutely *refractory*, that is, are incapable of fusion.    However,
those materials which melt at a very high temperature may,
for all practical purposes, be called refractory.

TABLE I

MELTING AND BOILING POINTS

| Substance | Melting Point Degrees F. | Boiling Point Degrees F. |
|---|---|---|
| Water...... | 32 | 212 |
| Mercury..... | −37.8 | 662 |
| Sulphur..... | 228.3 | 824 |
| Tin... | 446 | |
| Lead..... | 626 | |
| Zinc...... | 680 | 1,900 |
| Alcohol........ | −170 | 173 |
| Oil of turpentine | 14 | 313 |
| Linseed oil..... | | 600 |
| Aluminum.. | 1,400 | |
| Copper..... | 2,100 | |
| Cast iron...... | 2,192 | 3,300 |
| Wrought iron....... | 2,912 | 5,000 |
| Steel....... | 2,520 | |
| Platinum. . | 3,632 | |
| Iridium....... | 4,892 | |

44.  Fusion of Alloys.—It is an interesting fact that
mixtures of substances often melt at a lower temperature than
either one of the constituents    This is especially true of com-
binations of metals, called *alloys*.    Thus, a certain alloy
known as *Rose's fusible metal*, consisting of two parts of
bismuth, one part of lead, and one part of tin, melts at 200° F ,
that is, below the temperature of boiling water, although lead
melts at 620° F., and tin melts at 450° F.    There are many
other alloys that melt at low temperatures, and that find
employment in the industries in various ways, one important

use is as *fusible links* in fire-doors in buildings.   The door, mounted on an inclined overhead track, is held open by a weight which is attached to a chain containing a fusible link. A slight increase in temperature due to a fire in the building is sufficient to melt this link, which releases the weight and allows the door to slide down the track and close the opening Another use in buildings is in connection with *sprinkler systems*.   A sprinkler system consists of water pipes extending underneath the ceilings of the building and provided with outlets closed with plugs of metal that fuse at a low temperature.   In case of a fire the fusible plug melts, thus opening the outlets or so-called sprinkler heads and sending streams of water over the fire.

**45.**   In many industries it is necessary to liquefy materials that are more or less difficult to melt; in such cases other materials, called **fluxes,** are added in order to lower the melting point of the mass.   The flux to select for any given process is one that forms with the original material a mixture or alloy having a low melting point; different fluxes are of course used for different purposes.

**46.   Solutions.**—A solid may be changed into a liquid, not only by melting it, but also by dissolving it, as salt or sugar is dissolved in water.   Since the molecules of the solid body must be separated in opposition to the forces that hold them together, it is reasonable to suppose, and it can be easily proved, that a certain amount of heat will be required to do this.   A thermometer is placed in a vessel of water and some salt or sugar is added and stirred to make it dissolve more quickly; it will then be found that the temperature has fallen several degrees.   In fact, if any solid is dissolved in a liquid that does not act chemically upon it, it will be found that heat is absorbed and that the temperature of the mixture will be lower than if the solid did not dissolve.   It is this principle that is taken advantage of in the so-called *freezing mixtures.* Thus, a mixture of three parts of snow or finely powdered ice and one part of common salt will reduce the temperature from $32°$ to $-7.6°$ F., a range of $39.6°$; while a mixture of four parts

of potash and three parts of snow or powdered ice will lower the temperature from $32°$ to $-51°$ F., a range of $83°$.

47. The solution of a solid in water is generally much hastened by heating the water, and the hotter the water is, the greater is the volume of solid that a given volume of water will dissolve. A solution that will dissolve no more of the solid is called a **saturated solution.**

A solution containing less of the solid material than a saturated solution contains is called *weak* or *dilute*. These terms are, however, applied not only to solutions of solids but also to mixtures of different liquids. Not all liquids will mix, and some will mix only in definite proportions or within definite limits, but many liquids mix in any proportion. A perfect mixture of liquids is called a *solution* just as in the case of solids dissolved in liquids.

If oil is poured on water there is no tendency for the two liquids to intermix even after violent shaking; on settling, two distinct layers will form. But, if alcohol is poured on top of water, the latter, which is heavier, will be seen to rise and penetrate the alcohol against the force of gravity. Also, the alcohol will enter the water against the force of buoyancy, and the two liquids will continue to mix until each unit volume of the mixture contains the same proportions of water and alcohol    This intermixing between two different liquids is called **diffusion.**

48. **Freezing.**—If a liquid is cooled to a certain temperature it solidifies, or *freezes*    This temperature is the same as that at which the solid would be liquefied, but when a liquid freezes the term **freezing point** is used instead of melting point.

The freezing point is very different for different liquids; for water, it is $32°$ F , for mercury, $-37 8°$ F., and for alcohol it has the low value of $-170°$ F. Mixtures of water and alcohol freeze at lower temperatures than pure water; this fact is utilized in preparing non-freezing liquids for use in automobile radiators, and for similar purposes. The reason for not using pure alcohol is that alcohol is expensive, and that

it evaporates easily and boils away. Salt water is much cheaper than a mixture of alcohol and water, and since it freezes only at low temperatures, it is often used as a non-freezing liquid; however, salt water has a very injurious action on many metals and therefore it cannot be used in automobile engines. In refrigerator plants, salt water—called *brine*—is frequently utilized for cooling purposes, because it does not freeze at the low temperatures employed.

A small quantity of salt is sometimes mixed with concrete or mortar when there is danger of the concrete or mortar freezing before hardening; since the salt lowers the freezing point of the water used, the danger of freezing is greatly reduced or entirely eliminated, but this practice is not permitted in the best specifications.

**49. Expansion on Freezing.**—Although as a rule substances contract when freezing, water represents an exception. In freezing, water expands rapidly and with great force; milk bottles and water pitchers left outside in winter are often found cracked by the freezing of their contents. It is of great importance in construction work to remember this fact in the fall of the year, since a sudden frost may cause all exposed water pipes, tanks, and boilers to burst, thus ruining much valuable property, if the precaution is not taken to drain the water out each night, or to protect the water in the pipes and tanks in some way against freezing.

There are several liquids besides water that expand while solidifying. One of these is molten cast iron. The expansion of cast iron is a great advantage because the molten iron upon solidifying fills the molds more completely, thus making it easier to obtain dense and smooth castings, in fact, it is difficult or impossible to obtain perfect castings from any material that shrinks when solidifying. Shrinkage while solidifying must not be confused with contraction, which takes place in all solid materials while cooling.

**50. Vaporization.**—The passage from a liquid state into a gaseous state is called vaporization. Some liquids cannot be vaporized, because they are decomposed at tempera-

tures too low to vaporize them, but the majority of liquids are readily vaporized; such liquids are called *volatile*. Vaporization takes place at different rates of speed at different temperatures; the process is more rapid at higher temperatures. At a certain temperature, called the **boiling point**, the liquid boils and is gradually transformed into vapor. The boiling points of several substances are given in Table I.

It is not possible to heat a liquid in the open air to a temperature above its boiling point, because it then ceases to be a liquid and becomes a gas. At temperatures considerably below the boiling point, exposed liquids vaporize slowly; slow vaporization is called **evaporation**. At temperatures below the freezing point evaporation is very slow but it does not cease altogether, so that snow or ice on the ground will slowly evaporate   Several substances are known that pass from the solid into the gaseous state without first becoming liquid; this vaporization of solids is called **sublimation.**

**51.   Condensation.**—If a vapor is cooled to a temperature below the boiling point of the original liquid, it will return to the liquid state, this transformation is called condensation.   This accounts for the fact that moisture is deposited or *precipitated* on a cold object brought into a warm, moist atmosphere.   A similar effect takes place when the walls or windows of a building are cooled below the temperature prevailing within the building, because then the moisture of the atmosphere is precipitated on the cold surfaces.   If the walls are made of good heat conductors they will cool quickly when the outside temperature falls; it is, therefore, important that the walls and roofs of buildings should be constructed of insulating materials.   The phenomenon of precipitation of moisture on the walls and roofs of buildings is called *sweating*   Sweating cannot take place unless the air in the building contains much moisture, so that sweating can often be remedied by a more thorough ventilation whereby the moisture-laden air is changed for drier air.   Air containing so much moisture that it is on the verge of precipitation is said to be *saturated* with aqueous vapor.

**52.**   Liquids which will boil at a low temperature can be separated from a mixture or solution by means of heat, and the process is known as **distillation.**   The apparatus used is shown in Fig. 23 and is called a *still.*   The mixture is placed in vessel *a*, called a *retort*, and, due to the heat from flame *b*, the liquid to be segregated boils, while the remainder, or *residue*, does not change its form.   The vapor rises and passes through neck *c* and, as it is cooled, condenses and forms the
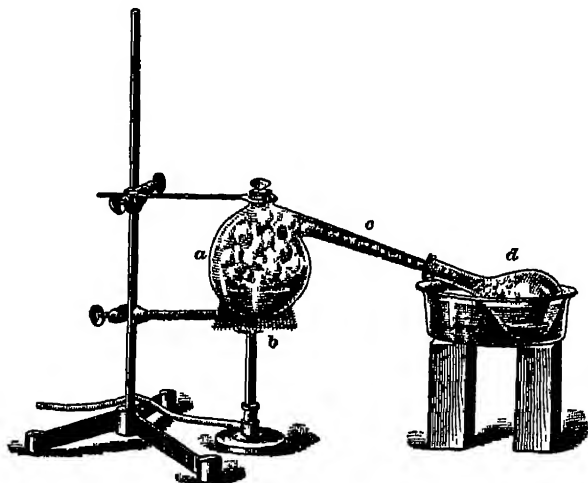


Fig 23

*distillate*, which is collected in receiver *d*.   The distillate is then very nearly pure unless the temperature is allowed to reach the boiling point of other ingredients of the mixture. After having separated the first liquid, the process can be continued and another liquid which has a higher boiling point can be similarly obtained in a different receiver.   When two or more liquids are separated from the same mixture the process is called **fractional distillation.**   An application of fractional distillation on a large scale is the refining of crude oil, which gives petroleum, gasoline, and lubricating oil at different temperatures.   When fractional distillation of a mix-

ture is carried on until all the distillable material has been segregated, the process is known as **destructive, or complete, distillation.**

**53.** A mixture of two solids may be separated by adding a liquid which dissolves one of the solids but does not affect the other. The liquid can then be segregated from the solid by a process called **filtration.** A filter is any substance which is sufficiently porous to allow a liquid to pass and sufficiently dense to hold back a solid   In the laboratories a special kind of paper is commonly used as a filter, while in the industries and in water-supply systems, large filters are constructed of sand and other materials. When a mixture of a liquid and a solid is poured on a filter, the liquid, called the *filtrate*, passes through and the solid, called the *residue*, remains behind.

For example, water added to a mixture of sugar and sand dissolves the sugar only. If the whole is then poured on a filter, the solution of sugar and water will pass as the filtrate while the sand will be held back as residue.

**54. Suspension of Solids in Liquids.**—Due to buoyancy, some solid particles float in a liquid although they will not pass through a filter; these particles are said to be held in suspension. A small particle may float while a larger particle of the same material may sink. The size of the particles which a liquid can hold in suspension depends on the speed of motion of the liquid. Thus, if a sample of sand of various sizes is put in a vessel of water and agitated violently, all of the sand will be held in suspension in the water. As the water gradually loses its speed of motion, the heaviest particles of sand will settle at the bottom of the vessel and each succeeding layer will contain smaller particles, with the finest grains on the top. In this way, the proportions of the various sizes in the sample of sand may be determined by the thickness of the layers. A similar method is employed for grading cement, except that alcohol is the liquid used, since water affects the cement chemically.

**55. Solidification.**—If a solid is dissolved in a liquid, and then the liquid is solidified by freezing, the solid is some-

times separated out. Thus, the ice formed by salt water is always entirely pure. However, some liquids retain a portion of certain solids when they solidify. For example, liquid iron dissolves carbon readily; upon cooling, a part of the carbon is separated out, but the rest of the carbon remains dissolved in the solid iron, forming a *solid solution*. The alloys previously referred to are solid solutions of one or more metals in another metal.

**56. Dialysis.**—If common salt is dissolved in water and the water is allowed to evaporate, the salt will remain in the form of *crystals*, which are particles having a definite geometrical shape  Crystals may also be obtained when a material is melted or vaporized and then allowed to solidify slowly by cooling. A **crystalloid solution** is one in which the solid can crystallize. Materials which do not possess the ability to crystallize are known as **colloids** and the solution of a colloid in a liquid is called a *colloidal solution*. Examples of colloids are glue, gelatine, and whites of eggs. A *dialyser*, consisting of a piece of parchment paper stretched over a hard-rubber ring, is suspended on the surface of a body of water as in Fig  24. When a solution containing a crystalloid and a colloidal substance is poured into the ring, the crystalloid part will pass through the parchment and dissolve in the water while the colloid will stay inside the parchment. This process of separation is called **dialysis**.
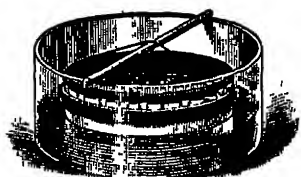


FIG. 24

## MAGNETISM

**57. Natural Magnets.**—Near the town of Magnesia, in Asia Minor, the ancients found an ore that possessed the remarkable property of attracting iron  This attraction they named **magnetism,** and a piece of the ore was called a **magnet.** The ore itself has since been named *magnetite*. It was also

found that a piece of magnetite, when freely suspended, would always take a fixed position with the same end pointing very closely toward the north pole of the earth   The ancients made use of this fact in steering ships, and the name **lodestone** (meaning *leading stone*) was given to this natural ore.

**58. Artificial Magnets.**—When a bar or a needle of hardened steel is rubbed with a piece of lodestone, it acquires magnetic properties similar to those of the lodestone, although the latter does not lose any of its magnetism.   Such bars are called artificial magnets.

Artificial magnets that retain their magnetism for a long time are called **permanent magnets.** A piece of hardened steel can be more or less permanently magnetized by rubbing it lengthwise with a permanent magnet, or by means of an electric current, as described later.   The common form of permanent magnet is a bar of steel bent in the shape of a horseshoe, as shown in Fig. 25, and then hardened and magnetized.
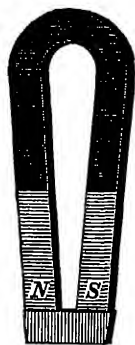


Fig. 25

A piece of soft iron called an **armature,** or *keeper*, is placed across the two free ends when the magnet is not in use to prevent the magnet from losing its magnetism.

**59. Magnetic Poles.**—If a bar magnet is dipped into iron filings, the filings will be attracted toward the two ends and will adhere there in tufts, as shown in Fig. 26, while near
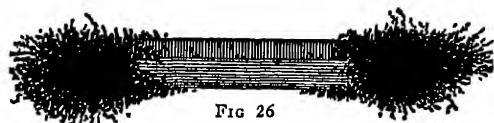


Fig 26

the center of the bar there is no noticeable attraction.   That part of the magnet where there is no apparent magnetism is called the **neutral region,** and the parts around the ends where the magnetic attraction is greatest are called **poles.**

Just like the piece of magnetite, a freely suspended bar magnet will also set itself in the direction of the earth's axis with the same end toward the north.   This end or pole is, there-

fore, known as the **north pole**. The opposite end of the magnet is called the **south pole**. The poles are usually marked $N$ and $S$, respectively. It is impossible to produce a magnet with only one pole  If a long bar magnet is broken into any number of parts, each part will still be a magnet with a north and a south pole at its ends.

**60.  Magnetic Attraction and Repulsion.**—If the bar magnet, Fig  26, is pivoted on a point, similar to a compass needle, and the north pole of another magnet is brought near the north pole of the pivoted magnet, the latter pole will be repelled. But, if a north pole is brought near the south pole of the pivoted magnet, there will be evidence of a mutual attraction between the poles. In general, *like magnetic poles repel each other; unlike poles attract each other*.

**61.  Magnetic Substances.**—Metals which are attracted by magnetism, although they do not become magnets themselves, are called *magnetic* substances. In addition to iron, and steel and its alloys, the following metals are magnetic: nickel, cobalt, manganese, cerium, and chromium. These metals, however, possess very feeble magnetic properties compared with those of iron and steel. Most other known substances are *non-magnetic*.

**62.  Molecular Theory of Magnetism.**—The fact that the parts of a broken bar magnet will each constitute a separate magnet may be explained by means of the molecular theory of magnetism. According to this theory each individual molecule in any bar of iron or steel is a magnet possessing a north and a south pole  Before the iron or steel bar is magnetized, these molecules are supposed to form an infinite number of minute magnets distributed in such a manner that the magnetic forces neutralize each other. The result is that the bar, as a whole, displays no apparent magnetism

When the pole of a strong magnet is brought near one end of the iron or steel bar, the molecule magnets are subjected to an external force and are compelled to point with like poles toward the pole of the external magnet. If the latter pole is

a north pole, it will be faced by the south poles of all the molecules. In case these molecules form parts of a hardened steel bar, they will, to a large extent, retain their new positions for a long time and thus the bar becomes a permanent magnet. But if the bar is made of soft iron, the mutual attraction between the poles of the molecules is supposed to be so great that the molecules will return to their previous positions as soon as the external magnetic force is removed.

**63. Magnetic Field.**—If a magnet is brought near a piece of iron, the iron will be pulled to the magnet before there is actual contact between the two bodies, which indicates
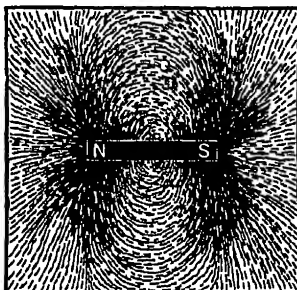


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that the magnetic forces act at a distance from the magnet. A simple experiment to determine the manner in which the magnetic forces act may be performed by sprinkling iron filings on a sheet of cardboard laid over a bar magnet and gently tapping the cardboard; as shown in Fig. 27, the iron filings will arrange themselves in lines which seem to issue from each pole in all directions and curve until they enter the other pole. Other experiments may be performed to establish the fact that the magnetic forces act in the space surrounding the magnet along well-defined lines, called **lines of force.** These lines of force should not be conceived as invisible threads, but merely as directions in which the magnetic forces act.

As illustrated in Fig. 28, where several lines of force are shown by dotted lines and their directions indicated by arrows,

it is commonly assumed that the lines of force pass out from
the north pole of the magnet, run through the surrounding
air, and reenter the magnet at the south pole, returning
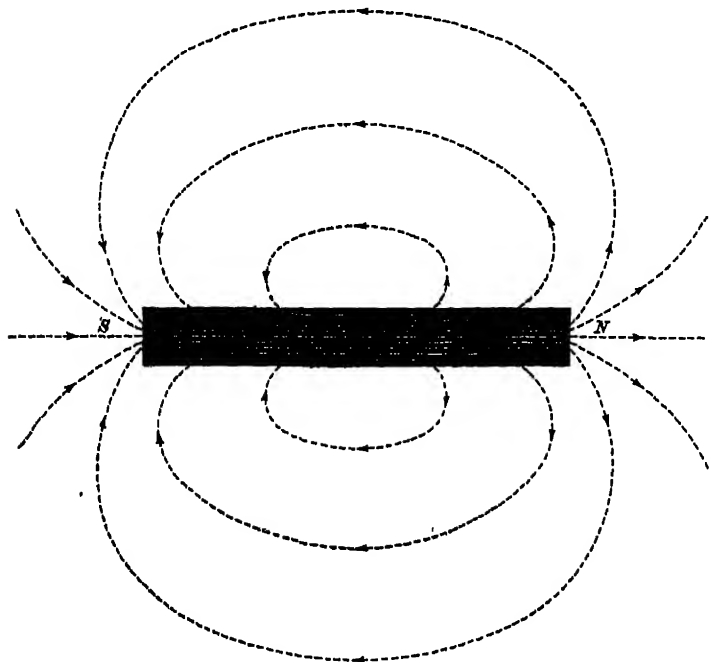


FIG. 28

through the magnet to the north pole.    The space around a
magnet in which the magnetic forces act is called its **mag-
netic field.**

## ELECTRICITY

### INTRODUCTION

**64.    Fundamentals.**—The exact nature of electricity is
not known, but its effects and the laws governing its behavior
are well understood.    Explanations of electrical phenomena
may be conveniently based on the assumption that electricity
acts like a fluid, although apparently there is no actual flow

of material such as in the case of water running through a pipe.    Just as hydraulic pressure tends to produce a flow of water to a lower level, it is said that electricity flows through conductors because of an electric pressure.

**65.    Electromotive Force.**—When a pump raises water to a higher level it does not create the water but simply produces a difference between the pressure acting on the water inside the pump and that acting outside the pump.    The resulting difference in pressures causes the water to rise; the water can then be stored in a reservoir for later use.

Similarly, any apparatus that raises electricity to a higher electric level simply produces a difference in electric pressure which causes the electricity to flow through a path provided to carry it.    Electric level is called **potential,** and a difference in potential between two points produces an electric pressure or **electromotive force,** abbreviated e. m. f.    It is this e. m. f. which causes a flow of electricity.    Some of the methods of developing e. m. f. will be briefly described later.

**66.    Measurement of e. m. f.**—The practical unit of electric pressure, or of e m. f., is the **volt,** just as the hydraulic pressure is measured in pounds per square inch. The e. m. f. in volts is sometimes called **voltage,** so that a voltage of 110 means the same thing as an e. m. f. of 110 volts. Voltages are measured by means of a **voltmeter,** usually of the form shown in Fig. 29. It is provided with a pointer and two scales,
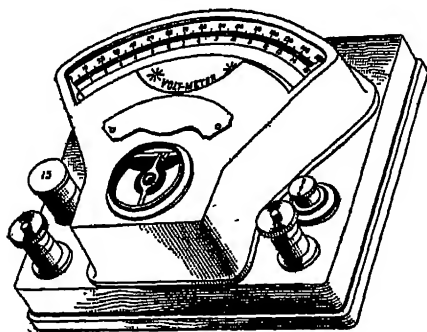


Fig 29

one for low voltages (from 0 to 15 volts) and one for high voltages (0 to 150 volts).    The instrument has three binding posts, of which the one to the right is always used, in conjunction with this the lower left one is used for the

150-volt scale and the upper one for the 15-volt scale.   On the right is a push button, which must be pressed whenever a reading is to be taken.

**67.   Electric Current.**—In considering the flow of water through a pipe, it is important to know the *rate of flow*, that is, the quantity of water which passes in a unit of time.   For example, this rate of flow may be given in gallons per second. The time involved is of as much interest as the total quantity flowing.   Suppose that 1,000 gallons of water flow through a pipe in 100 seconds.   The rate of flow is then $\frac{1,000}{100} = 10$ gallons per second.   If the flow through another pipe is 1,000 gallons in 50 seconds, the rate of flow in this case is $\frac{1,000}{50}$ $= 20$ gallons per second.   Although the total flow in both cases is 1,000 gallons, the rate of flow in the second pipe is twice as great as that in the first pipe.



FIG. 30

Similarly, a quantity of electricity may flow through one conductor in 10 seconds and the same quantity may flow through another conductor in 5 seconds. The rate of flow is twice as great in the second conductor as in the first one, because in 1 second twice as much electricity passes through the second conductor as through the first.   The rate of flow of electricity is called **electric current.**   In the example just stated, the current in the second conductor is twice that in the first conductor although the same total quantity of electricity passes through both.   The practical unit of electric current is known as the **ampere.**   An ampere represents a well-defined quantity of electricity flowing through a conductor in 1 second.
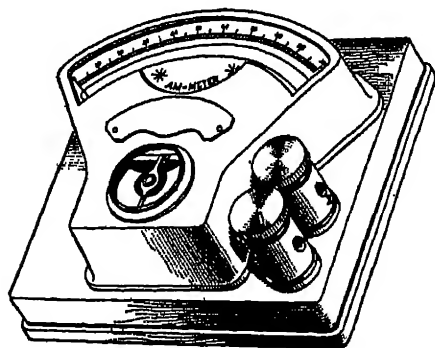
Electric current is measured by means of an instrument called an **ammeter,** of which a common type is shown in Fig. 30.   It has two binding posts, for connecting it in an electric circuit, to be defined later, and the current in amperes is indicated on a scale by the deflection of a pointer

An electric current always flows in the direction from a point at higher potential to a point at lower potential.   For convenience, the point at higher potential is called the **positive (+) terminal** and the point at lower potential is called the **negative (−) terminal.**

**68.   Direct and Alternating Currents.**—Electric currents are of two general classes, direct currents and alternating currents

A **direct current** is a flow of electricity always in the same direction, in a manner similar to that of a stream of water flowing through a pipe.

An **alternating current** is a flow of electricity that periodically reverses its direction.   The effect of an alternating current is similar to that produced by alternately compressing and releasing a rubber bulb, to which is attached a tube immersed in water.   As the water is forced out of the bulb and then drawn back, the flow in the tube changes direction. In an alternating current the reversals usually occur with such great rapidity that if the current is sent through an incandescent lamp the eye is unable to detect any variation in the brilliancy of the light

Whether a direct or an alternating current is to be used in any particular case depends on the purpose in view.   Each has its special advantages that make it particularly suitable for certain purposes, while in some cases either one may be used   For transmitting electric energy to a distant point, the alternating current is preferable, but for electro-chemical processes, the direct current must be used   Apparatus designed for one kind of current cannot, ordinarily, be used for the other kind   In practice it is customary to use the abbreviations A  C. for alternating current and D. C for direct current, both orally and in print

I L T 370—9

**69.  Electric  Resistance.**—Electricity  cannot  flow
through a substance without meeting some opposition.  That
quality of a substance by which it opposes the flow of elec-
tricity is known as **electric resistance.**  While this resis-
tance may differ in various substances, no substance is wholly
without resistance.  Substances through which electricity
flows readily are known as **conductors,** while those offering
very high resistance to its flow are called **non-conductors,**
or *insulators*

Electric resistance depends not only upon the material but
also upon the length and cross-sectional area of the conductor
through which an electric current passes.  Water flows more
readily through a short pipe of large diameter than through
a long pipe of small diameter; similarly, electric resistance is
greater in a long conductor than in a shorter one and is less
in a large bar than in a very fine wire of the same material.
The unit of resistance is called an **ohm.**

A complete conducting path, through which electric cur-
rent will pass between points at different potentials, is known
as a **closed electric circuit.**  If there is a break at any
point in the circuit, it is termed an **open circuit.**

**70.  Electric Relations.**—The strength of current in a
circuit depends not only on the electromotive force acting in
the circuit, but also on the resistance of the circuit.  The cur-
rent increases with increased electromotive force, but decreases
with an increase in resistance.  The relation between e. m f ,
current, and resistance in any circuit is given by the formula

$$I = \frac{E}{R}$$

where            $I$ = current, in amperes;
                 $E$ = e. m. f., in volts;
                 $R$ = resistance, in ohms.

For example, if the voltage is 110 and the resistance in the
circuit is 11 ohms, the current $I = \dfrac{E}{R} = \dfrac{110}{11} = 10$ amperes
Also, in order to drive a current of 20 amperes through a body

having a resistance of 5 ohms, the necessary e. m. f., $E = R\,I$ = $5 \times 20 = 100$ volts.

**71.   Electric Work and Power.**—Electricity possesses the capacity of doing work and therefore it is a form of energy. The rate at which electricity does work is called electric power just as the power of any machine is the work done by the machine in a unit of time    The unit of electric power is called the **watt.**   A watt is the power developed in a circuit in which the e. m. f. is 1 volt and the current is 1 ampere   If the e. m. f is 100 volts and the current 10 amperes, the power is $100 \times 10 = 1,000$ watts.   As a general formula,

$$P = E\,I$$

where    $P$ = power, in watts;
$E$ = e  m. f , in volts;
$I$ = current, in amperes.

Since a watt is a relatively small unit, electric power is generally expressed in kilowatts, a **kilowatt** is equal to 1,000 watts.   A kilowatt is the same kind of unit as a horsepower and values may be converted from one to the other by the relation that *1 H. P = 746 watts = .746 kilowatt*

When an average power of 1 kilowatt is developed for 1 hour, the work done is 1 **kilowatt-hour,** since work is equal to the product of power and time.   The kilowatt-hour is the unit of electric work and is the same kind of unit as the foot-pound of mechanical work, as shown in the following example:

EXAMPLE 1.—Find the value, in foot-pounds, equivalent to 1 kilowatt-hour.

SOLUTION.—         1 H. P. = .746 kilowatt

Hence,              1 kilowatt = $\dfrac{1}{.746}$ H. P.

Since 1 H. P. = 33,000 ft -lb. per min. = $60 \times 33,000 = 1,980,000$ ft.-lb. per hour, 1 kilowatt = $\dfrac{1,980,000}{746} = 2,654,000$ ft.-lb. per hour.

Therefore,    1 kilowatt-hour = 2,654,000 ft -lb.   Ans.

EXAMPLE 2 —Find the power, in kilowatts, used by a motor when the voltage is 220 and the current is 25 amperes.

SOLUTION —From the above formula,
$P = E\,I = 220 \times 25 = 5,500$ watts = 5.5 kilowatts.   Ans.

**72. Conductors and Insulators.**—Good conductors of electricity are made of materials having low resistance. Among such materials, arranged in order, with the best conducting material first, are silver, copper, gold, aluminum, zinc, brass, phosphor-bronze, platinum, tin, nickel, lead, German silver, steel, iron, mercury, carbon, and water. Copper, being plentiful and comparatively cheap, is extensively used; aluminum is also much employed for long transmission lines.

Insulators are non-conductors of such high resistance that, practically, no electricity will pass through them. Among the best known insulating materials are glass, porcelain, rubber, mica, ebonite, dry paraffined wood, paper, vulcanized fiber, asbestos, pure asphalt, air, and oils. Insulators are used to support conductors and to keep the electric current confined to circuits intended for it. For this reason copper wire, as used in various appliances, is covered with cotton, rubber, or a combination of the two materials. Electric-light and power wires on poles are supported by glass or porcelain insulators.

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### EXAMPLES FOR PRACTICE

1. A current of 10 amperes is to pass through a circuit having a resistance of 2 ohms. What e. m. f. will be required?          Ans. 20 volts

2. If the e. m. f. in a circuit is 220 volts and the resistance is 11 ohms, what will be the current?          Ans. 20 amperes

3. What is the H. P. developed by a direct-current motor if the e. m. f. is 500 volts and the current is 100 amperes, if motor losses are disregarded?
Ans. 67 H. P.

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### EFFECTS OF ELECTRICITY

**73. Magnetic Effect of Current.**—If some insulated copper wire is wound around a bar of soft iron, in the manner shown in Fig. 31, and an electric current is sent through the wire, the bar will become a magnet    As long as the current is maintained, the bar will remain magnetized, but when the current stops the bar will lose its magnetism entirely. The coil of wire and bar combined form what is known as an **elec-**

tromagnet; the wire winding is called the **exciting coil**, and the iron bar, the **core**.

A common form is the horseshoe electromagnet shown in Fig 32, consisting of a soft bar, bent to **U** shape, on which the exciting coil is wound as indicated. In both Figs 31 and 32 the direction of the current in the wire is indicated by arrows, and for this direction the poles are marked $N$ and $S$
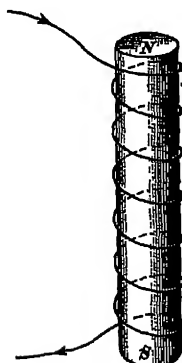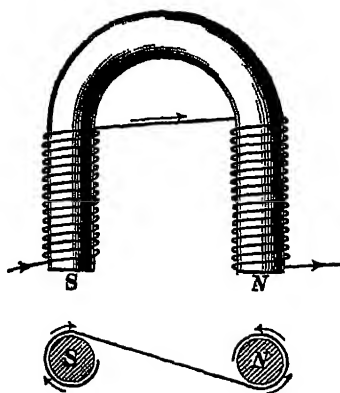


FIG. 31            FIG. 32

to indicate a north pole and a south pole, respectively. If the current in the wire is reversed in direction, the poles change also.

**74.** Electromagnets are used for many purposes, especially in motors and generators, as well as for all kinds of electrical apparatus. An interesting example is the crane shown in Fig. 33, where an electromagnet is used for handling heavy iron castings The electromagnet is enclosed in the metal hood $a$; the current is conducted by cables $b$ extending from the operator's cab $c$. The magnet is lowered to the piece to be lifted; when the operator, by means of a switch, turns on the current, the electromagnet at once attaches itself to the load, which can now be lifted without the use of hooks and slings When the load arrives at the desired point, it is instantaneously released by switching off the current. The magnetic crane is thus a very valuable appliance for handling

objects made of steel or cast iron, but it cannot be used for other materials unless they are attracted by the magnet.

**75.  Chemical Effect of Current.**—Electrical energy may be transformed into chemical energy   This phenomenon has found a wide application in modern industrial chemistry.
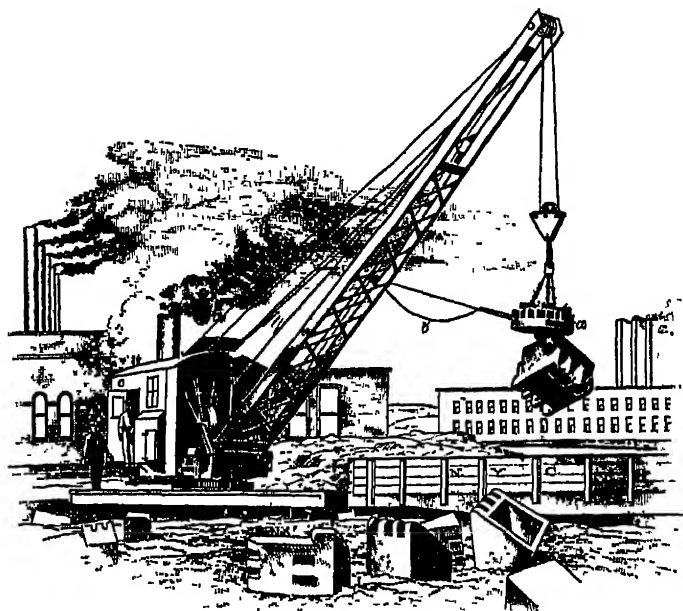


FIG. 33

A simple experiment may serve as an illustration of this kind of transformation.   The shallow basin *A*, Fig. 34, contains water to which a very small quantity of acid has been added in order to make the water a good conductor, as chemically pure water is an insulator.   Connected with the binding posts *B* are two platinum wires, which dip into the water and are knowns as *electrodes*.   The glass tubes *C* are also filled with acidulated water, and each is inverted over one of the electrodes.   The wires of an electric circuit are now connected to the binding posts so that the current must pass through the water.   As soon as the current is turned on, gas begins to

collect at the tops of the tubes; in the tube over the negative electrode the volume of gas evolved is double that evolved at the positive electrode.    The gas having the larger volume is called hydrogen and the other gas is called oxygen.    These two gases are the sole constituents of water, and combine in the exact proportions of 2 volumes of hydrogen to 1 volume of oxygen    It follows that 1 molecule of water must consist of 2 atoms of hydrogen and 1 atom of oxygen.    It is therefore to be seen that the electric current has the ability to decompose water by resolving it into its component elements.

The process of re-solving a substance into its constituent elements by means of an electric cur-rent is known as **electrolysis.**

**76.  Dangers of    Electrolysis.** While electrolysis is utilized in many industries, it is a phenomenon that gives the construc-



Fig. 34

tion engineer a great deal of trouble    The electric current used to propel street cars is almost invariably fed to the cars by means of a single wire, known as a trolley wire, connected to one terminal on the electric generator, to be explained later, while the current returns to the generator by means of the rails, a short wire being used at the power sta-tion to connect the rails with the second terminal    If the cur-rent followed the rails back to the generator, no damage would be done, but such is not always the case; since the current follows the path of least resistance, it frequently leaves the rail, passing through the ground to a water pipe or other good con-ductor buried in the earth, and electrolysis follows.    How-ever, in this case the electrodes are not made of a non-oxi-

dizable metal like platinum, but are usually pipes made of iron, lead, or some other metal easily attacked by oxygen, and the latter gas will combine with them and form compounds, such as rust, verdigris, etc. Thus, the pipes are eaten into in a remarkably short time, and entirely destroyed.
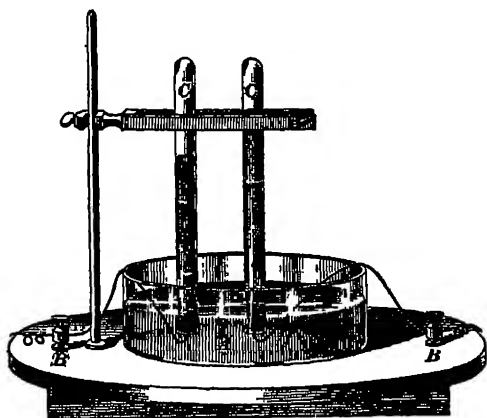
**77.** Electrolysis of underground pipes is a common occurrence in large cities. Another form of electrolysis which is equally destructive, but fortunately very rare, is known as *electrolysis in reinforced concrete*. Reinforced concrete consists of steel bars embedded in concrete; the steel bars, being good conductors, will lead a stray electric current to some part or parts of the concrete that, by reason of being moist, conduct a sufficient amount of electricity to cause corrosion of the reinforcement. Not only does the steel corrode and lose its strength, but it swells in corroding, thus bursting its concrete envelope. The remedy is obviously to insulate carefully all electric circuits in the building so that no currents can stray at any place.

### PRODUCTION OF ELECTROMOTIVE FORCE

**78.** It has been previously explained that in order to establish a flow of electricity in a circuit there must be a difference in electric level between the terminals of the circuit, or, in other words, there must be an electric pressure or electromotive force. An electromotive force may be produced by various means, but in this Section only those methods will be considered that convert mechanical or chemical energy into electrical energy.

**79. The Electric Generator.**—To convert mechanical energy into electrical energy, an apparatus known as an electric generator is employed. The elements of the generator are shown diagrammatically in Fig 35. $N$ and $S$ are the poles of an electromagnet whose coils, carrying an electric current from any suitable source, are omitted. There is a magnetic field between these poles and the lines of force are assumed to proceed from the north pole toward the south pole, as shown

by the dotted arrows.   Revolving between the poles $N$ and $S$ and carried by the shaft $b$, which is driven by some external source of mechanical energy, is a soft-iron cylinder $a$ known as the **armature core.**   On its convex surface the core is provided with a great number of conductors which form the **armature winding.**   The shaft, core, and winding constitute the **armature.**   Only two armature conductors are shown, both in the same loop, one at $c$ and the other diametrically opposite at $d$.   When these conductors revolve with the core, they cut across the lines of force and an e. m f. is thereby produced in each conductor.   The far ends of the conductors are connected to each other and the near ends are connected through collector rings $e$ and $f$.   Copper strips $g$
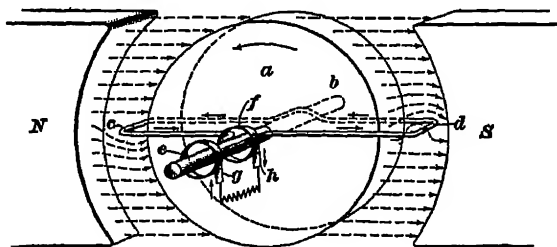


FIG. 35

and $h$, called **brushes,** are fixed to the stationary frame of the machine, and continually press against the rings $e$ and $f$   For the direction of rotation indicated by the curved arrow, the e. m. f.'s in $c$ and $d$ will act in the directions of the arrows.   In the actual armature the winding consists of many conductors connected together and active in generating e m f.'s.   One end of the conductor $c$ is connected to ring $f$ and one end of conductor $d$ connects to ring $e$.   Thus, a difference of potential exists between the two brushes.   When the circuit between $g$ and $h$ is closed, this difference of potential or e m. f. will establish an electric current in the direction shown by the arrows.   The part of the circuit through the rings and armature is called the **internal circuit** and the part outside of the machine, represented by the zigzag line between $g$ and $h$ in Fig 35, is called the **external circuit.**

**80.**   When the shaft makes a half revolution so that the conductors $c$ and $d$ exchange positions, the electromotive force in each case will be in the direction opposite to that which existed in the original position.   At the end of a complete revolution, the conductors are back in their first positions and the directions of the e m. f.'s are the same as at the beginning   It is thus seen that an alternating e m. f. is produced which changes direction twice in each revolution.   Therefore an alternating current will flow through the external circuit.   If a direct current is desired, a device called a commutator is installed instead of the collector rings $e$ and $f$.

**81.   Voltaic Cell.**—For the conversion of chemical energy into electrical energy a so-called **primary cell** is



FIG. 36

employed.   There are two forms, the *voltaic cell* and a modification known as a *dry cell*, but the chemical action is the same in both forms.
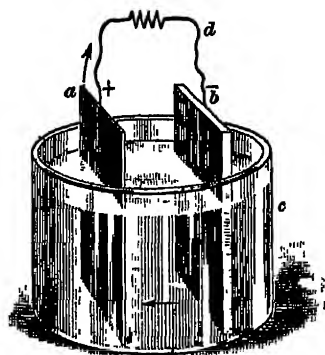
For the purpose of illustrating the principle involved, a voltaic cell is shown in a simplified form in Fig. 36.   The elements used in the cell are carbon, zinc, and sal ammoniac. Sal ammoniac is a white substance chemically known as ammonium chloride, or chloride of ammonia.   A carbon slab $a$, compressed to a dense form, and a zinc plate $b$ are partly immersed in a solution of sal ammoniac in water contained in a glass vessel $c$.   A difference of potential is produced at the surfaces where the carbon and zinc are in contact with the liquid.   Hence, if the carbon and zinc are connected by a copper wire $d$, the electromotive force is available to send a current through the wire from the carbon $a$ to the zinc $b$.   The current will return from the zinc plate through the liquid to the carbon, thus completing the circuit, as indicated by the arrows.

**82.**  The liquid in the cell shown in Fig. 36, in this case the solution of sal ammoniac and water, is called the **electrolyte,** and the solid plates are **electrodes.**  The electrical energy of the cell is produced by the difference in the chemical action of the electrolyte on the carbon and the zinc, which results in a gradual dissolving of the zinc.

The wire is usually attached to the plates by means of binding posts, not shown in the illustration.  The projecting part of the carbon plate is called the **positive** (+) **terminal,** or *cathode*, and that at the zinc is the **negative** (−) **terminal,** or *anode*.  As the current continuously passes through the wire from a to b, a primary cell develops a direct current. If the wire d connecting the terminals of the cell is cut and the ends separated, the current will cease.
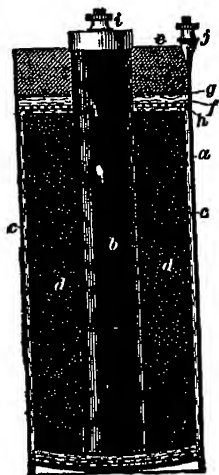


Fig. 37

**83.  Dry Cell.**—A dry cell such as is commonly used for supplying current for gasoline engine ignition is shown in Fig . 37, where the cell is cut in half to expose to view the materials of which it is composed.  The electrolyte is contained in a can a, made of sheet zinc, which takes the place of the zinc plate in the ordinary cell.  The carbon element is a carbon rod b, placed in the center of the can.  The electrolyte is a solution of sal ammoniac that is absorbed by a pulp-board lining c and the so-called *mix d*, composed chiefly of powdered carbon, and containing also certain chemicals intended to give longer life and more current.

**84.  Batteries.**—If two or more cells are grouped together, they form a **battery.**  The individual cells in a battery may be connected in various ways depending upon the purpose for which the battery is desired.  Batteries of primary cells are used mostly for electric bells and other forms of signaling, and sometimes for furnishing the current necessary for ignition purposes on stationary gasoline engines.

CONVERSION OF ELECTRIC ENERGY

**85. The Electric Motor.**—An electric generator may serve as a means for converting electrical energy into mechanical energy. It is then known as an **electric motor.** Assuming that the elementary generator, Fig. 35, is provided with a commutator and that a direct current from an outside source is sent through the brushes *g*, *h*, the armature will rotate and the shaft *b* will supply mechanical energy to any device suitably connected with it.    The principle of the action may be briefly explained as follows:

Let it be supposed that the current in the conductors *c* and *d* is in the direction indicated by the arrows; then, considering the armature *a* as an electromagnet, the top portion becomes a north pole and the bottom portion a south pole.    According to the rule in Art. 60, unlike poles attract, and similar poles repel, each other.    Consequently, the top, or north, pole of the armature will be attracted by the stationary south pole *S*, while the bottom will be repelled; also, the stationary pole *N* will attract the bottom of the armature and repel the top.    Hence, the armature will revolve in a direction opposite to that indicated by the curved arrow.

**86. Counter e. m. f.**—As soon as the armature *a*, Fig. 35, begins its rotation, it will tend to act as a generator and produce an electromotive force, which will act in a direction opposite to that of the current in the conductors *c* and *d*, and is therefore known as a **counter electromotive force.** This electromotive force acts in a direction opposite to that indicated by the arrows because the armature is now revolving in a direction opposite to that of the curved arrow.    As the speed of the armature increases the counter electromotive force also increases.

**87. Motor Starters.**—When a motor is started and the armature is beginning to revolve, it cannot produce any appreciable counter electromotive force; hence, there is nothing to prevent a great rush of current through the armature coils.    This would result in overheating the wires and in

burning off the insulating covering.   To avoid this, an adjustable resistance is introduced in the circuit, by means of which the starting current may be kept within safe limits.   As the armature gains in speed, the resistance is gradually reduced and is finally cut out entirely.   This adjustable resistance is known as a **rheostat**; it consists essentially of a system of wires made of special metallic alloys having a high electric resistance, with a handle to regulate the amount of resistance brought into action.

A specially constructed rheostat is also employed for adjusting the electrical energy supplied to a motor, thus controlling the mechanical energy delivered by it, as in the *controllers* of electric railroad cars, shop motors, cranes, etc.

**88.   Specifications.**—Electric motors can be had in many sizes; the smallest will furnish but a fraction of a horsepower while the largest will furnish many hundred horsepower.   Different motors are used for alternating current and for direct current, so that when ordering a motor it is necessary to specify whether A C or D. C. is used, as well as the voltage of the circuit.   The voltage on any circuit is supposed to be constant   Most city circuits have a voltage of either 110 or 220.   In order to make the specifications complete, the speed of the pulley on the motor must be indicated in revolutions per minute.   Different makes and sizes of motors have different speeds; a speed commonly used for the kinds of motors used on contractors' machinery, such as concrete mixers and the like, is 1,100 revolutions per minute.

**89.   Conversion of Electrical Into Chemical Energy.** By sending a current through a certain type of cell it is possible to produce chemical changes, and thus transform electrical into chemical energy.   Cells constructed for this purpose are known as **secondary batteries, storage batteries, or accumulators.**   This chemical energy is thus made available to be retransformed into electrical energy at a later time. A primary battery will deliver a current to a circuit as soon as the cell is put together, but a secondary battery must be

*charged;* that is, an electric current must be passed through it from some external source before it can be used.

Large storage batteries are often used in electric power stations to store electrical energy during slack periods and thus relieve the generators somewhat at the hours of greatest demand for current. Smaller storage batteries are also used for the purpose of furnishing motive power for electric automobiles, and for starting and lighting gasoline-driven automobiles

**90. Conversion of Electrical Energy Into Heat and Light.**—An electric current will raise the temperature of an
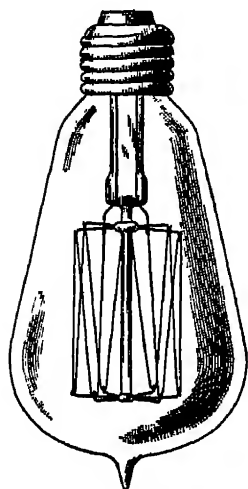
electric conductor, because a certain amount of electrical energy is required to overcome the resistance of the conductor, and this energy is converted into heat. As the amount of heat produced is directly proportional to the resistance of the conductor, it follows that by producing a conductor of very high resistance, as is the case with certain alloys, it is possible to obtain very high temperatures. This principle has been applied to electric heaters, cooking devices, etc., but an example of greater practical importance is the *incandescent lamp*, Fig. 38. In this a metallic filament or thin wire is caused to glow with a bright light by reason of the heat developed by the passage of the current. The filament is contained in an airtight glass bulb in which either a partial vacuum is produced or the air is replaced by a gas, such as nitrogen, to prevent the combustion of the filament.

Fig. 38

**91.** Another method by which light and heat may be obtained from an electric current is by establishing an arc or a spark. Generally speaking, when an air gap is made in an electric circuit, the current will cease; but, if the gap is made sufficiently short the current may bridge across either by

means of a spark or an arc    If the current passes across inter-
mittently, a **spark** is formed, if the current bridges the gap
continuously, an **arc** is formed    Which one of these forms
will be produced depends upon many factors, such as volt-
age, nature of the material at the break, width of gap, etc.
It is not the intention to describe these phenomena any fur-
ther than to state that to generate a spark, a so-called *high-
tension* current is required; that is, a current having a very
high voltage.    An arc is more advantage-
ously formed by a *low-tension* current;
that is, a current having a comparatively
low voltage, such as 110 to 220.    The arc
is usually formed by placing the electrodes
in contact and then drawing them apart.



FIG. 39

**92.**    The electric spark has many im-
portant applications, such as that of ignit-
ing the vapor in the gasoline motor of the
automobile or in stationary gas engines.

The electric arc is applied in the common electric arc lamps
used for illuminating streets, manufacturing plants, etc ; also
for producing the intense heat required in electric furnaces,
where the temperature is as high as 3,500° C    The arc in the
arc lamp is usually formed between terminals of special form,
called electrodes, and consisting of pointed rods of carbon.
If a direct current is used, the positive carbon becomes hollow
at the end while the negative one assumes a pointed shape.
In the case of an alternating-current arc both carbons become
blunt at the ends    In modern arc lamps the arc is enclosed
in an inner globe admitting just enough air to maintain com-
bustion.    Both ends assume then a flat shape, as in Fig  39.

# GENERAL PROPERTIES OF MATERIALS
## (PART 3)

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## ELEMENTS OF PRACTICAL CHEMISTRY

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### INTRODUCTION

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#### DEFINITIONS AND LAWS

**1. Physical and Chemical Changes.**—It was explained in *General Properties of Materials,* Part 1, that the condition of a body may be changed by either a physical or a chemical process. If a piece of the metal platinum is heated, it soon becomes white and emits light and heat. Its physical condition is now very much changed, but that it is still platinum can be shown by allowing it to cool. It will quickly resume its original silvery appearance and will not be permanently altered in any way. If, on the other hand, a piece of the metal magnesium is heated in the same way, it burns with a flash of dazzling white light and becomes a white substance which can be easily pulverized. The appearance of this substance is very different from the original magnesium and it is found that the weight of the powder is greater than the initial weight of the magnesium.

The hot platinum wire possesses properties which differ greatly from those it possessed when cold, but when it returns to its initial temperature it regains its original properties. The change has been a purely physical change.

§ 21

The powder which is obtained from the magnesium is composed of magnesium and oxygen which was added from the air. On cooling the substance does not become magnesium again; the change is permanent. Since the change has affected the molecules, it is a chemical change.

The conclusions that may be drawn are: *In a physical change, the molecules of the substance retain their identity. In a chemical change, each molecule of the substance is changed by the addition, loss, or exchange of one or more atoms.*

**2. Chemistry and Its Divisions.**—Chemistry is that branch of natural science that treats of the relations and combinations of atoms; it considers the changes that occur within the molecules and the laws and theories governing these changes.

Chemistry is divided into two parts: *organic* and *inorganic.* Organic chemistry is that part of chemistry which deals with a special large group of combinations that belong to the so-called hydrogen-carbon class. All other combinations belong to inorganic chemistry. As living organisms and their products are combinations that belong to the hydrogen-carbon class, it was formerly thought that hydrogen-carbon combinations were impossible without the intervention of organic life. Hence, the branch of chemistry treating of these combinations became known as organic chemistry. However, in more recent times it was found that many of the organic substances can be produced in the laboratory from substances that belong to inorganic chemistry. The line of division cannot therefore be drawn as rigidly as it was at first assumed.

Nearly all the chemical phenomena described in the following pages pertain to substances that belong to inorganic chemistry, as the substances that belong to organic chemistry are, on the whole, of little utility in the processes or for the purposes with which the construction engineer is concerned. The organic products used in construction are practically confined to only the so-called *bitumens* referred to later.

**3. Compounds and Elements.**—Experiments have shown that by far the greater number of substances with

which we are familiar may be decomposed into simpler substances. For example, if a small quantity of the substance known as mercuric oxide is heated in a test tube, it is decomposed into two substances—mercury and oxygen. It may be shown by experiment that the combined weights of the mercury and oxygen obtained are exactly equal to the weight of the mercuric oxide decomposed. Furthermore, if the mercury and oxygen are brought together again under proper conditions they will unite, forming exactly the amount of mercuric oxide previously decomposed. It appears, then, that mercuric oxide is composed of at least two simpler substances, mercury and oxygen, into which it can be resolved by proper chemical methods. The same may be said of by far the greater number of known substances. On the other hand, there are a number of substances that the most refined methods known to the chemist have failed to decompose. Mercuric oxide is resolved into two simpler substances by the mere application of heat, but no treatment has ever succeeded in producing anything simpler than mercury from mercury, or anything simpler than oxygen from oxygen. Whether or not these are capable of being decomposed is not known. All that can be said positively is that with his present knowledge and the appliances at his command, the chemist is unable to decompose them into anything simpler. The substances that can be decomposed into simpler ones are known as **compounds.**

Substances that cannot be decomposed into anything simpler are known as **elements.** The exact number of elements cannot be given, as there are indications that some of the substances usually thought to be elements are composed of more than one substance, and it is probable that there are still some undiscovered elements. A list of seventy-nine elements is given in Table I.

All the various forms of matter that come within our knowledge are composed either of these single elements or of combinations of them. About a dozen of these elements are quite common, and the rest are comparatively rare. Most of the substances with which we are familiar are composed of some of these common elements. Some compounds, like mer-

## TABLE I
### NAMES AND SYMBOLS OF CHEMICAL ELEMENTS

| Name | Symbol | Name | Symbol |
|------|--------|------|--------|
| Aluminum . | $Al$ | Neodymium.. . ...... . | $Nd$ |
| Antimony (stibium) .. | $Sb$ | Neon ...... ..... | $Ne$ |
| Argon ... . .. . . . | $A$ | Nickel ... ... .. . | $Ni$ |
| Arsenic ... ... . . | $As$ | Nitrogen. . ... .. .. | $N$ |
| Barium.. . .... . | $Ba$ | Osmium . . .. | $Os$ |
| Bismuth ..... . . | $Bi$ | Oxygen ........... | $O$ |
| Boron . ... . ...... | $B$ | Palladium . ........ | $Pd$ |
| Bromine . ........ | $Br$ | Phosphorus. .. . ... | $P$ |
| Cadmium.. .. ... | $Cd$ | Platinum. . . .. . . | $Pt$ |
| Cæsium ...... . . | $Cs$ | Potassium (kalium) | $K$ |
| Calcium . ..... .... | $Ca$ | Praseodymium . . | $Pr$ |
| Carbon ............ .. | $C$ | Radium .. ........ .. | $Ra$ |
| Cerium. .. ....... | $Ce$ | Rhodium. .. .... ... | $Rh$ |
| Chlorine .. . ..... . | $Cl$ | Rubidium.. . .... . | $Rb$ |
| Chromium . ... .. | $Cr$ | Ruthenium ... . | $Ru$ |
| Cobalt ........ .. | $Co$ | Samarium. . ... . | $Sm$ |
| Columbium (niobium) . | $Cb$ | Scandium . .... . | $Sc$ |
| Copper (cuprum). .. | $Cu$ | Selenium... .. .. | $Se$ |
| Erbium . ....... ... | $Er$ or $E$ | Silicon . ..... .. | $Si$ |
| Europium . ... .. | $Eu$ | Silver (argentum). .. | $Ag$ |
| Fluorine .. ... . | $F$ or $Fl$ | Sodium (natrium) .. | $Na$ |
| Gadolinium . . . | $Gd$ | Strontium . . .. ... | $Sr$ |
| Gallium. ........ . | $Ga$ | Sulphur . .... . . .. | $S$ |
| Germanium . ... .. . | $Ge$ | Tantalum ....... ... | $Ta$ |
| Glucinum (beryllium) .. | $Gl$ | Tellurium ........ ... | $Te$ |
| Gold (aurum) . . . . | $Au$ | Terbium. ........ ... | $Tb$ |
| Helium..... ........ | $He$ | Thallium . . . ... | $Tl$ |
| Hydrogen . . ..... | $H$ | Thorium ........... | $Th$ |
| Indium. ... ... ... . | $In$ | Thulium ............ | $Tm$ |
| Iodine . . . ......... | $I$ | Tin (stannum).. ..... | $Sn$ |
| Iridium . . ....... . | $Ir$ | Titanium.. . .. ..... | $Ti$ |
| Iron (ferrum) . . .... | $Fe$ | Tungsten (wolfram) .. | $W$ |
| Krypton . .. ..... | $Kr$ | Uranium . ..... .... | $U$ |
| Lanthanum .. .. | $La$ | Vanadium. . . ...... | $V$ |
| Lead (plumbum) .. . . | $Pb$ | Xenon. .............. | $Xe$ |
| Lithium. ... . .... . | $Li$ | Ytterbium.. ........ | $Yb$ |
| Magnesium .... .... | $Mg$ | Yttrium... .. ........ | $Yt$ |
| Manganese .... .. | $Mn$ | Zinc ................ | $Zn$ |
| Mercury (hydrargyrum) | $Hg$ | Zirconium . ........ | $Zr$ |
| Molybdenum... ...... | $Mo$ | | |

curic oxide, are composed of two elements, others contain a greater number, but most compounds are simple, and comparatively few contain more than four or five elements. It is estimated that about 200,000 different chemical compounds are known to chemists; hundreds of new compounds are discovered each year. Even the expert chemist is supposed to be familiar with only comparatively few of these compounds and the requirements of the construction engineer are still more limited in their scope. In the following pages there is given only sufficient information to familiarize the engineer with the chemical terms that he is likely to encounter in his daily work.

**4. Mechanical Mixtures and Chemical Compounds.**—The preceding article states that all known substances are made up of combinations of elements; but it is to be noted that the combinations may be of two kinds: mechanical mixtures and chemical compounds.

If fine copper filings and powdered sulphur are placed together and mixed thoroughly, the mixture will apparently have some properties differing from those of either copper or sulphur; thus, the color is somewhat greenish. If, however, a little of the mixture is examined under the microscope, the particles of copper and sulphur may be distinctly seen lying unaltered, side by side, indicating that the mass is merely a mechanical mixture.

If, now, the proper mixture of copper and sulphur is placed in a test tube and heat is applied, it will soon begin to glow, and it becomes evident that a pronounced change is taking place. If the mixture is removed from the flame it will continue to glow until every particle of it has been changed, the action growing more vigorous as it proceeds. If, when cool, the substance is removed from the tube, instead of being the yellowish-green powdered mixture, it will be found that it is a black, homogeneous, brittle solid. Examined under the microscope, it shows no trace of either copper or sulphur. No physical treatment can separate it into its constituents. In fact, all the properties are changed; a chemical change has taken place, and a new compound, called copper sulphide, has been formed.

From these examples it may be readily seen that a mechanical mixture is a collection of substances, either elements or compounds, which retain their original identity, and which may be separated by mechanical means.  A chemical compound, on the other hand, is made up of elements so combined that each has lost its original identity and cannot be separated except by chemical action.

**5.  Solutions and Mixtures.**—By the term *solution* is generally meant a liquid in which a solid is dissolved.  A solid may be dispersed all through a liquid in a finely divided form, in which case an ordinary suspension of the solid in the liquid results, and if left undisturbed the suspended matter will gradually settle out.  If the subdivision of the solid is so fine as to assume the molecular form, the combination of the solid and the liquid is said to constitute a solution.  A mixture has an opaque appearance and the suspended particles show a tendency to separate and to settle on the bottom of the vessel. In the case of a solution this tendency is absent and the liquid is clear and transparent.  The term solution includes also liquids in which one or more other liquids are dissolved. Gases may also be dissolved in liquids.  Generally, the substance dissolved is known as the **solute** and the liquid in which it is dissolved is known as the **solvent.**

**6.**  It is not necessary at this place to enter into a detailed examination of the nature of a solution, as in some respects the required explanations must be based on several hypotheses. Suffice it to say that in a simple solution, the chemical combination effected between the solvent and the solute is so loose that the latter may be recovered by evaporating the solvent. However, in a compound solution the solvent combines with the solute and forms a new compound of so stable a nature that the solute cannot be recovered by evaporation.

A solution containing only a small amount of the solute is known as a **dilute solution,** while a **concentrated solution** contains a relatively large quantity of the solute.  The total amount of solute that may be dissolved in a given quantity of solvent depends on the nature of both materials as well as

on the temperature of the solvent.   A solution is said to be saturated when the solute is unable to give up any more material to the solvent.   This amount is increased or decreased by increasing or decreasing the temperature of the solvent.

**7.  Symbols and Formulas.**—Each element is indicated by an abbreviation of its full name, called its symbol.   These abbreviations usually consist of the initial letter of the Latin name of the element.   As, however, there are seventy-nine elements, and only twenty-six letters in the alphabet, a large number of the symbols are composed of the initial and another distinctive letter selected from the name.   Thus, it is seen in Table I, which gives the names and symbols of the elements, that the three elements carbon, chlorine, and copper (cuprum) all have names commencing with the letter C; carbon has the letter $C$ for its symbol, while $Cl$ and $Cu$ stand for chlorine and copper, respectively.

It has been previously stated that a molecule is composed of atoms.   However, atoms cannot exist by themselves but must combine with other atoms to form molecules.   Thus, it is believed that the molecule is the smallest particle of matter which can exist as a separate body and that the atom only exists in combination with other atoms.

When several atoms of the same kind are combined to form a molecule, the molecule is that of an element.   When the molecule consists of atoms of different kinds, the molecule is that of a compound.   The symbols for the elements given in Table I represent in each case that of 1 atom.

As each molecule of a compound is a combination of atoms of various elements, the composition of a molecule may conveniently be expressed symbolically by placing side by side the symbols of the constituent elements, provided the molecule contains only one atom of each element.   This combination of symbols is called a *formula*.   Therefore, *a formula is a group of symbols showing the composition of a molecule.*   For instance, common salt consists of one atom of sodium, whose symbol is $Na$, and one atom of chlorine, whose symbol is $Cl$; accordingly, $NaCl$ is written for its formula.

When the molecule of a compound contains more than one atom of any element, the number of atoms of each such element is indicated by a small Arabic numeral written after the respective symbol and partly below the line. For example, a molecule of water contains 2 atoms of hydrogen and 1 atom of oxygen. The formula is therefore written $H_2O$.

The multiplication of a molecule is usually expressed by placing an Arabic numeral in front of the formula; thus, $2H_2O$ means 2 molecules of water.

**8. Chemical Affinity.**—Since little is known about the size, shape, and arrangement of the atoms and molecules it is not possible to state just what takes place when an atom of one element unites with an atom of another element to form a molecule of some compound. It is, however, convenient to assume that atoms have a certain attraction for one another so that, for instance, a magnesium wire remains a magnesium wire by virtue of the attraction between the several magnesium atoms. But when these atoms come in contact with other atoms for which they have an attraction stronger than the one that is holding them together, the stronger attraction prevails. Thus, if magnesium is heated in oxygen, the magnesium atoms separate in spite of their mutual attraction and unite with the oxygen atoms by virtue of the stronger attraction between magnesium atoms and oxygen atoms. This attraction between atoms is called chemical affinity.

**9.** As a rule, the more dissimilar the substances, the stronger their affinity, but if atoms of different kinds are not present, atoms of the same kind combine among themselves. Thus, 2 atoms of the gaseous element hydrogen unite to form 1 gaseous molecule of hydrogen, and 2 atoms of the gaseous element chlorine unite to form 1 gaseous molecule of chlorine. If, however, the hydrogen and the chlorine gases are mixed, and then exposed to the sunlight, the molecules of hydrogen and the molecules of chlorine are broken up, and each atom of hydrogen unites with one atom of chlorine, forming a compound known as hydrochloric acid. This phenomenon is explained by chemists by assuming that under certain circum-

stances the affinity between an atom of hydrogen and an atom of chlorine is greater than the affinity between one atom of hydrogen and another atom of hydrogen or between one atom of chlorine and another atom of chlorine. The circumstances under which *chemical changes* or, more properly, *chemical reactions* take place are varied; heat, light, and electricity are all powerful in causing chemical reactions. Often it is merely necessary to dissolve the substances in liquids and mix the solutions either hot or cold.

**10. Law of Definite Proportions.**—When an experiment is performed, similar to that referred to in Art 4, in which small quantities of copper filings and powdered sulphur are mixed for the purpose of producing copper sulphide, the question naturally arises as to the effect of varying the relative quantities of the two ingredients. That is, will all the copper combine with all the sulphur and will the quantity of copper sulphide produced, in all cases, be equal to the combined weight of the copper and the sulphur, irrespective of the relative proportions of these substances?

Numerous experiments in the whole range of chemistry have proved that this is not the case. Referring to the special example under consideration, experiments show that a given quantity of sulphur requires a definite quantity of copper to produce the copper sulphide. If this quantity is exceeded, the excess of copper remains intact, and if, on the other hand, the quantity of copper is insufficient, a portion of the sulphur is left uncombined. Conversely, it has been found that if the compound copper sulphide is decomposed, the constituents are always present in the same relative proportions. Numerous compounds have been examined by the most refined methods of chemistry, and in no case was any variation found in the relative proportions of the constituents of any one compound. The conclusion drawn from these experiments is expressed in the following statement of fact, known as the **law of definite proportions:**

*In any chemical compound, the proportion, by weight, of the constituents is always the same*

**11. Law of Multiple Proportions.**—According to the law of definite proportions, the ratio of the weights of the constituents in a given compound must always be the same. These same constituents, however, are at liberty to combine in other proportions and form different compounds. For example, copper and oxygen may unite in the ratio 63.6 : 8 and form cuprous oxide; but copper may also unite with oxygen in the ratio 63.6 : 16 and form cupric oxide. Again, in the compound carbon monoxide the ratio of carbon to oxygen is 6 : 8; but in the compound carbon dioxide the ratio is 6 : 16.

It is to be noted that in these examples the weight of oxygen combined with a given weight of carbon is twice as great in the case of carbon dioxide as in the case of carbon monoxide. The same ratio exists between the weights of oxygen that go to produce cupric and cuprous oxides. Other ratios apply to other compounds. For example, oxygen may combine with nitrogen in weights proportional to the numbers 1, 2, 3, 4, and 5, forming in each case a different compound.

From investigating a number of similar cases, Dalton formulated the result of his observations in the following statement, known as the law of multiple proportions:

*When a fixed weight of one element unites with several weights of another element to form definite compounds, these several weights will bear a simple ratio to each other.*

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**12. Metallic and Non-Metallic Elements.**—The elements are usually classified in two groups, one of which contains all the metals, and the other all the elements that are not metal; that is, the elements are classified as *metallic* and *non-metallic*. It is not easy to define the term *metal;* from remote antiquity the elements gold, silver, copper, tin, iron, and lead have been classified as metals and thus has arisen the usual conception of a metal as a hard, heavy, lustrous, malleable, ductile, and tenacious substance. In modern chemistry this classification has, however, been extended so as to include substances that possess only a few of these properties. The

generally accepted chemical definition of a metal is as follows:
*A* metal *is an element which unites with the element oxygen to form a base.*

NOTE—The definition of the term *base* will be given later.

**13.  Periodic System.**—Another method of classification, known as the periodic system, divides all the elements according to their properties into so-called families. This method of classification is of great interest, theoretically, and has led to investigations and discussions of much scientific importance because when the tabulations of the elements were first made, certain vacant places occurred, and it was predicted that not only would elements some time be found to occupy the vacant spaces, but the properties of these unknown elements were foretold with considerable detail. When, later on, certain new elements were discovered it was found that these fitted into the vacant places in the families, and that they actually had the properties foretold. Although these investigations are of great interest to the scientific chemist, they have little practical importance for the construction engineer. In this Section the old classification into non–metallic and metallic elements will be retained, even though less acceptable from a modern point of view.

## NON-METALLIC ELEMENTS

**14.**  Under non-metallic elements will be described the gaseous elements oxygen, hydrogen, chlorine, fluorine, and nitrogen, as well as the solid elements sulphur, phosphorus, carbon, and silicon. This list of non-metallic elements includes only those of importance to the engineer; there are many other non-metallic elements of great importance, both commercially and scientifically, to which no reference is made in this Section.

### GASEOUS NON-METALLIC ELEMENTS

**15.  Specific Gravity of Gases.**—In chemistry, as in physics, the weight of a unit volume of purified and dried air is chosen as the standard of comparison when speaking of the

specific gravity of gases.    Thus, when the specific gravity of chlorine is said to be 2.4910, it means that one unit volume, as 1 cubic foot, of this gas weighs 2.4910 times as much as 1 cubic foot of air under the same pressure and at the same temperature.    In the following pages, wherever the specific gravity of a gas is stated, the specific gravity of air is always referred to as the unit; but where the specific gravity of a liquid or a solid is mentioned, the unit is the specific gravity of water.

**16.    Oxygen,** $O_2$, specific gravity 1 1054, is a transparent, colorless, odorless gas.    The word oxygen is derived from two Greek words that literally mean *acid former*, as it was formerly believed that oxygen was a constituent of all acids. The chemist Davy, however, showed that hydrogen and not oxygen was necessary for an acid.

Oxygen is found in both a combined and a free state.    In the air it is mixed, but not combined, with nitrogen in the proportion of about 21 per cent. of oxygen and 78 per cent. of nitrogen, other constituents of the air being carbon dioxide, aqueous vapor, etc.    Oxygen is an important constituent of all animal and vegetable substances, and is contained in all mineral substances.    In granite, slate, clay, limestone, and other rocks, except coal, the quantity of oxygen is nearly 50 per cent.    It combines with practically all other elements. Water dissolves about 4 per cent. of oxygen.    About 47 per cent. of the earth's crust is composed of oxygen, while it constitutes about 89 per cent. of water.

**17.    Hydrogen,** $H_2$, specific gravity .0695, is the lightest of all known substances.    As one of the two elements constituting water, it is present everywhere in the crust of the earth.    Hydrogen exists in plants and animals, and in numerous minerals, such as petroleum, bitumen, and many others. The amount of water chemically combined in the rocks or mechanically held in their crevices is estimated to exceed many times the amount of water in the oceans.

Hydrogen, in combination with carbon, forms the hydrocarbons found in the rocks that contain the remains of animal

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and plant life, such as the rocks from which petroleum and natural gas are obtained.

Hydrogen, combined with the minerals, forms only .21 per cent. of the crust of the earth. The amount of it present in water, however, increases the importance of this element.

**18. Chlorine,** $Cl_2$, specific gravity 2 4910, is a nearly transparent gas, of a greenish-yellow color. It is never found in a free state, and has a great affinity for hydrogen and most metals, with which it forms so-called *chlorides*, of which sodium chloride (common salt), $NaCl$, and magnesium chloride, $MgCl_2$, are the most common and widely distributed. Sodium and magnesium chlorides are found in sea-water, and all soils of the earth contain some sodium chloride. Chlorine forms about .01 per cent of the material of the entire globe, but about 2 per cent. of the oceans.

**19. Fluorine,** $F_2$, specific gravity 1.31, is a colorless gas very similar to chlorine, and, like it, is never found in a free state, because of its great affinity for other substances; it attacks even glass. In nature it is found widely distributed in the mineral fluorspar, so called because it is used as a flux in many industrial processes. Fluorspar is a compound of fluorine with the metal calcium; the compound is in chemical language termed fluoride of calcium, $CaF_2$.

**20. Nitrogen,** $N_2$, specific gravity .9674, is an odorless, colorless, and tasteless gas. It does not support combustion and is not combustible itself. While it does not exert any poisonous influence on the animal anatomy, animals are quickly suffocated in an atmosphere of pure nitrogen, because it lacks the property of oxygen of supporting life. In its free state, nitrogen is a remarkably inert element; that is, it combines directly with only very few substances. Its compounds, however, are among the most energetic known, including strong acids, violent explosives, and active poisons.

COMPOUNDS OF THE GASEOUS NON-METALLIC ELEMENTS

**21. Water.**—When hydrogen burns in an atmosphere of oxygen, the product is water, whose molecule, $H_2O$, contains 2 atoms of hydrogen and 1 of oxygen. Hydrogen and oxygen, mixed in the proportions of 2 to 1, as, for instance, 2 cubic feet of hydrogen to 1 cubic foot of oxygen, form a highly explosive mixture; if a lighted match is applied to the mixture or an electric spark is passed through it, the gases immediately combine with a violent explosion and form water. Conversely, if an electric current is passed through water, the water is split up into hydrogen and oxygen.

Water occurs abundantly in nature, both in a free state as water, and also in chemical combination with other substances. Many chemical compounds, when dissolved in water and solidified by evaporating the solvent, are found to have united with one or more molecules of water to form new compounds with properties different from those of the original compounds. The compounds so formed are called **hydrates.** If a hydrate is heated it gives off the molecules of water it contains and is thereby changed into a so-called *anhydrous* form, which has the same composition as the hydrate except that it contains no water  The molecules of water entering into the hydrates are sometimes referred to as *water of crystallization*, but it must not be inferred from this name that crystallization is dependent upon the presence of water, since practically all chemical compounds, when solid, and in a stable physical condition, are of a crystalline form. Thus, the mineral gypsum is a hydrate that forms large crystals containing 2 molecules of water; its chemical formula is $CaSO_4 \cdot 2H_2O$. When the water of crystallization is driven off by heat, the crystals disintegrate to a white, chalky powder that apparently is *amorphous,* or non-crystalline, although, upon closer investigation, it is found to consist of minute crystals.

**22. Dehydration.**—The process of driving off the water of crystallization is called dehydration; the fact that dehydration changes a solid body into a powder is of the greatest

importance in the construction of fireproof buildings, because the heat of a conflagration in such a building will naturally dehydrate the outer layers of those materials that contain water of crystallization, and thereby reduce their tensile or compressive strength.   The process of dehydration, however, is one that consumes much heat, and so the dehydration itself tends to keep the materials cooler than they would be otherwise, and prevents the action from penetrating very far from the surface.   Gypsum is much used in plastering walls; plastered walls are therefore not strictly fireproof.   Portland cement and concrete made with Portland cement are also dehydrated to some extent in hot fires, although not as much as gypsum.   In order to prevent excessive dehydration in case of fire, concrete walls and floors are often built somewhat thicker than would otherwise be required.

**23.   Oxidation.**—Many substances burn vigorously when heated in an atmosphere of oxygen.   A metal burning in oxygen unites with the oxygen and forms a so-called **oxide,** but many metals also form oxides slowly by uniting with the oxygen of the air at ordinary temperatures.   The union of a substance with oxygen is called **oxidation,** irrespective of whether it proceeds rapidly or slowly; rapid oxidation is usually accompanied by development of large amounts of heat and light and it is then called **combustion.**

A familiar example of slow oxidation is the rusting of iron in moist air; in the presence of moisture, which usually contains a small amount of acid, a complicated chemical action takes place finally forming an oxide of iron   The oxides of the elements form compounds with hydrogen that contain both oxygen and hydrogen and therefore are called **hydroxides,** or, more commonly, but incorrectly, *hydrates.*

The oxides and hydroxides of the metals are called **bases.** In chemistry, the bases are very important, since they form one large group of compounds that are characterized by their reactions with another large group known as the acids.   The term acid is defined in the following article; suffice it here to say that acids and bases unite to form compounds known as

*salts,* of which gypsum, table salt, and saltpeter furnish familiar examples.

**24. Acids.**—The term acid is familiar, because in everyday language it is applied to everything that is sour.  A number of substances have a sour taste, but to the chemist the sourness of an acid is but an incidental property.   In chemistry, an acid is defined as *a compound containing hydrogen, which may be replaced by the metal of a base to form a salt.*

Acids can be divided into two groups: (*a*) a large group which comprises acids containing oxygen, and (*b*) a smaller group comprising acids that do not contain oxygen.  The acids of group (*a*) are hydroxides of non-metallic elements, just as the bases are hydroxides of metallic elements.  This distinction is so important that in chemistry a non-metallic element is defined as an *acid-forming element,* and a metallic element is defined as a *base-forming element.*

**25. Neutralization.**—A solution of the vegetable dye litmus is blue in color.  Solutions of acids have the power of changing this color to red, and solutions of bases in water have the power of restoring the blue color.  For example, if two or three drops of hydrochloric acid are added to a neutral litmus solution, the color of the solution will be changed to a bright red.  If, now, the base known as sodium hydroxide is dissolved in water and a few drops of this solution are added to the reddened litmus solution, the original blue color will be restored.  By adding acid, the color of the solution may again be changed to red.  When the red color is obtained, it is known as an *acid reaction,* and when the blue color is restored, it is know as an *alkaline reaction.*  If the acid is very carefully added to the blue solution, the blue color will become less distinct at a certain point, and the solution, while still blue, will be found to have a slight reddish tint.  At about this point the solution is said to be *neutral;* that is, it is neither basic nor acid, the base and acid having neutralized, or destroyed, the properties of each other.  This process of destroying basic and acid properties by allowing a base and an acid to act on each other is known as **neutralization.**

**26. Hydrochloric Acid.**—If equal volumes of hydrogen and chlorine are brought together in a jar, there will be no action so long as the mixture is kept in the dark, but if the mixture is exposed to bright light, or if a flame is brought to the mouth of the jar containing it, or if an electric spark is passed through the mixture, the gases will suddenly combine with a loud explosion. The result of this chemical action is that 1 atom of hydrogen unites with 1 atom of chlorine to form the compound $HCl$, which is a colorless, pungent, acid gas called hydrogen chloride. Of this gas, 1 cubic foot of water at 32° F. dissolves more than 500 cubic feet, forming a solution known as hydrochloric acid, which is strongly acid, though the dry gas is neutral. At higher temperatures less gas is dissolved in the water.

The ordinary concentrated hydrochloric acid of commerce contains 38 per cent of $HCl$ dissolved in water and is called **muriatic acid.** The solution has a specific gravity of 1.20, and finds extensive use in the industries. Muriatic acid is recognized by its peculiar odor and also by the dense fumes emitted from the bottle. It is a very strong acid; in common with other strong acids it is violently poisonous and causes deep burns if accidentally spilled on the flesh.

**27. Nitric Acid.**—Nitric acid is a compound of hydrogen, oxygen, and nitrogen; its formula is $HNO_3$, and its specific gravity is 1.42. It is, when pure, a colorless liquid, but usually it has a faint yellow tinge owing to impurities caused by its partial decomposition  It is a fuming, corrosive, and strongly acid liquid with a characteristic odor. Nitric acid has a wide application in the chemical industries. In contact with such vegetable compounds as cotton and sugar it forms violently explosive compounds. All the metals in common use, except gold, platinum, and aluminum, are dissolved by nitric acid. Gold is, however, dissolved by a mixture of hydrochloric and nitric acids known as *aqua regia.*

### SOLID NON-METALLIC ELEMENTS

**28.   Phosphorus,** $P_4$, specific gravity 1.83, is a yellowish, translucent, crystalline, waxy solid and is never found in a free state because of its affinity for oxygen and other elements.   Combined with oxygen it forms the **phosphates,** of which phosphate of lime is the most common.   Its compounds are quite abundant and widely diffused, the principal one being calcium phosphate, which occurs in certain minerals, such as phosphorite and apatite.

Phosphorus, in minute quantities, enters into the bones and tissues of many animals and the tissues of many plants.   It constitutes only about .1 per cent. of the crust of the earth.

**29.   Sulphur,** $S$, specific gravity 2.046, is found in a free state in the neighborhood of volcanoes.   It occurs also in combination with many metals, as in iron pyrites, $FeS_2$, copper pyrites, $FeCuS_2$, and in combination with metals and oxygen as calcium sulphate, or gypsum, $CaSO_4 \cdot 2H_2O$.

Sulphur is found in the tissues of many animals and plants, and in the rocks in small quantities.   It forms only about .03 per cent of the crust of the earth.   Sulphur is a lemon-yellow, brittle, crystalline solid that melts at 114.5° C. to a limpid yellow liquid.

**30.   Carbon,** $C$, is known in three entirely dissimilar forms:   (1) *diamond,* having a specific gravity of 3.5; (2) *graphite,* having a specific gravity of 2.5; (3) *amorphous carbon,* having a specific gravity of about 1.75.   That these are different forms of one and the same element can be shown by burning the same quantity of each in oxygen; the same amount of a gas, called carbon dioxide, $CO_2$, will always be produced.   If, however, the same weight of each form of carbon is burned, a different amount of heat is evolved, and from this fact it is concluded that the three forms of carbon differ from one another by being possessed of different amounts of energy.

The diamond is the hardest of all known substances; a diamond will scratch all other substances, even hardened steel,

without itself being scratched. Diamonds are therefore employed in the industries for cutting glass and as cutting edges on certain kinds of drilling instruments used for boring deep wells. The diamonds so used are smaller and much less expensive than those used as gems.

Graphite is much softer than the diamond. The finer grades found in nature are used in the manufacture of lead pencils; the coarser grades of graphite are used for making the melting pots known as crucibles, which are employed in chemical laboratories. An interesting use of graphite in mechanical engineering is as a lubricant, either alone or mixed with grease. Grease is a carbon compound to which reference is made later.

Amorphous carbon forms an important part of lampblack, charcoal, coal, and coke. Of these, only coal will be considered here.

**31.   Coal** is classified as *anthracite, bituminous,* and *lignite,* according to its properties. All vegetable matter contains a large proportion of carbon. In past ages huge quantities of various forms of vegetation were in some unknown manner brought together in strata where their carbon contents still remain in the form of coal.

Anthracite coal is a black, hard, somewhat lustrous substance and has entirely lost its vegetable structure; lignite, also called *brown coal,* consists of ancient deposits of vegetation that have scarcely been changed. There is a plainly marked gradation from anthracite at one end of the scale to lignite at the other; bituminous coal occupies an intermediate position. Anthracite and bituminous coal are often called *hard coal* and *soft coal,* respectively. Hard coals are clean to handle, burn without smoke, and form but little soot. Soft coals are easily crumbled, causing much black dust in handling; they burn with much smoke and form a great deal of soot.

**32.** The carbon in coal is present in two different forms, namely, *fixed carbon* and *volatile carbon compounds.* **Fixed carbon** is carbon not combined with any other element; **volatile carbon compounds** are gaseous compounds that

can be liberated by heating the coal. The volatile compounds can be separated by distillation. In the distillation of coal, the latter is heated in a vessel called a retort; the retort is closed so as to exclude the air, as otherwise the coal would burn. From the retort issues a pipe that carries the volatile matter away; in the retort remains a mass of fixed carbon, known as **coke.** The coke burns with a steady glow until the whole of the carbon is destroyed, leaving a residue known as **ash,** which consists of the mineral substances present in the coal; the volatile matter is known as **gas.** This gas, after purification and cooling, is used for illuminating and for other purposes. The liquids obtained by condensation are water and so-called **coal tar,** a black, viscous, heavy liquid to which reference is made later.

**33.** **Silicon,** *Si*, has a specific gravity of 2.49. It can be produced in the chemical laboratory both as a crystalline body and as an amorphous powder, but in both forms it is merely a chemical curiosity and of no practical utility. In nature, however, its compounds with oxygen are widely distributed, forming such important substances as quartz and flint, both of which are common minerals. Silicon also forms compounds containing oxygen combined with such metals as potassium and aluminum, and these compounds form some of the most common and widely distributed rock formations that compose the solid crust of the earth.

Under the influence of air and water the rocks of the earth continually disintegrate, forming clay and sand. Clay is the product of weathering of rocks containing combinations of silicon and aluminum known as alumino-silicates, of which many are known; they are all of a very complex chemical composition. Geologically, these compounds are known as feldspars. Sand is also a product of weathering of rocks, but, being formed of harder particles, the grains remain larger than those that compose clay; clay is an extremely fine granular mass. Sand may consist of small pieces of feldspar, often mixed with small pieces of quartz, or it may consist entirely of small pieces of quartz, in which case it is called silica sand.

Silicon compounds form our most common building materials; many natural building stones, as well as sand, bricks, and cement, are silicon compounds.

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**OXIDES OF SOLID NON-METALLIC ELEMENTS**

**34. Nomenclature of Oxides.**—Oxygen unites with other elements to form the so-called oxides, as already stated. Many elements form more than one oxide; thus, one oxide may contain only 1 atom of oxygen in the molecule, another 2 atoms of oxygen, a third 3 atoms, and so forth. These oxides are distinguished from one another by placing such Greek prefixes as *mono* (1 atom), *di* (2 atoms), or *tri* (3 atoms) before the word oxide, so that various oxides become known as *monoxide, dioxide,* and *trioxide.* Thus, sulphur combines with oxygen to form a number of different oxides of which sulphur dioxide, $SO_2$, and sulphur trioxide, $SO_3$ are the most important.

**35. Sulphur dioxide,** $SO_2$, is formed when sulphur burns in the air, 1 atom of sulphur uniting with 2 atoms of oxygen taken from the air. It is a colorless gas with a very disagreeable smell; it is poisonous to animals and especially to vegetation. Coal often contains sulphur; when this coal is burned, sulphur dioxide is formed, which not only is fatal to the surrounding vegetation, but also injures the iron of boilers and grates, because compounds containing iron and sulphur are formed that lack entirely the strength of the iron. Sulphur dioxide is a powerful bleaching agent and is used for this purpose in the industries, for instance, in bleaching straw hats   The effect is, however, not permanent, since the original colors are gradually restored under the influence of the oxygen of the atmosphere, which accounts for the fact that straw hats bleached white by sulphur dioxide slowly return to the original yellow color of the straw.

**36. Sulphur trioxide,** $SO_3$, is formed in small quantities when sulphur dioxide, $SO_2$, is made by burning sulphur in air. It can be obtained by combining sulphur dioxide with

oxygen at a certain temperature and under favorable conditions. It has the remarkable property that it can exist at the same temperature in two distinct forms, one being a crystalline solid and the other a liquid.

Sulphur trioxide combines with water to form a compound known as sulphuric acid, $H_2SO_4$, one molecule of $SO_3$ uniting with one molecule of $H_2O$. Sulphur trioxide can therefore be considered as sulphuric acid less its water, for which reason sulphur trioxide is often called the anhydride of sulphuric acid. If Portland cement is analyzed it is found to contain a small quantity of sulphuric acid combined with other substances; the precise composition of these compounds will not be considered here. For the present purpose it is sufficient to state that a chemical analysis of Portland cement is often furnished to the construction engineer by a chemist who makes a specialty of such work; on this report will be found a statement of the quantity of sulphuric anhydride, $SO_3$, contained in the cement. The words *sulphur oxide* or *sulphur trioxide* are also sometimes used instead of sulphuric anhydride, and to such use there can be no objection, since these terms mean one and the same thing. The amount of sulphuric anhydride contained in good Portland cement is always a small quantity, not exceeding 2 per cent. by weight of the total weight of the cement.

**37. Sulphuric acid,** $H_2SO_4$, is a dense, colorless liquid of oily consistency, having a specific gravity of 1.84 at 15° C. It is characterized by its strong attraction for water, with which it mixes accompanied by the evolution of great heat. For this reason water must never be poured into the acid; in mixing the two, small quantities of acid are gradually poured into the water so that the process is always under perfect control, or otherwise a violent boiling, similar to an explosion, will occur. Since sulphuric acid is destructive to all organic substances, an explosion of sulphuric acid must by all means be avoided; the flying acid will burn the flesh, possibly blind the operator, and not infrequently set the building on fire.

Even when greatly diluted by water, sulphuric acid destroys textile fabrics very rapidly, because the water will evaporate, leaving behind an acid strong enough to destroy the fiber of the fabric. Sulphuric acid must therefore never come in contact with any ropes, cables, or hoisting apparatus used on the works.

Sulphuric acid unites with most of the bases; it dissolves many metals in a cold state and most of the other metals when applied hot. It finds extensive use in all the industries and it is, after water, the most commonly used liquid.

**38. Various Carbon Oxides.**—Oxygen forms two compounds with carbon, known as *carbon monoxide, CO,* and *carbon dioxide, $CO_2$.* They are both obtained by burning carbon in air; if insufficient oxygen is furnished, the monoxide is formed, and if a surplus of oxygen is present the dioxide is formed. They are both colorless gases. **Carbon monoxide,** if pure, is odorless and is extremely poisonous, producing a painful headache even when present in small quantities in the air; 1 per cent. of it in the air is said to prove fatal. It burns in the air with a characteristic soft blue flame, which may be observed hovering over any coke or charcoal fire or over any clear and smokeless coal flame.

**39. Carbon dioxide,** sometimes called *carbonic acid gas,* is formed by the complete combustion of carbon, and it is therefore a product of all ordinary fires. It is a tasteless, odorless gas, heavier than air. It is very injurious when inhaled, because it prevents the escape of the carbonic acid gas discharged by the lungs and the required absorption of oxygen from the air. A small quantity is constantly present in the air, and, since all animals inhale the oxygen of the atmosphere and exhale carbon dioxide, its quantity may become too great for comfort in poorly ventilated rooms containing many people.

**40. Carbonic Acid.**—Carbon dioxide is soluble in water, the solution being known as carbonic acid, $H_2CO_3$, which is formed by adding one molecule of water, $H_2O$, to one molecule of the gas, $CO_2$; carbon dioxide is therefore sometimes

called *carbonic anhydride.* Carbonic acid is the common so-called soda-water having a familiar refreshing taste. The colder the water is, the greater is the volume of gas it will dissolve; heating the water drives off the gas, and boiling removes the carbon dioxide entirely from the solution. Carbonic acid is a very weak acid, having but little effect upon metals or other substances, except the metal oxides (bases) already referred to. With these it forms salts called *carbonates;* a familiar example is bicarbonate of soda, $NaHCO_3$. When baking powder, which is prepared from bicarbonate of soda, is mixed with bread dough and heated, carbon dioxide and water are given off, leaving carbonate of soda, $Na_2CO_3$; the carbon dioxide, being a gas, expands and thus raises the bread dough. The carbonates constitute a large portion of the crust of the earth; marble, limestone, and chalk consist of calcium carbonate, $CaCO_3$.

**41. Silicates.**—Oxygen forms with silicon but a single compound, silicon dioxide, $SiO_2$, also called *silica,* which is a very hard substance found in nature as quartz, flint, and sand. Silica is the anhydride of silicic acid, $H_2SiO_3$, which is a gelatinous, that is, glue-like, mass. The acid is not of importance to the construction engineer, but the salts of the acid, the so-called silicates, are of very great importance, because, as already stated, the silicates form the mineral known as feldspar, from which clay and sand have been formed in past ages.

**42. Cement.**—Certain silicates are of especial interest to the construction engineer because it is to them that cement owes its useful property. *Cement* is a grayish powder that, when mixed with water, hardens to a stonelike, very strong substance. Cement is used to bind sand and stone together to form concrete; the strength and utility of the concrete depends largely upon the binding power of the cement. Several kinds of cement are sold and used in construction work, but the following remarks are applicable to only one kind of cement, namely, *Portland cement,* which is by far the most important variety.

Portland cement is manufactured by mixing proper proportions of limestone and clay, heating (*burning*, as it is called) the mixture, and then grinding the product to a fine powder. Limestone is carbonate of calcium, $CaCO_3$, and clay is a complex combination of silicon, aluminum, and oxygen, in connection also with other elements of less importance. When the mixture of limestone and clay is burned, a product is formed containing (1) compounds of silica and calcium, called *calcium silicates;* (2) a compound of aluminum, oxygen, and calcium, called *calcium aluminate;* (3) a compound of calcium and oxygen called *quicklime,* or *calcium oxide;* and (4) a number of other compounds and ingredients of less importance, and not considered here.

**43.** The calcium silicates in Portland cement are of two kinds, namely, *tricalcium silicate* and *dicalcium silicate,* so called because the former contains 3 molecules and the latter 2 molecules of calcium oxide. Since calcium oxide is written $CaO$, 2 molecules of calcium oxide is written $2CaO$, and 3 molecules is written $3CaO$. The anhydride of silicic acid is written $SiO_2$; hence tricalcium silicate has the formula $3CaO \cdot SiO_2$. Dicalcium silicate, similarly, has the formula $2CaO \cdot SiO_2$. Obviously the tricalcium silicate is formed from the dicalcium silicate by the addition of 1 molecule of calcium oxide, $CaO$, changing $2CaO$ to $3CaO$; therefore, perfectly burned cement contains less $CaO$ and $2CaO \cdot SiO_2$, and more $3CaO \cdot SiO_2$. This is important because the tricalcium silicate gives the cement its cohesive strength; when mixed with water it hydrates and forms minute crystals that harden slowly and attain great strength. The dicalcium silicate also hydrates and forms crystals but the process is very slow and the resulting substance is not so strong.

The calcium aluminate already referred to has the formula $3CaO \cdot Al_2O_3$; this compound both hydrates and hardens quickly, but attains little strength. The *free lime, CaO,* will in time absorb carbon dioxide, $CO_2$, from the atmosphere and form calcium carbonate, $CaCO_3$, but the process is so slow,

the quantity involved so small, and the resulting strength so low that this compound has little effect upon the strength of the cement.

**44. Set of Cement.**—To summarize, the most important facts already stated in regard to the setting of cement when mixed with water are: The calcium aluminate, $3CaO \cdot Al_2O_3$, hydrates as well as hardens quickly; this hardening is called the **initial set** of the cement and results in a stiffening of the mass, but gives only a very low strength. This action may be completed in less than 1 hour. Next, the tricalcium silicate, $3CaO \cdot SiO_2$, hydrates and hardens; the beginning of this hardening is called the **final set** of the cement. The final set takes place in less than 10 hours, but the hardening goes on for a long time, so that the cement continues to gain in strength for weeks or months, aided probably to a considerable extent by the slow hydration and hardening of the dicalcium silicate, $2CaO \cdot SiO_2$.

**45.** The methods employed in discovering the facts concerning the composition of cement and the action taking place during the period of setting are extremely complicated and difficult, calling for the highest degree of chemical skill. Only the best equipped laboratories, such as that of the United States Bureau of Standards, can therefore undertake investigations of this kind. The construction engineer is sometimes interested in having cement submitted to a chemical analysis, and when this need arises a sample of the cement is sent to a laboratory for the purpose of determining, among other things, the total amount of silica, $SiO_2$, and alumina, $Al_2O_3$, present in the cement. However, for the reasons already given, the chemist cannot be called upon to indicate how these compounds are combined in silicates and aluminates. Besides, this information would not serve any particularly useful purpose, even if it were obtainable, since the required knowledge for predicting the strength of cement from its chemical analysis is lacking. Moreover, it is possible to obtain the strength of any given cement by submitting samples to simple strength tests.

**46. Hydrocarbons.**—The elements hydrogen, $H$, and carbon, $C$, form what are known as **carbon compounds.** The number and importance of these compounds is so great that a special branch of the chemical science, known as organic chemistry, has been developed to treat of them, as already stated   To give even a brief and elementary description of the most important of these compounds and their chemical composition would require a thick volume.   Hence, the succeeding descriptions will be limited to those hydrocarbons that are of special interest to the construction engineer.

The hydrocarbons used in construction work are employed chiefly in road construction and in waterproofing; the most important are collectively known as bitumens.   A **bitumen** is a mixture of hydrocarbon compounds, either found as such in nature or artificially prepared by the distillation of bituminous coal.   The mixtures of hydrocarbons in bitumen are so complex, and the various compounds contained in the mixtures are so numerous that it is most difficult to separate them into individual compounds; the practical chemist engaged in the examination of bitumens usually limits himself to the study of certain series or families of hydrocarbons contained in bitumen; all the members of any such family have many properties in common, so that the predominance of one family over another largely determines the general properties of the mixture.

According to the predominating properties, bitumens are classified as *paraffins* and *asphalts*.   The physical difference between these is that paraffin is greasy and asphalt is sticky. The asphaltic bitumens are widely used in roadbuilding, since they have the property of binding the grains of sand and stone together to form a solid body capable of supporting the traffic; the paraffins are used in engineering work chiefly as waterproofing agents.

**47.**   The bitumens occurring in nature are gases, liquids, and solids.   Frequently all three forms are found mixed or

dissolved in one another, forming a heavy, dark, yellowish-green liquid known as *crude oil*, from which the gaseous hydrocarbons readily escape under ordinary conditions of pressure and temperature. When crude oil is distilled by being heated to between 40° C. and 150° C. the so-called *low-boiling products* evaporate and can be collected by condensation; the liquid thus collected is known as **gasoline.** At a temperature between 150° C. and 300° C. the liquid known as **kerosene** is obtained. The products remaining after the temperature has reached 300° C. are so-called *high-boiling products*. If the crude oil is of the paraffin class, heavy lubricating oils, greases, and paraffin wax remain, but if the crude oil is of the asphalt class, a black mass consisting of solid hydrocarbons remains; this mass is known as **asphalt.** From this explanation it will be understood that no distinctive boundary line exists between gasoline and kerosene, except the arbitrarily fixed boiling point of 150° C ; both liquids are more or less indefinite mixtures of many chemical compounds, and have no special definite chemical formulas of their own. This is true also of the paraffins and asphalts that remain after the more volatile products have been removed by distillation. Asphalt is, however, not always obtained by distillation, since it is found in solid deposits of more or less pure hydrocarbons; asphalts from such solid deposits are collectively known as *native asphalts*.

**48.** The chemical composition of the hydrocarbons is very interesting; the compounds of the paraffin series may serve as an example. The names and formulas of these compounds are given in Table II. It will be noticed that each succeeding compound is formed from the preceding one by adding 1 atom of carbon and 2 atoms of hydrogen, so that the difference between succeeding compounds in the table is always expressed by the quantity $CH_2$.

The greater the number of atoms of carbon and hydrogen that are present, the higher becomes the boiling point  The simplest compounds are gases, then follow liquids and the more complex compounds; the latter are all solids, as is plainly

indicated by the boiling points given in the fourth column. When the boiling points are given for the more simple compounds, which at ordinary temperatures exist in a gaseous form, it is to be understood that these gases at lower temperatures assume a fluid state and that the boiling temperatures refer to the fluids.

**49.**   In organic chemistry there are many series similar to that given in Table II.   Whenever the compounds of a

### TABLE II
#### COMPOUNDS OF PARAFFIN SERIES

| Name of Compound | Formula | Characteristic | Boiling Point Degrees C. |
|---|---|---|---|
| Methane .......... | $CH_4$ | Gas | $-$ 160.0 |
| Ethane ........... | $C_2H_6$ | Gas | $-$ 93.0 |
| Propane .......... | $C_3H_8$ | Gas | $-$ 45.0 |
| Butane ........... | $C_4H_{10}$ | Gas | 1.0 |
| Pentane .......... | $C_5H_{12}$ | Liquid | 36.4 |
| Hexane .......... | $C_6H_{14}$ | Liquid | 68.9 |
| Heptane .......... | $C_7H_{16}$ | Liquid | 98.4 |
| Octane ........... | $C_8H_{18}$ | Liquid | 125.6 |
| Nonane .......... | $C_9H_{20}$ | Liquid | 149.5 |
| Decane ........... | $C_{10}H_{22}$ | Liquid | 173.0 |
| Undecane ......... | $C_{11}H_{24}$ | Liquid | 195.0 |
| Dodecane ......... | $C_{12}H_{26}$ | Solid | 214.0 |
| to |  |  |  |
| Hexacontane ...... | $C_{60}H_{122}$ | Solid |  |

series show a great resemblance in their chemical properties and have a constant difference, similar to $CH_2$, between consecutive members, the series is said to be *homologous.*   One such series is obtained by fractional distillation of the substance coal tar, which is referred to in the succeeding article.

In fractional distillation a thermometer is inserted in the vessel where the evaporation is carried on.   As the temperature increases one component after another will be vaporized.

By changing the receiver when certain temperatures are reached, it is possible to obtain a series of liquids separated according to their boiling points.

**50. Coal tar** is a thick, black liquid obtained as a by-product in the manufacture of illuminating gas from bituminous coal; the black color is due to suspended particles of carbon. The coal is heated in retorts, as already explained; the gas obtained is cooled, and one of the condensation products is coal tar. This liquid coal tar contains many different hydrocarbons, some of which are extremely valuable; these are removed by fractional distillation of the tar, and there remains in the retort used for the distillation a heavy, black liquid that solidifies when cooled. This substance is called **coal-tar pitch.** The properties of this pitch vary according to the methods and temperature used in the fractional distillation. The best variety is known as *straight-run coal-tar pitch,* which is pitch without the addition of foreign substances, as distinguished from pitch thinned with petroleum or other crude-oil products; pitch so thinned is said to be *fluxed* and the oil used for thinning is called the *flux.* Coal-tar pitch is extensively used for waterproofing purposes and for road construction of the kind known as **tar macadam,** which consists of stone and sand mixed with coal-tar pitch.

**51.** One important property common to all bitumens is that they are soluble in carbon disulphide. **Carbon disulphide,** $CS_2$, is a colorless, mobile liquid that can be formed by passing sulphur vapor over red-hot charcoal; when pure, it has an agreeable aromatic odor but the ordinary commercial carbon disulphide has a most disagreeable and rancid smell due to various impurities. It must be handled with care, since it is very volatile, boiling at 46° C.; the fumes are injurious to health if constantly inhaled, even in small quantities, and they form with air a highly inflammable and dangerous mixture.

Carbon disulphide, or, as it is sometimes called, carbon bisulphide, has remarkable power to dissolve such substances as fats and greases, which consist of hydrocarbons, to which class bitumens also belong. However, the impurities con-

tained in bitumens are not soluble in carbon disulphide and the purity of a sample of bitumen can therefore be tested by dissolving the sample in carbon disulphide; the undissolved residue consists of undesirable impurities.

**52.** It has already been stated that the compounds of the paraffin group form a homologous series in which the formula of each compound is obtained by adding $CH_2$ to the formula of the preceding one. If in each of the formulas thus obtained, 1 atom of hydrogen is replaced by the molecule $CO_2H$, there results a homologous series of fatty substances. The members are collectively known as **fatty acids,** because like an inorganic acid they have the characteristic property of uniting with a base to form a salt. One of these organic acids, acetic acid, is contained in the vinegar used on the family table; its formula is $CH_3(COOH)$. This formula is given in the way in which it is usually written by chemists; why it is written in this manner cannot be explained here, as it will involve certain theories that lie outside the scope of this discussion. For ordinary purposes the formula may as well be written $C_2O_2H_4$. Vinegar is a liquid, and so are the members closest to it in the scale, but as more of the molecules $CH_2$ are added, the members become more solid. Thus, by adding 16 times $CH_2$ there is obtained a solid substance called stearic acid, $C_{17}H_{85}(COOH)$, which occurs in tallow; stearic acid is of interest since it is sometimes used in the waterproofing of concrete, in the form of a powder to be mixed with cement.

**53.   Glycerine.**—Just as there are organic acids having properties somewhat similar to inorganic acids, so there are organic bases having properties similar to the inorganic bases; of these only glycerine, $C_3H_5(OH_3)$, will here be mentioned. It is a clear, colorless, sirupy liquid having a remarkable attraction for water, and is usually found chemically combined with stearic acid. When so combined it is important to remove the glycerine from the stearic acid before it is used in waterproofing, since otherwise the glycerine will attract water, thereby defeating the purpose of the waterproofing. The

powder referred to in the preceding article is a patented preparation used in waterproofing which is obtained by removing the glycerine from the stearic acid.

**54. Acetylene.**—In addition to the compounds already described, there are many other hydrocarbons of great importance chemically and industrially, but of these only one will be described here. Acetylene, $C_2H_2$, is a hydrocarbon that can be prepared by direct union of the elements carbon and hydrogen, as pure hydrogen unites directly with carbon at temperatures exceeding 1,100° C. It is a colorless gas of a pleasant odor when pure, but, owing to impurities, it usually has an offensive smell reminding one of decaying onions; care should be taken, when this odor is noticed, to avoid inhaling the gas, since acetylene is poisonous. Since it is combustible and burns with an extremely bright flame when issuing from a special burner provided with a very small opening, it is much used for illuminating purposes on construction work carried on at night. It is obtainable compressed in steel cylinders, but can be made cheaply on the works by the action of water on calcium carbide, $CaC_2$. One molecule of calcium carbide combines with 2 molecules of water to form 1 molecule of acetylene, $C_2H_2$, and 1 molecule of calcium hydroxide, $Ca(OH)_2$. The calcium carbide used is a compound of calcium and carbon, obtained by heating lime and coke in an electric furnace. It is a hard, brittle, crystalline solid having a specific gravity of 2.2, white when pure, but usually discolored by impurities to a dark gray or bronze color. It is sold packed in tin cans in order to protect it against the moisture of the atmosphere.

**55.** An important use of acetylene is in the oxyacetylene welding torch, a device used for producing a very high temperature by burning acetylene in oxygen. The acetylene and oxygen issue under pressure from a special burner provided with minute openings; when ignited, the flame of the mixture reaches a temperature of from 2,400° to 3,000° C. This temperature is high enough to melt iron and steel rapidly, and the flame can therefore be used in cutting steel plates and rods in

places where other means cannot be used conveniently. It can also be used in welding (that is, uniting metals by means of heat); ordinarily, welding of iron is a difficult process, because the iron unites with the oxygen of the atmosphere to form an iron oxide entirely lacking in the essential properties of iron. The result is that the welded places are very much weaker than the original iron   When the oxyacetylene flame is employed, the acetylene absorbs all the oxygen by uniting therewith, thus preventing the formation of iron oxide and allowing an intimate contact of chemically pure metallic surfaces, which is an essential requirement for the production of a strong welded joint.

Acetylene is not the only gas that can be used in blowpipe welding and cutting; hydrogen is sometimes used instead in the so-called oxyhydrogen blowpipe, in which hydrogen burns in oxygen. The temperature of the oxyhydrogen flame is, however, lower than that of the oxyacetylene flame, reaching about 2,000° C.

**56.** As already stated, when hydrogen burns in oxygen, water is formed. When acetylene, $C_2H_2$, burns in oxygen the carbon unites with oxygen to form carbon monoxide, $CO$, which again unites with more oxygen to form carbon dioxide, $CO_2$. The hydrogen contained in the acetylene combines as usual with the oxygen to form water. This reaction is characteristic of all hydrocarbons; when they burn with sufficient oxygen, the final products are always carbon dioxide and water, but if there is not sufficient oxygen, carbon monoxide may be formed, and also free carbon may be liberated. Carbon liberated in this manner is the familiar smoke and soot issuing from poorly constructed chimneys, and from poorly managed fires.

## METALLIC ELEMENTS

**57. Classification.**—The elements generally known as metals, such as copper, lead, and iron, are not the only elements so classified by the chemist. Chemistry includes as metals the so-called alkaline elements, the earth elements, and alkaline-earth elements, occupying an intermediate position. For the purpose of the following descriptions it will, therefore, be convenient to classify the metals as (1) *alkalies,* (2) *alkaline-earth metals* and *earth metals,* and (3) *heavy metals,* the last group including all those elements ordinarily called metals, and which are especially distinguished from the two first groups by having a higher specific gravity and a more metallic appearance. It must, however, not be inferred that this classification is either scientific or exhaustive; in more complete textbooks on chemistry it is customary to classify the elements, whether metallic or non-metallic, according to the periodic system already referred to. This classification is, however, not suitable for the purpose in view, which is that of describing only a few elements of special interest to the engineer.

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### ALKALIES

**58. General Characteristics.**—The alkalies are soft metals having a silvery luster and a specific gravity very nearly equal to that of water or even lower. They are easily melted and volatilized. Their hydroxides—that is, compounds with hydrogen and oxygen—are soluble in water and form the strongest bases known, uniting with all acids to form salts that can be decomposed only with difficulty. The two most commonly occurring elements of this group are potassium and sodium, neither of which occurs in a free state in nature, since they decompose water violently, uniting with the oxygen and liberating the hydrogen, which generally takes fire. They are both lighter than water; a small piece of either element will therefore float upon the surface of the water, darting to and fro while burning with almost explosive violence. Since they

absorb moisture and oxygen from the atmosphere, they must be stored under naphtha or petroleum.

**59. Potassium,** $K$, occurs in nature in the salt potassium nitrate, $KNO_3$, also known as saltpeter, which is widely used in the industries. Another important potassium salt is the carbonate, $K_2CO_3$, used industrially under the name *potash*. Both saltpeter and potash are of great importance in farming, since they form indispensable ingredients of most commercial fertilizers. A solution of potash is called *lye;* aside from many important commercial uses, lye, in combination with an aluminum compound known as *alum*, is sometimes used in waterproofing concrete surfaces, and in the purification of water.

In speaking about potash and potash solutions it is important to remember that potash is a carbonate, $K_2CO_3$; it must not be confused with the so-called caustic potash, which is potassium hydroxide, $KOH$.

**60. Sodium,** $Na$, occurs in nature in enormous quantities as sodium chloride, $NaCl$, which is the common table salt, called *sea salt* when it is derived from evaporation of sea-water. Sodium chloride occurs also in large deposits in the form of *rock salt*, which, when clean, consists of transparent crystals. Sodium is also found in nature in large masses of sodium nitrate, $NaNO_3$, sometimes called *Chile saltpeter* to distinguish it from ordinary saltpeter, which is $KNO_3$.

**61. Sodium oxide,** $Na_2O$, is a white, fusible substance that unites with water to form sodium hydroxide, $NaOH$, commonly called *caustic soda*. Caustic soda unites with carbonic acid to form sodium carbonate, $Na_2CO_3$, which is one of the most commonly used salts; it is commercially known as *soda*, or *soda ash*. This salt again unites with carbon dioxide to form bicarbonate of soda (or sodium bicarbonate), $NaHCO_3$, which product is used as *baking soda*.

**62. Soap.**—The alkaline bases caustic potash and caustic soda unite not only with inorganic acids but also with organic acids. The salt of an alkaline base and a fatty acid is called

soap. There are two kinds of soap, namely, *soft soap*, which is a compound of a fatty acid and caustic potash, and *hard soap*, which is a compound of a fatty acid and caustic soda. They are both made by boiling fat with the alkaline base dissolved in water; this process is called saponification.

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### ALKALINE-EARTH METALS AND EARTH METALS

**63. Classification.**—The term *earth*, as used in chemistry, refers to those compounds that constitute the crust of the globe; these compounds nearly all contain the element aluminum, which, being a metal, is therefore classified as an earth metal. Several other elements belong to this group but, since these are comparatively rare and not used in construction work, no reference will be made to them here. Although the earth metals differ in many respects from the alkalies, there are certain points of similarity. These are especially noticeable in a group of metals that seems to bridge the gap between the earth metals and the alkalies and are consequently known as *alkaline-earth metals*.

Many of the alkaline-earth metals are comparatively rare, and only two, namely, calcium and magnesium, are of special interest to the engineer. In the following articles reference will be made, first to calcium and its compounds, since calcium is closely similar to potassium and sodium; next to magnesium, which closely resembles calcium; and finally to aluminum and its compounds, since these are more distantly related to the alkalies.

**64. Calcium,** *Ca*, is a light metal having a specific gravity of only 1.54. It has a light-yellow, brilliant color, is about as hard as gold, and is very ductile and malleable. In perfectly dry air it is not changed, but where there is a trace of moisture present it oxidizes slowly, and therefore it cannot be stored where the atmospheric air has access to it. The metal itself is not employed to any extent in the useful arts, since it decomposes water, uniting with the oxygen while hydrogen is being liberated; but its compounds are of great

importance, since *lime*, which is the oxide, *gypsum*, which is the sulphate, and *limestone*, which is the carbonate, are all much used in the building trades. Calcium is found as a carbonate in many minerals in nature, such as limestone, chalk, and marble, and in many combinations containing phosphorus, fluorine, and silicon. It is an important constituent part of the bones of animals, shells of eggs and of oysters, and in nearly all spring and river waters.

**65. Calcium oxide,** $CaO$, is a white, hard, amorphous substance having a specific gravity of about 3. It has a great affinity for water, which it attracts from the atmosphere, forming therewith calcium hydroxide, $Ca(OH)_2$. This hydroxide is a soft, white, bulky powder, slightly soluble in water. The solution is a feebly alkaline liquid known as *lime water*. Lime water has a great affinity for carbon dioxide, $CO_2$. When a small quantity of carbon dioxide comes in contact with lime water, calcium carbonate, $CaCO_3$, is instantly formed, and remains suspended in the lime water, thereby giving to the formerly clear solution a cloudy, milky appearance; lime water is thus useful in ascertaining the presence or absence of carbon dioxide, since the solution becomes milky when the gas is present.

**66. Lime and Mortar.**—Commercially, the processes just referred to are of the greatest importance. Limestone, that is, carbonate of lime, $CaCO_3$, is heated on a large scale in a so-called kiln, or lime kiln. This process of heating is in chemistry called *ignition* and in lime-manufacturing *calcination* or *burning*, although in point of fact it is the fuel used for heating that is ignited or burned, and not the limestone. When a sufficiently high temperature has been reached (about 1,500° F.), the molecules of $CaCO_3$ split into molecules of calcium oxide, $CaO$, and carbon dioxide, $CO_2$; the carbon dioxide, being a gas, escapes and there remains only the oxide. This oxide is known as quicklime, which is ordinarily sold in a white or gray lumpy form. Before using, the quicklime is *slaked*, that is, mixed with water, converting it into the hydroxide $Ca(OH)_2$, which is called *slaked lime;* slaked lime prepared

by special machinery in a particularly careful manner is commercially known as *hydrated lime*.  Slaked, or hydrated, lime, mixed with sand, is called *lime mortar;* lime mortar forms the binding material used between brick and stone in the ordinary brickwork used in residences and small houses.  Exposed to the elements, the slaked lime absorbs carbon dioxide from the atmosphere and is thereby gradually transformed into an artificial limestone constituting the hard mortar upon which the strength of the masonry depends.  The hardened mortar is, in fact, an intimate mixture of $CaCO_3$ (that is, artificial limestone) with sand; the sand is merely a filler and takes no part in the chemical process of hardening.

**67.  Calcium sulphate,** $CaSO_4$, occurs in nature in combination with 2 molecules of water as the compound $CaSO_4 \cdot 2H_2O$, called gypsum.  When gypsum is heated to above 200° C. it loses the 2 molecules of water, passing into the anhydrous form $CaSO_4$.  If, however, gypsum is heated to only 120° C., it loses but part of the water, forming a compound having the formula $(2CaSO_4)H_2O$, and known as *plaster of Paris*.  Plaster of Paris has the important property that, when mixed with about one-third of its own weight of water, it forms a white plastic mass which in a few minutes hardens into a porous, hard body.  In hardening it expands slightly and it is therefore well adapted for casting, since the expansion causes it to fill the mold completely.  Plaster of Paris is, however, not only used for casting but also for wall plaster, often in combination with other substances such as glue, table salt, or alum.  Alum increases the hardness of the plaster; a mixture of plaster of Paris and alum in certain proportions is called *Keene's cement*.  The mixture with glue is called *stucco*.  If heated above 200° C. the plaster of Paris loses its ability to harden when mixed with water; such plaster is said to be *dead-burned.*

Unlike Portland cement, plaster of Paris does not continue to gain in strength with age, and as the greatest strength obtained is but a fraction of that obtainable with Portland cement, plaster of Paris is not used in engineering work for

structural purposes.   Another serious objection is that it can be used only indoors, since it is slightly soluble in water and therefore, if left out of doors, it would in time be washed away.   For indoor work it is, however, admirably suitable for statuary, molded stucco cornices, and wall plaster.

**68. Magnesium,** *Mg,* specific gravity 1.70, is a less abundant element than calcium, but is almost invariably associated with this element.   It is a silver-white metal with a high luster, and is never found in a free state.   Magnesium carbonate, known as magnesite, $MgCO_3$, is found alone in nature, but usually occurs with calcium carbonate in dolomite and magnesian limestones.   The sulphate and carbonate may be found in many natural waters.   As silicate, magnesium enters into the composition of numerous minerals, as soapstone, serpentine, and meerschaum, while asbestos, hornblende, and many other silicates contain this element.   Magnesium forms about $2\frac{1}{2}$ per cent. of the crust of the earth.   Magnesium carbonate has properties similar to those of calcium carbonate; if heated, it gives off carbon dioxide, leaving magnesium oxide, $MgO$, known as *calcined magnesia.*   It is a white powder of a highly infusible nature, used to a great extent in making crucibles and for the lining of electric furnaces.   When mixed with water, it hardens slowly and forms magnesium hydroxide, $Mg(OH)_2$.

**69. Aluminum,** *Al,* is one of the most abundant and most widely distributed constituents of the earth's crust.   It does not occur free in nature, but is found, in its most common form, as a silicate in all clays and in many minerals.   Aluminum oxides, hydroxides, and other aluminum compounds also occur in the crust of the earth.

Aluminum is a white metal capable of being polished, in which state it has a somewhat bluish luster.   It is ductile, malleable, very light, its specific gravity being 2.6, and it melts at about 654° C.   It is unaltered by the air and thus becomes available for use as a metal for industrial and mechanical purposes.   Nitric and sulphuric acids scarcely attack it, but it is readily dissolved by hydrochloric acid and it is also imme-

diately attacked by boiling solutions of potassium hydroxide or sodium hydroxide, the hydrogen of the hydroxide being liberated and the aluminum uniting with the base to form so-called *aluminates*.  Since aluminum thus unites with a base, it is in that respect more similar to the non-metallic elements than to the metals, but, as it also unites with acids to form salts, it is generally classified as a metal.  It has a great affinity for oxygen, but the reaction does not take place at ordinary temperature; once the reaction is started by heating the metal to above 800° C., enormous quantities of heat are generated by the combustion, so that aluminum powder can be used to advantage as fuel in welding iron and steel.  Sometimes a temperature exceeding 3,000° C. is obtained.

**70.**  Aluminum forms with oxygen *aluminum oxide, $Al_2O_3$,* generally known as *alumina,* which occurs in nature as a mineral; alumina is extremely hard and is used as a polishing material under the name of *emery.*  Aluminum forms salts with the acids, as already mentioned, and these salts have the property of uniting with salts of other metals, forming so-called *double salts* such as potassium-aluminum sulphate, generally known as potash alum, $AlK(SO_4)_2 \cdot 12H_2O$; potash alum is a chemical union of sulphuric acid with potassium, aluminum, and water.  There are many other similar double salts of aluminum, all of which are collectively known as *alums;* however, when the word alum is used without any further explanation, potash alum is always meant.

The silicates of aluminum form a large part of the crust of the earth, especially as rather complicated double silicates of aluminum, iron, magnesium, and sodium.  Reference has already been made to the aluminum-silicate salt contained in hardened Portland cement.

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### HEAVY METALS

**71.  Properties.**—The group of metals to be subsequently described are distinguished from those already considered by having a much higher specific gravity; they may, therefore, be classified as **heavy metals.**  Arranged according to their

specific gravity, which is indicated by a number in parenthesis after the name of each metal, the heavy metals of most importance are: *Vanadium* (6.025), *chromium* (6.9), *zinc* (7.1), *manganese* (7.4), *iron* (7.8), *nickel* (8.8), *copper* (8.9), *silver* (10.5), *lead* (11.4), *mercury* (13 6), *gold* (19.3), and *platinum* (21.2). Of these, iron is the only metal that is extensively used for construction purposes. It is seldom used pure, but is usually combined with carbon and other elements such as manganese, phosphorus, and silicon. The alloy of iron and carbon that is of principal importance to the engineer is *steel*. The addition to steel of small quantities of vanadium, chromium, tungsten, manganese, or nickel greatly improves its strength and other desirable properties. Although iron possesses great strength, it does not resist the attacks of the atmosphere well, for which reason it is always given a protective coating to exclude the atmosphere. This coating may be of paint, cement mortar, or concrete; for special purposes coatings of zinc or tin are used.

**72.** Zinc and tin, mixed with copper or lead, form alloys, such as *bronze, brass,* and *Babbitt.* These alloys are not much used for construction purposes, but are nevertheless of importance to the construction engineer because they are indispensable in the bearings and journals of the machinery used for handling the construction materials. Mercury is used by the engineer merely in connection with such instruments as thermometers and barometers. Copper, nickel, silver, and gold form the metal of which coins are made, but these metals are not used in engineering work except as a medium of exchange. Silver is employed in certain parts of surveying instruments; copper is much used as wire in electrical instruments and machines and in transmission lines on account of its conductivity, and as sheets for certain waterproofing purposes Platinum, being very expensive, is not used in engineering work, except incidentally in electrical apparatus.

Although the heavy metals are extensively used in the industrial, electrical, and chemical arts, their chemistry is of little interest to the construction engineer. Certain iron compounds

form an exception to this rule, for which reason they will be given a brief consideration.

**73.** **Iron,** $Fe$, when chemically pure, is a brilliant, silver-white metal, capable of receiving a high polish. In its pure form it is too soft for practical uses, for which reason it is always used in a form containing compounds or alloys of various other elements, as previously explained. Iron occurs in nature as oxides, such as hematite, $Fe_2O_3$, and magnetite, $Fe_3O_4$; as carbonates, such as siderite, $FeCO_3$; as disulphides, such as pyrite, $FeS_2$; and in many other combinations. It is not found in the free state, except as meteoric iron, which is almost pure iron falling from the sky in masses and originating in an unknown manner.

Iron is soluble in cold diluted solutions of hydrochloric acid and sulphuric acid, as well as in hot nitric acid, salts being thereby formed of the acid and iron. Iron may combine with oxygen and hydrogen to form an acid, called ferric acid, $H_2FeO_4$; but this acid has never been isolated and is known only through its salts, the so-called *ferrates*.

**74.** Iron is affected by the atmosphere, forming under certain circumstances a reddish-brown compound called *rust*. The process of rusting is called *corrosion*. Just what conditions are necessary for the corrosion of iron is not definitely known, nor has the chemical composition of rust been definitely established; however, it is a familiar fact that a moist atmosphere and the presence of acid fumes greatly accelerate the rusting. The process of rusting is a very complicated one in which various oxides and carbonates of iron are formed; the action is accelerated by the water from the atmosphere condensed on the surfaces of the metal, as this water absorbs from the atmosphere oxygen and carbon dioxide, which unite with the iron. Iron oxide is *hygroscopic,* that is, it attracts moisture from the atmosphere, so that when once the oxide has been formed at any one point, corrosion proceeds faster at that point than anywhere else.

# FOUNDATIONS

## INTRODUCTION

**1.** It is obvious that the weight of any artificial structure must be, directly or indirectly, transferred to and carried by some part of the natural surface of the earth. The word *foundation,* as used in connection with engineering structures, is commonly applied both to the natural material, or the portion of the earth's surface, on which a structure rests, and to the lower part of the structure itself that is in contact with or contiguous to the natural surface. To prevent confusion, the term *foundation bed* will be here applied to that part of the surface of the earth on which the structure rests.

The **foundation** of a structure is not only that part of the structure that is directly in contact with the foundation bed, but also as much of the lower part of the structure as may have to be modified to properly connect the structure with the foundation bed. Thus, if a bridge pier rests on level solid rock, equal in strength to the material of which the pier itself is composed, it may be safely designed as shown in Fig. 1, in which case the foundation is restricted to, at most, the lowest course of masonry. But if it is necessary to distribute the weight of the structure over a larger area of foundation bed, as much of the lower part of the structure as must be modified to comply with this requirement may be properly called a part of the foundation. Thus, in Fig. 2, all that part of the structure from $a$ up to $b$ may be properly called the foundation.

**2. Main Factors Governing the Construction of Foundations.**—The stability of an engineering structure

depends entirely upon the ability of its foundation bed to carry the weight of the structure. The foundations of a structure may fail in two distinct ways; namely, either by crushing of the materials composing the foundation bed or by unequal settlement.

Crushing of the foundation bed will occur if the pressure of the foundation upon the foundation bed exceeds the strength of the material composing the soil. To avoid overloading, the strength of the soil that is to be the foundation bed is often determined experimentally, and the foundation is then designed so as to bring on the soil a pressure of only one-eighth or one-
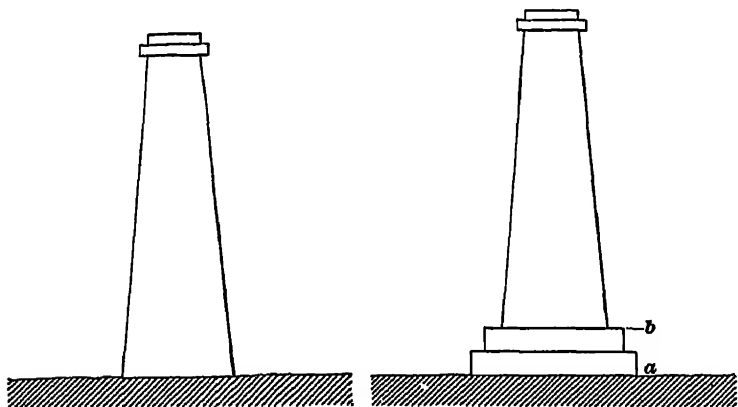


Fig. 1  Fig 2

tenth the strength so found. Thus, if the foundation rests upon material having an ultimate strength, as found by experiment, of 20 tons per square foot, a pressure of only 2 tons per square foot may be used, and the foundation is then said to have a factor of safety of $20 \div 2 = 10$.

Even if the pressure on the foundation bed is kept within safe limits, failure of the structure may result from unequal settlement. Unequal settlement will occur if the pressure of the foundation upon the foundation bed is greater at one point than at another, because all soil is compressible in proportion to the pressure, so that the soil is compressed most where the

pressure is greatest. The structure will then settle more at one point than at another and cracks will result which may ultimately cause the collapse of the structure. Unequal settlement may be avoided by distributing the weight of the structure uniformly over the foundation bed.

## FOUNDATION SOILS

### MATERIALS

#### CLASSIFICATION

**3.** The foundation bed of a structure may consist of any of the materials that are found on the earth's surface and are suitable for the purpose. The materials usually regarded as suitable are: (1) *Solid rock,* including shale in its natural geological position; (2) *loose rock,* which is rock broken into masses of comparatively small sizes; (3) *earth,* including clays, loams, and bogs, in whatever condition they may be found; (4) *sand,* including gravel.

In some cases, two or more, or even all, of these materials may be met with in the same foundation bed.

#### ROCK

**4.** Rock in its undisturbed geological position forms the best foundation bed and is always to be preferred when it is available. Nearly any sound durable rock, including shale of average hardness, found in its natural position, may be safely used for the support of any ordinary structure.

**5. Crushing Strength of Solid Rock.**—The supporting power of a rock foundation bed may be considered as approximately equal to the resistance to crushing of the material of which the rock is composed, modified by a suitable factor of safety, as already explained. Samples selected for crushing tests are likely to be superior to the average, and as, in

large areas, imperfections are likely to exist, the factor of safety should be a liberal one: it should not ordinarily be less than 8, nor, except in rare cases, more than 15. If the rock appears to be of uniform good quality, without cracks or seams, over and for a considerable space around the area to be occupied by the foundation, a load equal to one-tenth of the ultimate crushing strength of the material may be used. Crushing tests are usually made on small cubes, and the crushing strength is stated in pounds per square inch of the surface exposed to pressure. The units commonly applied to foundations are the square foot and the ton (2,000 pounds), and it is usually

### TABLE I
#### CRUSHING STRENGTH OF ROCK

| Kind of Rock | Ultimate Crushing Strength | | | | | Safe Foundation Load, in Tons per Square Foot. Factor of Safety of 10 | | |
|---|---|---|---|---|---|---|---|---|
| | Pounds per Square Inch | | Tons per Square Foot | | | | | |
| | From | To | From | To | Average | From | To | Average |
| Granite .... | 10,000 | 20,000 | 720 | 1,440 | 1,080 | 72 | 144 | 108 |
| Limestone .. | 6,000 | 18,000 | 430 | 1,300 | 870 | 43 | 130 | 87 |
| Sandstone .. | 4,000 | 15,000 | 300 | 1,080 | 690 | 30 | 108 | 69 |
| Shale .... | 400 | 14,000 | 30 | 1,010 | 520 | 3 | 100 | 52 |

assumed that the strength per square foot of any material in its natural bed is 144 times the crushing strength per square inch of the same material tested in the laboratory.

The crushing strength of the different kinds of rock, as well as of different varieties of the same rock, varies within wide limits. Table I gives the approximate crushing strength of the various kinds of rock more frequently met with, and the foundation loads that they may usually be depended on to carry.

#### LOOSE ROCK

**6. Description of Loose Rock.**—Under the operation of natural laws, the solid strata or deposits of rock have, in many cases, been broken up, and the resulting pieces moved

from their original position by the action of gravity, water, and ice. The individual masses of this broken-up material may be of any size or form. Masses of considerable size and consisting of pieces of angular shape lying near the rock from which they were originally broken are commonly called **loose rock.** Other pieces of various sizes that have been carried for considerable distances by water or ice are called **boulders,** and may or may not be rounded or water worn. In a deposit of such materials, there will exist cavities between the separate masses of rock, which may be voids or may be filled with smaller fragments of the same material, or with earth or sand. If filled with a material that has hardened into rock and has cemented the fragments into a solid mass, the rock is called **breccia, conglomerate,** or **pudding stone.**

**7. Loose Rock as a Foundation Bed.**—Loose rock in any of its forms may make a satisfactory foundation bed. The celebrated cantilever bridge at Niagara Falls rests on a loose-rock foundation bed composed of large masses of rock detached from the adjoining cliffs, the spaces between the large fragments of rock being filled with smaller fragments and with earth. The pressure on this foundation bed is about 3 tons per square foot. Foundation beds of this kind, owing to the comparatively recent geological age of the formations and the conditions under which the deposits have been made, require careful examination, and, if possible, should be avoided for important structures.

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<p style="text-align:center"><strong>EARTH</strong></p>

**8. General Description.**—Under the general head of earth are here included the different clays, loams, and other soils, in whatever condition they may be found. In hardness and capacity to sustain weight they vary all the way from the indurated clays to soft mud, often mixed with more or less of the organic matter found in swamps and bogs. Nearly all these materials are porous in structure, and, when confined and subjected to sufficient pressure, are compressed to a greater or less degree. When moist or wet, they are generally

*plastic;* that is, they possess the property of yielding or flowing when exposed to pressures exceeding certain limits, these limits being different for different materials   This property of yielding under pressure is so characteristic of the class of materials under consideration that the term "compressible materials" is generally applied to them in connection with foundation work.   The different degrees to which these materials are compressible, and the difficulties and dangers that this property introduces into the problems of earth foundation beds, make them a subject of great interest and importance to the engineer.

It sometimes becomes necessary to found important structures on clay or earth of a quality far from satisfactory to the engineer, and he will often find his knowledge, experience, ingenuity, and skill taxed to the utmost by the problems presented.   For the less important, and in fact for the great majority of ordinary structures, earth foundation beds, if intelligently made use of, may be safely relied on.

**9.   Strength.**—When exposed to gradually increasing pressure, as when tested in a testing machine, clay and earth usually yield gradually by flow and deformation of the mass, rather than by distinct crushing, and for this reason crushing tests on small samples are of comparatively little value.   Their strength is largely affected by the quantity of water they contain; and the extent to which they may be exposed to water in the foundation beds is an important element to be considered in determining their sustaining capacity.

The loads that may safely be placed on earth foundation beds are determined largely by observation and experience. The soil known as *hard pan,* which is either a very compact clay or a gravel well cemented with clay, can sustain a weight of 8 to 10 tons per square foot of surface.   Ordinary comparatively dry clays can carry from 2 to 4 tons per square foot.   In some standing structures founded on clay, the weight on the foundation bed is greatly in excess of these figures, and no settlement or indications of failure have been observed.   However, excessive loads on clay foundation beds are not sanctioned by good practice.

Experience seems to indicate that the strength of earth foundation beds of the various general kinds may be taken approximately as given in Table II. Owing, however, to the

## TABLE II

### SAFE LOADS ON EARTH FOUNDATION BEDS

| Kinds of Material | Loads, in Tons per Sq Ft. | |
|---|---|---|
| | From | To |
| Hard pan ................. .. ....................... | 8 | 10 |
| Ordinary clays and clay soils, not submerged in water.... | 2 | 4 |
| Clay, soft and plastic ................................. | 1 | 2 |
| Ordinary soils, comparatively dry..................... . | 2 | 4 |
| Ordinary soils, wet.................................... | 1 | 2 |
| Swamp and bog material............................... | ½ | 1 |

greatly varying individual character of materials that may be classed under the same general name, the engineer must in each case be guided largely by judgment based on experience and actual tests.

### SAND AND GRAVEL

**10. Ordinary Sand and Gravel.**—Sand and gravel are but slightly compressible, even when saturated with water, and are capable of carrying very great loads. The high frictional resistance of the separate grains or pebbles on each other tends to prevent the movement or flow that is characteristic of clay. The material is, however, readily eroded by flowing water, and, when utilized for foundation beds, great care must be taken to protect it from direct contact with currents of water, whether of ordinary streams or those caused by wave action. Sand and gravel are both likely to undergo slight initial compression when subjected to pressure; this fact should be taken into consideration, and proper allowance should be made for it in designing structures to rest on this kind of foundation beds. Clean dry sand can bear a load of from 2 to 4 tons per square foot.

I L T 370—13

**11. Quicksand.**—One of the most treacherous and troublesome materials with which the engineer has to deal in foundation work is quicksand. It is difficult to give a satisfactory definition of this material, though its physical properties are easily described. In most respects it does not differ essentially from ordinary fine sand, and when removed from its native bed and freed from water it cannot usually be distinguished from sands that do not possess its peculiar properties. The separate grains are always small and generally more rounded than those of ordinary sand. But when saturated with water, a mass of this material seems to lose more or less of its internal cohesion or friction, and its power to support weights is almost zero. A bar may be worked to the bottom of a bed of quicksand several feet deep with very slight exertion, and men and animals attempting to walk over such a deposit are often hopelessly engulfed. This description applies to well-developed examples of the material. There is no well-defined demarcation between ordinary sand and quicksand, and the one may pass into the other by imperceptible stages; in other words, sands may possess the peculiar properties of quicksand in very different degrees.

In general, quicksand has practically little or no value as a foundation bed; however, it may sometimes be so treated as to make it serve to support ordinary structures. If a bed of quicksand is drained, it loses its distinctive property; it becomes more compact and resisting than ordinary sand, and may make a convenient and secure foundation bed.

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### MIXED SOILS

**12.** It is often found that the site of a structure is occupied by several of the materials described—either in separate deposits or mixed together in various proportions. If rock is found, it may extend over only a part of the area of the foundation, or it may contain large fissures filled with other materials; or the rock, although extending over the whole area, may not be all of the same kind or quality, in which case different parts of the foundation bed require different methods of

treatment. Again, the area may be occupied by a mixture of earth and loose rock or boulders, or part of it may be clay and part of it sand or quicksand. Such complications are of frequent occurrence, and conditions are always likely to be found that are without precedent in the engineer's experience, and that will tax to the utmost his judgment and skill.

## EXAMINATION AND TESTS OF FOUNDATION BEDS

**13.** It is generally unsafe to trust to surface appearances in judging of the character, strength, and soundness of foundation beds. A stratum of rock may, when its surface is uncovered, appear to be of satisfactory character, without serious fissures or other defects, and may give the impression of being continuous and solid downwards to an indefinite depth. This appearance may, however, be deceptive; the layer of rock may be comparatively thin, and may be underlaid with a body of soft clay or other unresisting material, and may therefore not be capable of sustaining the structure designed to rest on it. Not a few important engineering structures have failed from the existence of such unforeseen conditions. Likewise, an apparently satisfactory bed of clay may be too thin, or may be underlaid with a stratum of sand that the swift current of a stream may sometime reach and carry away, undermining the clay and causing the structure founded on it to fail. It is therefore necessary that the engineer should carefully investigate not only the material that he purposes to use for a foundation bed, but, as far as may be possible, all the conditions surrounding it, and satisfy himself that these conditions are not such as may ultimately result in the destruction or impairment of the proposed structure.

### EXAMINATION AND TESTS OF ROCK FOUNDATION BEDS

**14. Geological Examination.**—A general knowledge of the geology of the region—such as the character and succession of the local formations, the inclination or dip of the

strata, and the prevalence of faults and other disturbances—should be acquired, and this knowledge should be utilized in the study of the immediate locality of the work. In a region where the geological formations have been subjected to upheaval and are greatly broken up or much inclined, greater caution is necessary than where the strata remain in their original nearly horizontal and unbroken condition.

Where the structures to be erected are of secondary importance and the weight to be carried is comparatively small, the character of the foundation bed may often be determined accurately enough from the nature of the neighboring strata and their outcrop in near-by streams or ravines. For all important works, however, special tests should be made, as hereafter described.

**15. Drill Test and Drilling Machines.**—Where the foundation bed is rock, it may be most satisfactorily examined by sinking drill holes into it. The number and depth of these drill holes depend on the apparent conditions and the importance of the structure. In the case, for instance, of a bridge with short spans carried on comparatively low piers, the same thoroughness of investigation is not called for as in the case of a long suspension bridge with high piers, carrying very heavy loads. In the former, one or two drill holes on the site of the pier, carried to a depth of 10 feet, or even less, might be sufficient; while in the latter, drill holes not only within, but for some distance outside of, the boundaries of the pier and carried to a depth of from 20 to 30 feet, or more, may be advisable.

**16.** In the great majority of cases, drill holes of the size of, and made with the apparatus commonly used for, those in quarry and rock-excavation work, will answer every purpose. In the more important works, particularly where there is reason to expect defects and irregularities, and where the holes are to be carried to a considerable depth, some one of the forms of drilling apparatus that yield complete samples, or cores, of the material passed through should be used. These machines are generally power driven. The drilling tool is a

hollow rod that, when revolved, makes an annular cut into the rock, leaving an undisturbed section, or core, within the tool; this core is occasionally broken off and brought up with the tool. The recovered cores may, after examination and record, be preserved for future reference.

**17.** Test drilling will, if properly conducted, disclose any subterranean defects in the foundation bed, and will furnish data for a second judgment of the capacity of the material. The defects that may thus be disclosed are: (1) Beds or pockets of clay or other soft material dangerously near the surface; (2) large cavities that might weaken the foundation bed, and (3) thin layers of clay or soft shale, which, in cases where the strata are much inclined and other conditions permit, might allow the structure and its foundation to slide or move laterally. Such a possible condition is illustrated in Fig. 3. The weight of the pier *P* and its load might cause the foundation bed to slide
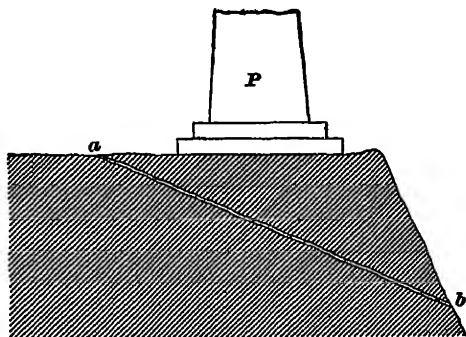


Fig. 3

along the inclined joint *a b,* particularly if this joint is lubricated by a thin layer of slippery clay. This is not a purely supposititious case, since the destruction of not a few structures has been caused by similar conditions.

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**EXAMINATION AND TESTS OF OTHER FOUNDATION BEDS**

**18.** All that has been said of rock foundation beds applies with equal force to loose rock, earth, and sand foundation beds; in fact, these usually require the exercise of greater care and caution than ordinary rock foundation beds. The formations consisting of these materials, as they are now found, are much more recent geologically than rock formations, and are

far less stable in character. It is not at all uncommon to find beds of firm strong clay that prove comparatively thin and are underlaid with deposits of much softer clay, alluvium, or quicksand, saturated with water and having very little capacity to carry loads. Two methods are in common use for testing such foundation beds; namely, *test pits* and *borings*.

**19. Test pits,** or **wells,** are ordinary excavations similar to the common well, carried to such depth as may be thought necessary, within or near the boundary of the foundation. The method of sinking such test pits requires no special description.

**20. Borings.**—The method of boring is, in general, similar to the rock drilling previously described, but is usually quicker and less expensive. The bore holes are really wells or test pits of a small diameter. Several methods of making such borings are in use, some of which are so simple as to require no description here. Frequently, the procedure and the apparatus used are almost the same as for rock drilling. This method is objectionable because of the difficulty of obtaining good samples and determining the character of the materials passed through, but it may give results that are satisfactory in many cases. Thus, a drill hole sunk into a bed of clay or other material may show that no important change in its character occurs within such a distance from the surface as would render it unsuitable for foundation purposes. But even if the results from this method of making borings were always satisfactory, difficulties are frequently met with that make it inapplicable. A layer of sand or other soft material may be encountered that has not sufficient stability to maintain the wall of the bore hole; as a result the material caves or runs into the hole, filling it and obstructing the operation of the drill. To prevent this, the hole must be *cased;* this is done by driving down, inside the hole, an iron tube. When borings are to be carried to a considerable depth, it is generally best to begin with the expectation that casing will be necessary, and to make the bore hole large enough to allow the use of iron pipe not less than 3 inches inside diameter.

**21.** For these larger holes, two well-defined methods of boring are employed. In the one, suitable earth augers are used, which excavate and bring up the material in its natural condition; in the other, called **wash boring,** the material is loosened partly by the auger or drill, and partly by a jet or current of water, which also carries the loosened material to the surface. In this method, the drill rod is hollow, and a current of water is forced through it by pumps or other devices. The water escapes at the working end of the drill and rises through the ring-shaped space between the drill and the casing, carrying with it the excavated material, which may be collected and examined. As the drill progresses downwards, the casing is, from time to time, forced downwards by a pile driver or other suitable device, additional lengths of pipe being added by means of screw joints or couplings, as required. The first method yields the most satisfactory samples of the material passed through; the second is more expeditious, and, with a proper equipment, more economical. Whatever method may be employed, the results of borings should be carefully observed, accurately recorded, and samples preserved for future reference.

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## AREA OF THE FOUNDATION BED

**22. Required Area of Foundation Bed.**—In the case of foundations for ordinary structures, where weight is the only force to be resisted, and where that weight is uniformly distributed over the whole of the foundation, the first problem of importance will be: Over what area of the foundation bed must the weight be distributed, in order that the safe bearing capacity of the material of the foundation bed shall not be exceeded?

Where the material is solid rock in its natural geological position, it is usually unnecessary to solve this problem, since almost any sound rock is sufficiently strong to carry the concentrated weight of any well-designed structure. But where earth or sand foundation beds must be utilized, care should be taken to distribute the weight to be carried over such an area

that the resistance of the material shall safely exceed the applied weight. Thus, if the estimated weight of a bridge pier is 1,200 tons, the weight that will come on it from the dead load of the bridge, 500 tons, and the weight of expected live load, 300 tons, the foundation bed will be depended on to sustain a load of $1,200+500+300=2,000$ tons. If the foundation bed is clay and may be safely loaded with 2 tons per square foot, the area required will evidently be $2,000 \div 2 = 1,000$ square feet.

EXAMPLE.—The foundation for a standpipe has the form of a cylinder with a diameter of 24 feet and a height of 5 feet. The weight of the material of the foundation is 100 pounds per cubic foot, and the weight of the standpipe is 500 tons. If the foundation bed is clay that may be loaded with 1 5 tons per square foot, what is the required area?

SOLUTION.—The weight of the foundation is $.7854 \times 24^2 \times 5 \times 100 = 113\ 098$ T., or 113 T., nearly. The total load on the foundation bed is $500+113=613$ T. The required area of the foundation bed is, therefore, $613 \div 1.5 = 408.7$ square feet. Ans.

**23.** If the plan of the structure is irregular, so that it is difficult to secure a uniform pressure over the whole area of the foundation bed, or if lateral forces must be taken into consideration, or if the character or sustaining capacity of the material of the foundation bed varies over parts of it, the question becomes more complicated. Clay and earth foundation beds are usually more or less compressible; the amount of compression or settling is in direct proportion to the intensity of the applied pressure, and a slight variation in the settlement of different parts of the foundation bed may distort or disrupt the structure erected on it.

The effect of a load not uniformly distributed on a foundation that rests on a compressible bed is illustrated in Fig. 4, which is a longitudinal elevation of a masonry-arch culvert $a$ under a railroad embankment $b$, shown in cross-section above it. The pressure on the foundation varies with the height of the embankment over it, being greatest directly under the middle of the embankment. If the foundation of such a culvert is made of the same width throughout, the intensity of

pressure on the foundation bed will increase as the middle section is approached, unequal settlement may follow, the structure may take the distorted form shown by the dotted lines, and the distortion may be sufficient to fracture the masonry. If the width of the foundation is increased properly from the ends toward the middle of the structure, so as to distribute the load evenly per unit area, then, whatever settlement may take place will be the same throughout the whole length of the structure, and the masonry will not be injured. The walls of heavy buildings are often cracked through the failure of the designer properly to proportion the area of the foundation to the pressure on it.

It is always desirable to avoid foundation beds of compressible material, or, if this cannot be done, to make the loading so
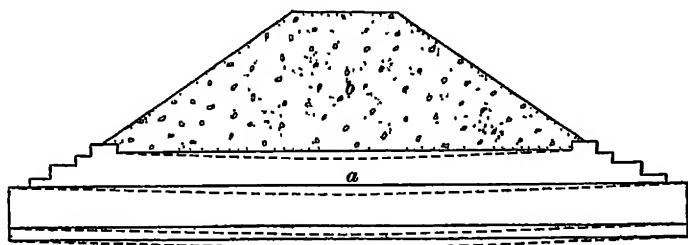


Fig. 4

light that settlement, if any, will be so slight as to be negligible. But as this is not always possible, and as it can seldom be determined in advance whether a material will be compressed under a given load, and, if so, how much, it is important that in earth foundation beds every possible effort should be made to distribute the pressure as evenly as possible over the whole area covered.

**24. Depth of Foundation Below Surface of Ground.**—Foundations in earth should be carried to such a depth below the ground surface that frost will not reach the underlying bed. Nearly all moist earth expands, or heaves, with freezing, and repeated freezing is likely to soften and disintegrate it The depth of foundations may, however, be dictated by conditions other than frost, because often a good

material cannot be found except at greater depths than are necessary to provide against frost.

The penetration of frost varies with the latitude. In the American Gulf States, ice seldom forms; while in the Lake region, the ground sometimes freezes to a depth of 5 or even 6 feet. Ordinarily, in the northern parts of the United States, foundations 4 feet below the ground surface may be considered safe from injury by frost.

**25. Effect of Weight and Friction of Superimposed Earth.**—Not only is the soil likely to be firmer and harder a few feet below the surface, but the surrounding earth, by its weight, tends to counteract any movement and therefore to increase the bearing capacity of the foundation bed. Thus, if a wall is founded at a depth of 5 feet below the surface, the weight of the surrounding earth above the foundation bed tends to prevent any flow of the clay from under the foundation. Such movement of the material can only take place by overcoming the weight and internal friction of the banks of clay around the foundation and causing an upheaval of the surface around the structure.

This counter pressure of the superimposed earth is a factor of much importance in the strength of foundation beds in plastic soil, but its exact value is difficult to compute. In important work, a test of the actual bearing capacity of the soil is made by loading a platform of a definite area and measuring accurately the amount of settlement under increasing loads.

In testing the bearing capacity of the soil on which the New York State Capitol at Albany was erected, a measured load was applied to a square foot and also to a square yard. For the first test, a timber mast 12 inches square held in a vertical position by guys was fitted with a cross-frame to hold the weights. A hole 3 feet deep was dug in the blue clay at the bottom of the foundation. The hole was 18 inches square at the top and 14 inches square at the bottom. Small stakes were driven in the ground on lines radiating from the center of the hole. The tops of the stakes were brought exactly to the same level, so that any change in the surface of the

ground adjacent to the hole could readily be detected by means of a straightedge. The foot of the mast was placed in the hole and the weights applied. No change in the surface of the adjacent ground was observed until the load reached 5.9 tons per square foot, when an uplift of the surrounding earth was observed in the form of a ring with an irregular curved outline, as shown by the dotted line in Fig. 5. Similar experiments were made by applying the load to a square yard, with practically the same result.
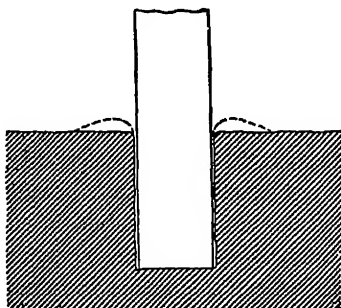
**26.** The experiments just described illustrate a point that seems to be often overlooked in designing heavily loaded foundations in compressible soils. By reference to Fig. 6, which represents a structure founded on clay on a sloping hillside, it is evident that the high wall of clay on the upper side of the structure will exert a greater pressure to counteract the flow of the clay from under the foundation than will the low wall on the lower side; that, under loads that might cause yielding on the lower side, the foundation might settle there while remaining firm on the upper side. In such situations, care should be taken to keep the loads on the whole of the foundation within such safe limits that the clay where least confined or counterbalanced will not yield by flowing. If, for any reason, this is not possible, and loads of questionable magnitude must be used, the danger may be averted by dressing down the high bank, as illustrated by the dotted line in Fig. 6.
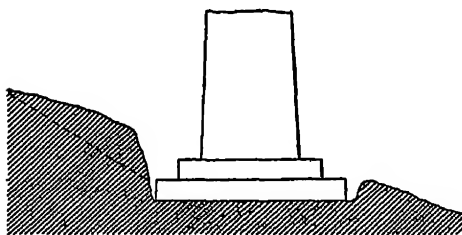
FIG. 5

FIG. 6

# PREPARATION OF THE FOUNDATION BED

## PRELIMINARY CONSIDERATIONS

**27.** Before the foundation can be built, it is necessary to excavate the site to proper width, length, and depth. The excavation for the cellar of an ordinary building or structure is not usually difficult, because in the great majority of cases a suitable foundation bed is encountered at the desired depth of foundation. However, when structures are erected in localities where a suitable foundation bed cannot be obtained except at great depth below the surface, the excavation of the site and its preparation for the construction of the footings may become very difficult, especially if the site is under water, which is frequently the case in the construction of bridge piers and similar structures. Tall buildings frequently require deep excavations because they are designed with two or more subcellars, and springs or subterraneous streams are then often encountered.

According to the nature of the problems to be solved, foundation beds may be divided into two groups, namely, *dry* and *wet,* the former being prepared at no great distance below the surface and without special difficulties, and the latter being prepared usually at greater depth and calling for special excavation or construction.

**28.** Sheet Piling.—Common to both wet and dry excavations is the necessity of preventing the banks of the adjoining land from caving into the excavation. Quite often, especially in shallow excavations, the banks can be sloped back to stand at their natural angle, but where this is impossible because the land is already occupied or impractical because of the quantity of earth thereby added to the excavation, it becomes necessary to brace the banks. This is conveniently and efficiently done by means of sheet piling. **Sheet piling** usually consists of

planks 3 or 4 inches thick, 12 inches wide, and of length to suit conditions at the site. The sheet piles are driven side by side to form a wooden wall, as in Fig. 7 at *a,* all around the excavation. To support the sheet piling against the pressure of the earth, horizontal timbers *b* called *wales* are used; the wales are in turn supported by inclined or horizontal timbers *c* called *struts*. The purpose for which sheet piling is used makes it important that the joints between adjoining piles should be as close and as nearly water-tight as possible. Several devices for securing this condition are in use, a number of them being
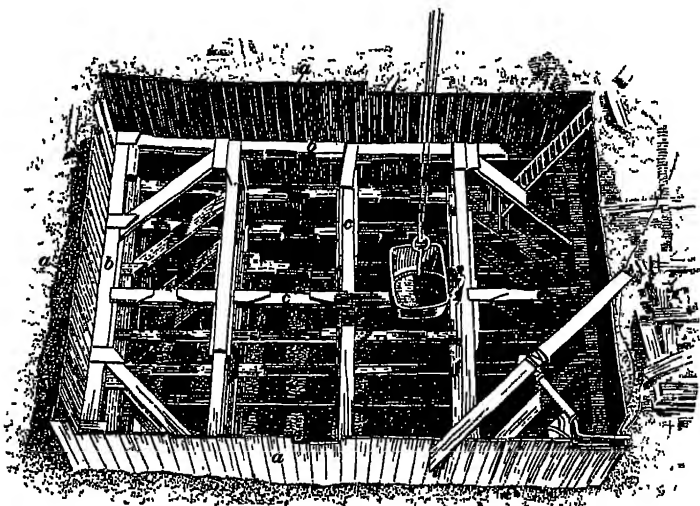


FIG 7

shown in Fig. 8. In this figure, (*a*) represents a horizontal section through plain sheet piling, which depends for close joints on the care and accuracy with which it is driven. In (*b*) are shown two contiguous rows of piling, one row breaking joint with the other; three rows of plank are sometimes used, the outside rows being of thinner plank than the central or main row. In (*c*), (*d*), (*e*), (*f*), and (*g*) are shown various forms of "tongued-and-grooved" sheet piling; the object of this construction is not only to make a tight joint, but also to assist in guiding the piles truly in the same plane. The form (*d*) is

very frequently used, the tongue and groove being formed by spiking narrow strips on the edge of the pile. The form $(g)$ is a patented form known as the "Wakefield" pile; its construction is evident from the figure without further description.
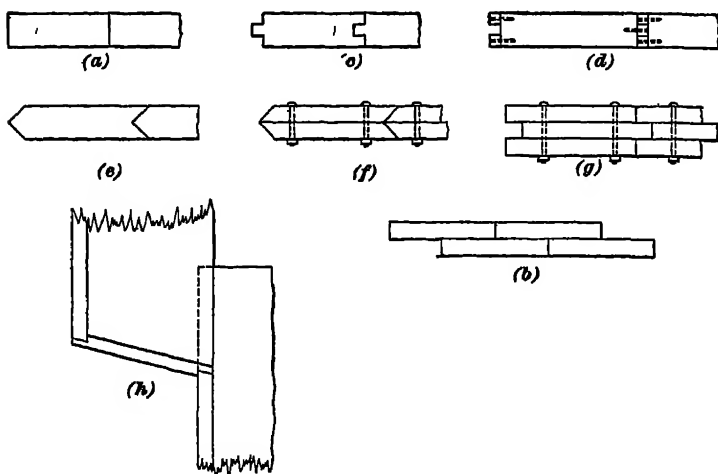


FIG. 8

The points of sheet piles are usually sharpened to facilitate driving, and one side is often sloped, as shown in $(h)$, to cause the piles to come into close contact with one another.

Sheet piling of steel is also often used, while sheet piling of reinforced concrete is only occasionally employed.

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## DRY FOUNDATION BEDS

**29. Rock Foundation Bed.**—The preparation of a shallow and dry foundation bed is generally a very simple matter. In rock it is not usually considered necessary to excavate below the surface to a greater depth than may be found needful to remove disintegrated and weather-worn rock, and to bring the surface into proper shape to receive the foundation. Where a stone-masonry structure is to rest directly on sloping rock, the rock must be leveled off to receive the courses of masonry, or else cut into benches or terraces, according to cir-

cumstances. The dressing of the rock into benches may some-
times be avoided by the use of concrete for leveling up inequali-
ties between the rock and
the masonry, as illustrated
in Fig. 9. This is, however,
permissible only where the
general slope of the rock is
not sufficient to induce or
permit sliding of the con-
crete on the surface of the
bed rock. In the case of
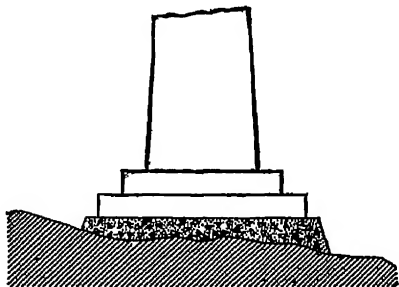dams and retaining walls,
which are liable to failure



FIG. 9

by sliding, sloping surfaces are to be avoided. In the case of
arches, the foundation bed is often sloped as at $a$ in Fig. 10
because the weight of the arch has a tendency to cause the abut-
ments to spread apart. This tendency is resisted by the sloping
foundation bed, which acts as a shoulder against the pressure.

**30. Earth Foundation Bed.**—In the preparation of
earth foundation beds, the material is excavated to a sufficient
depth to guard against the action of severe frost, and the
whole area is graded to a level plane. In very soft and com-
pressible material, measures are sometimes taken to solidify
or reinforce the foundation bed before the structure is begun.
This may be accomplished by driving a large number of short
wooden piles over the area, to compress the soil into a more
compact mass; but, unless these piles are below the permanent
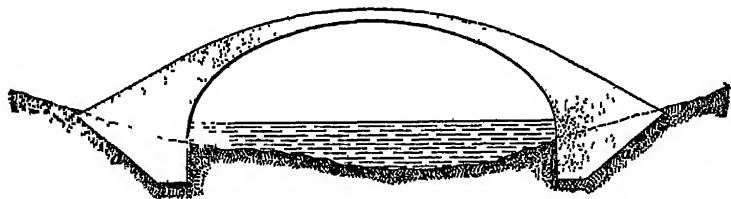surface of the water, they may in time decay and become use-



FIG. 10

less   To avoid this possibility, piles are sometimes driven and
at once pulled out, and the holes are filled up with clean sand or

gravel. Sand, dry earth, or crushed rock spread over the surface are sometimes used, but even if rammed into the soft material, they are not likely to prove satisfactory under heavy structures, as the comparatively thin and flexible layer between the soft material and the base of the structure may not sufficiently prevent the shifting of the soft material below.

**31.** The effect of a deep excavation on the bearing capacity of soft and plastic soils has been referred to in previous articles, and very satisfactory foundation beds for ordinary structures may often be thus secured in material that, at or near the surface, has little supporting power. Deep excavations for foundations are sometimes partly refilled with sand or gravel. This gives good results, which are probably partly due to the compressing effect of the overlying sand or gravel on the soft material, and especially to the fact that a wide and thick layer of good material will distribute the weight of the structure over a large area of the softer strata underneath.

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### WET FOUNDATION BEDS

**32.** The chief obstacle encountered in foundation work, and the most common one, is water, which is sometimes met with in shallow foundations, and almost always in deep foundations. Where the foundation of a structure must be placed in water, one of two general methods is followed: either the space to be occupied is freed of water, after which the work proceeds as on land, or some device for sinking the foundation through the water is employed. These two methods of constructing foundations are · (1) *the cofferdam method,* in which the foundation bed is freed of water, and (2) *the caisson method,* in which the foundation is sunk through the water.

In order to consolidate the unwatered foundation bed, *piling* is often used. The subject of piling will, however, be treated in a separate Section, partly because of the importance of the subject, and partly because special reference is to be made to concrete piles, which are outside the scope of this Section.

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**33. Cofferdams.** — A cofferdam is a water-tight structure enclosing a space from which the water may be pumped out, leaving a comparatively dry area on which a foun-
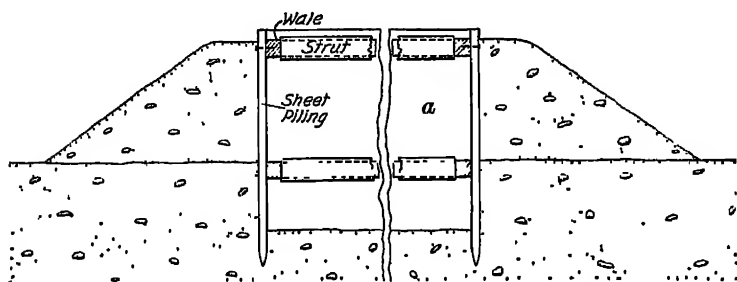


Fig 11

dation may be constructed as on dry land. A great variety of cofferdams have been devised and used. The simplest and most primitive, suitable for use in very shallow water, is a bank of earth, clay, or sand, rising above the surface of the water, surrounding the area to be built on.

Next in order is a simple timber structure surrounded by a bank of earth. Fig. 11 shows a cross-section of a cofferdam



Fig. 12

of the latter form  A wall of sheet piling is driven around the space *a* to be unwatered, the tops of the sheet piles are spiked

I L T 370—14

to a timber, called a *wale* or *waling piece,* and a bank of earth is then deposited around the outside of the structure.

**34.** A form of cofferdam very frequently used is illustrated in *Fig.* 12. It consists of an inner and an outer line of sheet piling with waling pieces at the top, tied together with iron bolts. Frequently, each wall is composed of two or more lines of sheet piling, as shown at *m* and *n,* the individual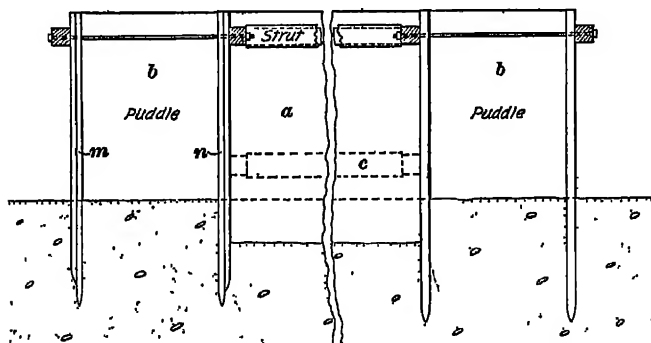 piles breaking joint for the purpose of increasing the strength of the structure and of making the walls themselves as nearly watertight as possible. The space *b* between the two walls is then filled with earth or other suitable material to exclude the passage of water. This material is called *puddle.* The water may then be pumped from the interior *a,* and, if necessary, additional wales and cross-struts, as shown by dotted lines *c,* may be added as the water is lowered, to resist the pressure inwards.

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### CAISSONS

**35.** The previously described cofferdams are not adapted for excavations of great depth nor for use in very deep water, the usual limit of depth being about 15 feet. For greater depth a caisson is usually preferable. The term **caisson**, as used in foundation work, is applied to any movable box-like structure that is used to exclude water from a foundation bed during construction. There are three kinds of caissons in common use; namely, (1) the *box caisson,* which is a box with four sides and a bottom, but without top; (2) the *pneumatic caisson,* which is a box with four sides and a top, but without bottom; this type derives its name from the fact that compressed air is constantly pumped into the box to keep the water out; and (3) the *open dredging caisson,* which is a box with four sides having neither top nor bottom, but containing pockets or compartments with bottoms.

**36.** **Box caissons** can be used only where little or no excavation is required, because the bottom of the caisson prevents any excavating from the inside. Whatever digging has to be done must be undertaken from the outside and before the

caisson is finally placed. Box caissons are frequently built on dry land, launched into the river or lake as a ship is launched, and towed to the final destination and there sunk by the weight of the masonry of the structure which is being built within the four protecting sides. A box caisson for a concrete foundation is used as a form for the concrete, which is simply filled into the huge box forming the caisson until it sinks.

**37. Pneumatic caissons,** Fig. 13, consist of a box-like structure, built as indicated at *A*, enclosing a chamber *B* called
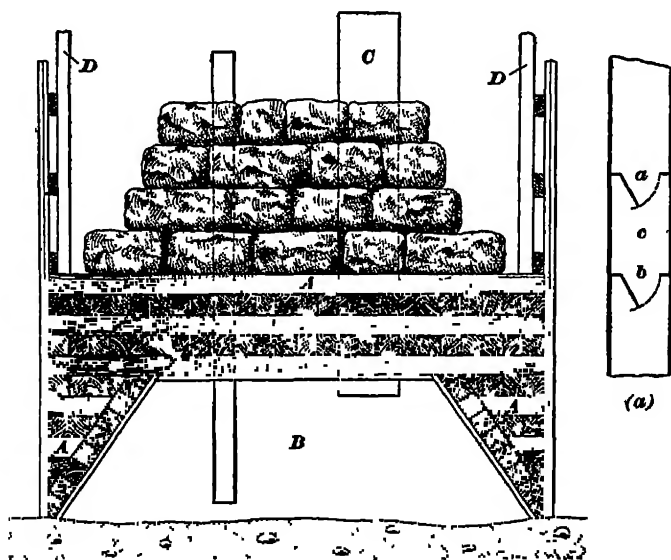


Fig. 13

the *working chamber.* Compressed air is used to force the water out of the working chamber, and men, working in the compressed air, are thus enabled to excavate the earth from within the caisson, which, forced down by the weight of the masonry on its cover, gradually sinks down in the earth because the walls are sharpened at the bottom into *cutting edges,* which are usually shod with iron. The cover, called the *deck* of the caisson, is formed of several, often a dozen or more, courses of timbers crossing each other at angles usually of 90°, and the

whole is thoroughly tied together with bolts. In large caissons, one or more trusses or partitions crossing from side to side give the structure additional strength and stiffness. The walls and roof of the working chamber are made as nearly water-tight and air-tight as possible by lining them with courses of plank, the joints between which are calked with oakum and pitch. On the deck is usually, though not always, constructed a water-tight enclosure *D D,* called the *cofferdam,* the object of which is to permit the laying of the masonry below the surface of the water by unwatering the space within the cofferdam. Through the deck, communicating with the working chamber, are inserted a number of large tubes or shafts *C* built of steel plates, which are carried up as the sinking of the caisson and the building of the masonry progress. The most important of these shafts is called the *air-shaft,* which is used for the entrance and exit of the men to and from the working chamber, and sometimes for the bringing out of excavated material. Two or more such air-shafts may be provided in a large caisson. Situated in the air-shaft is the *air lock,* the use of which will be explained presently. Other tubes and pipes are for the forcing out of water and semifluid excavated material or for conveying concrete and other materials into the working chamber.

The principle of the air lock will be understood from Fig. 13 (*a*). The lock consists of a chamber *c* separated from the upper and lower portions of the air-shaft by doors *a* and *b,* which are fitted to close air-tight. A man wishing to enter the working chamber passes down the air-shaft through the upper door of the air lock, the lower door being closed. When he is inside the air lock, the upper door is closed, the compressed air from below is slowly admitted into the lock through valves, and when the pressure in the lock has become equal to that in the working chamber, the lower door is opened and the man continues his descent. If he wishes to return, the operation is reversed: the upper door being closed, the lower one is opened; he enters the lock and the lower door is closed; the lock is then connected through valves with the atmosphere; the compressed air in the lock escapes until the pressure is the

same as that outside, when the upper door is opened and the man continues his ascent.

**38. Open dredging caissons** are box-like structures divided by cross-walls into compartments. Some of these compartments have bottoms and are called *pockets;* these are gradually filled with concrete to make the caisson sink. Other compartments have no bottoms and are called *wells;* in these the water usually stands at about the level of the water outside and the soil from the bottom is dredged out by means of buckets. Fig. 14 (*a*) is a vertical section through such a caisson and (*b*) is a horizontal cross-section. The pockets, as indicated at *a*, are filled with concrete, and the single well *b* is enlarged at the bottom, as at *c*, so that it occupies nearly the entire area of the caisson. When, by continued dredging, the caisson has been sunk to the desired depth, the surface of the river bottom or other foundation bed is made level, as nearly as possible, by means of the excavating buckets, and a layer of concrete rich in cement is deposited over the entire area in order to make a waterproof seal at the bottom. So far, it will be understood, the excavating and concreting have been going on



Fig. 14

in the water which fills the wells, but after the seal has been placed the wells are pumped dry and filled with concrete, which, together with the concrete of the pockets, becomes a permanent portion of the masonry of the structure itself.
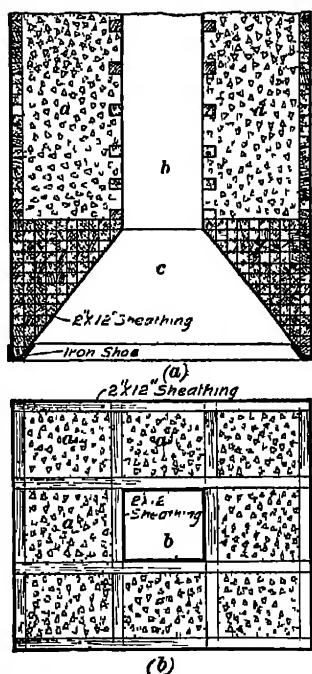
## CONSTRUCTION OF FOUNDATIONS

### FOOTINGS

**39. Necessity for Footings.**—The office of the foundation is to form a suitable connection between a structure and the foundation bed. If, as is sometimes the case, the material of the foundation bed is equal in strength, hardness, and durability to the material of which the structure itself is built, no foundation is required in the sense in which the word is here used, except in so far as the name may be applied to the line or surface of contact between the material of the foundation bed and the material of the structure. It is common to construct the lower courses of piers, walls, and like structures, with steps or projections, as shown in Fig. 15, but these projections have no utilitarian value if the material of the foundation bed is equal in strength and durability to that of the structure. Thus, in Fig. 15, if the structure is of granite or limestone and the foundation bed is of the same or equally good material, the offsets in the footing courses might safely be dispensed with, and the lines of the structure continued down to the foundation bed, as shown by the dotted lines. These projecting courses may, however, possess a certain esthetic value, which justifies their use.
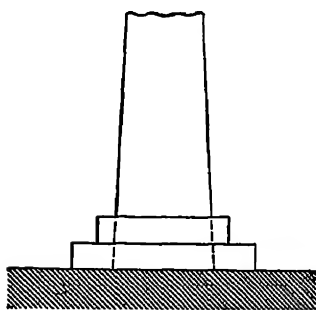


Fig. 15

Where the material of the foundation bed is inferior in strength to the material of the structure, it obviously becomes necessary to accommodate the one to the other, in order to secure equal strength throughout. The material of the structure may readily carry a load of 20 or more tons per square

foot, while that of the foundation bed may be capable of carrying only 2 tons per square foot. The problem is, therefore, to distribute the load carried by the structure over such an area of the foundation bed that its safe load shall not be exceeded. This is ordinarily accomplished by the use of what is commonly called the method of **spread foundations.** Essentially, this method consists in enlarging the lower end or base of the structure so that it may cover an area of foundation bed the resistance of which will be at least equal to the weight of the structure. The usual method of accomplishing this is by stepping out, or enlarging, the base of the structure by offsets, called **footing courses,** or simply **footings.** The material employed in these footing courses may be the same as, or different from, that used in the body of the structure, the kind of material used being determined mainly by the conditions that are to be met in each particular case.

<center>———</center>

### TYPES OF FOOTINGS

**40.** There are two kinds of footings in common use, namely, *wall footings,* which extend the entire length of a wall or other continuous structure, and *column footings,* which support an individual, concentrated load, such as that carried by a column. In Fig. 16 (*a*) is shown a portion of a footing plan of the corner of a building, which illustrates the difference between the two types, the building having continuous brick walls *a* for the exterior construction, while the interior portion of the building is supported by isolated, individual columns *b*. The footing under the brick wall projects at either side of the wall at *c* and *d,* so that a vertical section at *A B* would be as indicated in (*b*), where *a* is the wall and *c* and *d* are the projections. The footing under the column *b* projects at all four sides of the column, as indicated by the letters *e, f, g,* and *h* in the plan. Usually the offset *c* projects as far as the offset *d,* and all four projections *e, f, g,* and *h* are alike, but this rule is not without exceptions.

A brick wall, such as *a,* or a concrete wall of similar construction, is sometimes built with projections opposite the

interior columns; such projections, called *pilasters*, are indicated at *i* and *k*   They serve the purpose of reinforcing the wall where extra weights are supported, and in order properly to transfer such extra weights to the foundation bed, the footing must be provided with projections *i'* and *k'* corresponding to those on the wall.  If the projection *i'* is great in comparison with the projection *c,* the wall footing at the pilaster resembles a column footing, especially if pilasters are built on both faces of the wall as shown in the plan view, in Fig. 17 (*a*),



(a)

(b)          (c)

Fig. 16

where the wall *a* is comparatively thin and is provided with only a small footing *b,* while the pilaster *c* is large and is provided with a correspondingly large footing *d.* The construction indicated in (*a*) is often used but cannot be recommended because it is difficult so to proportion the footings that they will settle uniformly.  A better construction is
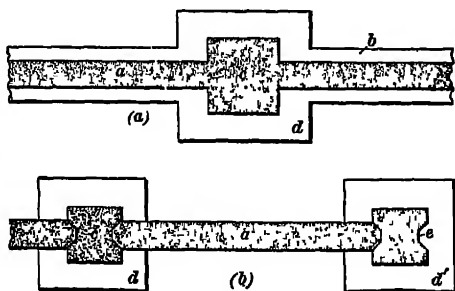


Fig 17

obtained by substituting individual columns for the pilasters and individual column footings for the continuous wall foot-

ing. This construction is indicated in (b), where the column c, independent of the wall a, rests upon its footing d; the wall a, resting upon shoulders in the footings d and d', spans the interval between footings d and d' without any wall footing. A wall of this kind is said to be *self-supporting;* self-supporting walls, sometimes called *curtain walls,* are easily built of reinforced concrete. In order to prevent the comparatively thin curtain walls from tipping sidewise, slots or recesses e are left in the sides of the columns to receive the ends of the curtain walls. The curtain walls are, of course, no part of the foundation, but the question whether curtain walls should be used or whether solid walls should be used greatly influences the arrangement of the entire foundation and foundation bed. Buildings with curtain walls are often of the so-called *skeleton construction* in which columns and beams are used rather than continuous walls, and such buildings usually have individual column footings both for the exterior and interior portion of the building, whereas buildings with solid walls usually have continuously extending wall footings, at least for the exterior construction.

In engineering structures such as dams, retaining walls, and bridges, the wall footing is very commonly used, whereas column footings are rarely used outside of buildings. There are, however, some engineering structures, such as chimneys and standpipes, which consist principally of a huge vertical shaft or cylinder, where the entire foundation is constructed as a single block resembling a large column footing. In the following, wherever reference is made to wall footings and column footings, foundations of the type shown in Fig. 16 (b) and (c) are meant.

**41. Materials Used for Footings.**—The materials of which footings are constructed are timber, brick or stone masonry, plain concrete, reinforced concrete, and structural steel.

**42. Timber footings** are ordinarily constructed for temporary structures only, because timber in contact with soil decays rapidly. However, timber entirely submerged in water

will not decay, although in salt water it may be destroyed by various marine organisms. Where these do not exist, timber footings are sometimes, but not often, used for permanent structures. Oak, long-leaf yellow pine, and Norway pine are the best kinds of timber for this purpose.

Fig. 18 shows the ordinary arrangement of timbers in the footing. The first footing course is composed of heavy squared timbers placed close together on the foundation bed. At right angles to the timbers of the first course are then placed the lower timbers $n\,n'$ of the next footing course. On top of this latter layer is placed at right angles to it a floor layer $p\,n'$ of heavy timbers on which rests the column $D$. In Fig. 18 $a\,b, c\,d$, and $p\,q$ are termed the *offsets* or *projections*.
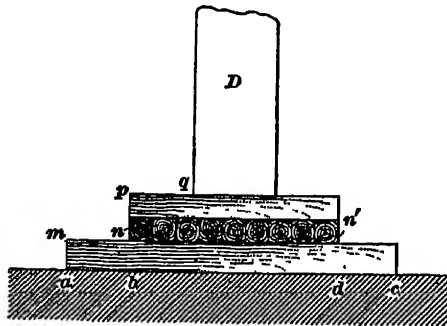


Fig. 18

**43. Brick - Masonry Footings.** The use of brick-masonry footings is practically restricted to wall footings in small structures, particularly residences, where only little spreading out of the lower courses is required. For foundation purposes, concrete is preferred to brick masonry in most cases, because concrete is cheaper, and, on account of the absence of joints, is stronger and more durable.

**44. Stone-Masonry Footings.**—Stone has been frequently employed in the construction of foundations, particularly for bridge abutments and piers, dams, retaining walls, and occasionally for low buildings. The stones usually employed are granite, limestone, sandstone, and trap rock. The kinds of masonry most used for foundations are *squared-stone* and *rubble* masonry.

In **squared-stone masonry** all stones are roughly squared and roughly dressed on all sides and the thickness of the mor-

tar joints between any two stones is ½ inch or more. If well laid in good mortar, masonry of this type has proved to be strong and durable. The greatest danger to stone masonry is from water percolating into the joints, and freezing there in winter, because water, when freezing, expands so that each fissure and joint is constantly being enlarged by the repeated freezing of the water until finally entire disintegration of the
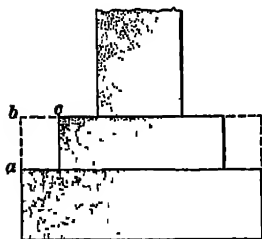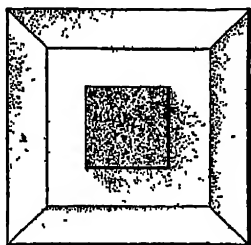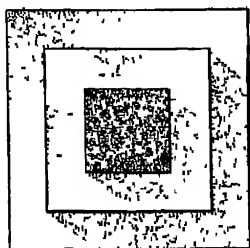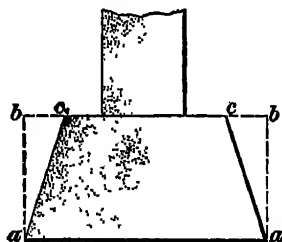


Fig 19                    Fig 20

structure ensues. This danger can be avoided only by frequent inspection and conscientious repairing of all defective joints.

In **rubble masonry** the stones are not squared. Many foundations of sea walls and breakwaters as well as foundations for large dams and retaining walls are of this type.

Stone masonry, like brick, has been superseded by concrete as material for foundations, except in those cases where stone is cheap and plentiful. Even where the stone may be obtained free of charge, the cost of dressing and handling the blocks is frequently much greater than the cost of reducing them to broken stone and making them into concrete.

**45. Plain-Concrete Footings.**—Footings of plain concrete are used only where the offset or projection of the footing course is slight and where great weight is no objection. If the projection exceeds one-half of the depth of the course, reinforced concrete should be used, because otherwise the projection is likely to break off.

Plain-concrete column footings are constructed as *stepped* footings, as in Fig. 19, or as *sloped* or *tapered* footings, as in Fig. 20. The purpose of the step or the slope is to save the concrete indicated by the dotted lines *a b c* in Figs. 19 and 20, but unless this amount is considerable the higher cost of the more complicated forms usually more than offsets the saving
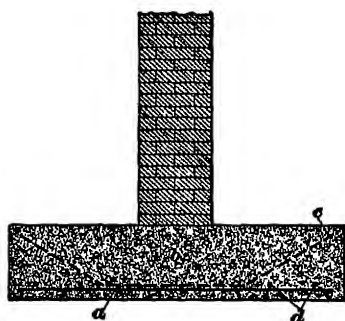


FIG. 21

in concrete. This is especially true of the tapered footing, for which a tapered mold is required. Tapered molds are expensive to build and awkward to handle and, in addition, they have a tendency to float because the liquid concrete within presses upwards on the sloping sides. It is therefore often necessary to load the forms for tapered footings with large stones or to brace them with timbers, all of which costs money; contractors therefore generally prefer vertical sides without steps or slopes.

The cross-section of a wall footing also may have vertical, stepped, or sloping sides, the first two named being preferred as cheaper and more easily built.

When starting a concrete footing on the foundation bed, it is advisable to wet the ground before depositing the concrete, as otherwise the ground, if dry or porous, may rob the concrete of the water required for its proper hardening, but an excess of water should not be used.

**46. Reinforced-Concrete Footings.**—As has been explained, plain-concrete footings should not project more than

half the depth of each step, because of danger that the step will break off if the projection is greater. By the use of concrete reinforced with steel to take the tension stress, however, the footing may be spread over a greater width without unduly increasing its depth. Since in ordinary walls and column footings the tension stresses occur at the under side of the footing, the reinforcement is usually placed near the bottom
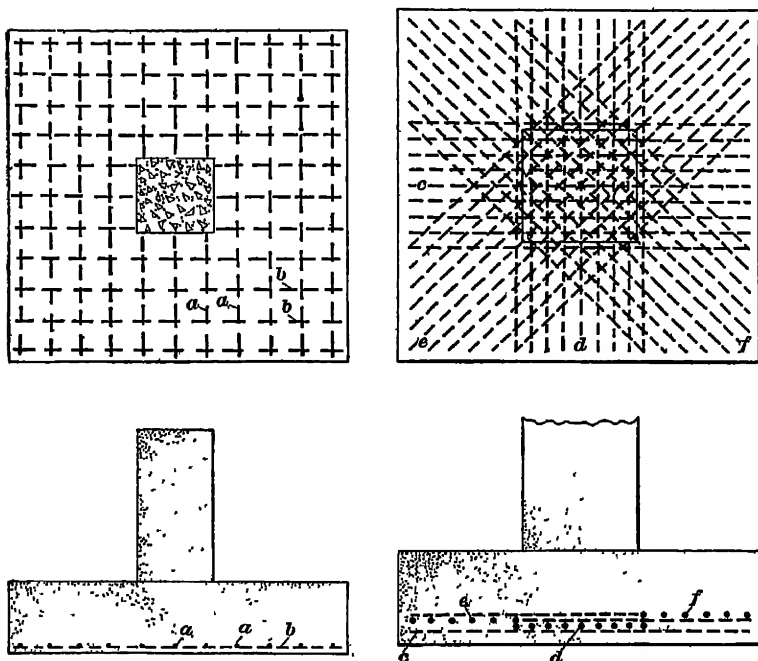


FIG. 22



FIG. 23

of the concrete, but not too close to the surface because of the danger of corrosion. It is customary to protect the steel with from 2 to 4 inches of concrete.

In wall footings, the steel reinforcement, as shown in Fig. 21, consists of tension rods *a* laid crosswise near the bottom of the footing *b*. Sometimes every second rod is bent or trussed as indicated by the dotted line *c*. So-called distributing rods *d* are sometimes used lengthwise in the footing; these enable the

footing to bridge over any soft local spots without breaking, and they also serve to space correctly the tension rods, which are wired at regular intervals to the distributing rods.

Column footings may be square or rectangular in plan. In either case the sides of the footing may be vertical as in Figs. 22 and 23 or sloping or tapered as in Fig. 24. The tapered footing usually terminates downwardly in a portion *a*, Fig. 24, with vertical sides; this portion, from 4 to 12 inches deep, is always cast in one piece with the remainder of the concrete and is a separate course in appearance only. The purpose of this vertical portion is to avoid a thin edge such as would result at *m n o* if no vertical portion were used. Such a thin edge, called a *feather edge*, is objectionable in all kinds of concrete work: (1) because it is difficult to fill concrete properly into such a form; (2) because even if the form is properly filled, the edge is likely to be broken off when the form is removed, and (3) because the edge dries out
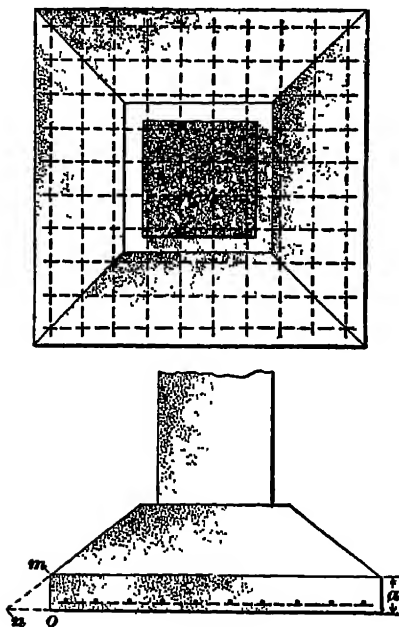


Fig. 24

too quickly in hot weather and therefore attains less strength than the main body of the concrete.

In square column footings, the reinforcement may consist of two equally strong layers of rods crossing one another at right angles, as in Fig. 22, where the layers are marked *a* and *b*, respectively; or, as in Fig. 23, four layers, *c, d, e,* and *f* may be used. This construction is preferred by some engineers because here all the bars pass underneath the columns. In rectangular footings, Fig. 25, usually two layers are used.

**47.** Reinforced-concrete footings are often used over either wood or concrete piles in the manner shown in Fig. 25. In this illustration the piles $a$ project only 3 inches into the concrete $b$, but in heavy bridge piers or retaining walls the piles may project a foot or more into the concrete. Two layers of reinforcement are customarily used; these cross at right angles a couple of inches above the upper ends of the piles.
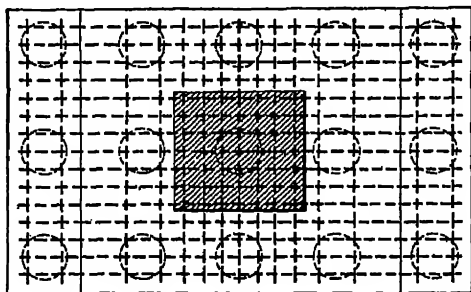
**48.** If the soil of the foundation bed is very compressible and heavy loads are to be sustained, the entire foundation bed may be covered with a thick layer of concrete the function of which is to equalize the pressure. In order to distribute the load uniformly, the layers must be many feet thick or be reinforced with steel. A foundation consisting of a plate of reinforced concrete extending over the entire foundation area is called a *raft;*
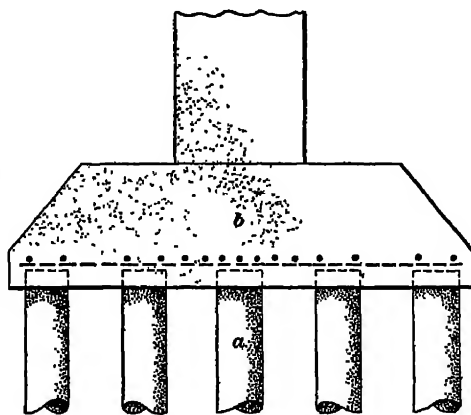


Fig. 25

an example of a raft foundation without piles is shown in Fig. 26, but piles are often used under footings of this kind. The raft of Fig. 26 consists of a slab $a$ reinforced with beams $b$ crossing at right angles; the columns of the structure to be supported are located at the intersections $c$ of these beams. The spaces between beams are filled with a cin-

der fill *d* upon which the concrete slab *e* rests. This slab is a thin concrete plate, usually 3 or 4 inches thick, which forms the basement floor.



Section A B    Section C D
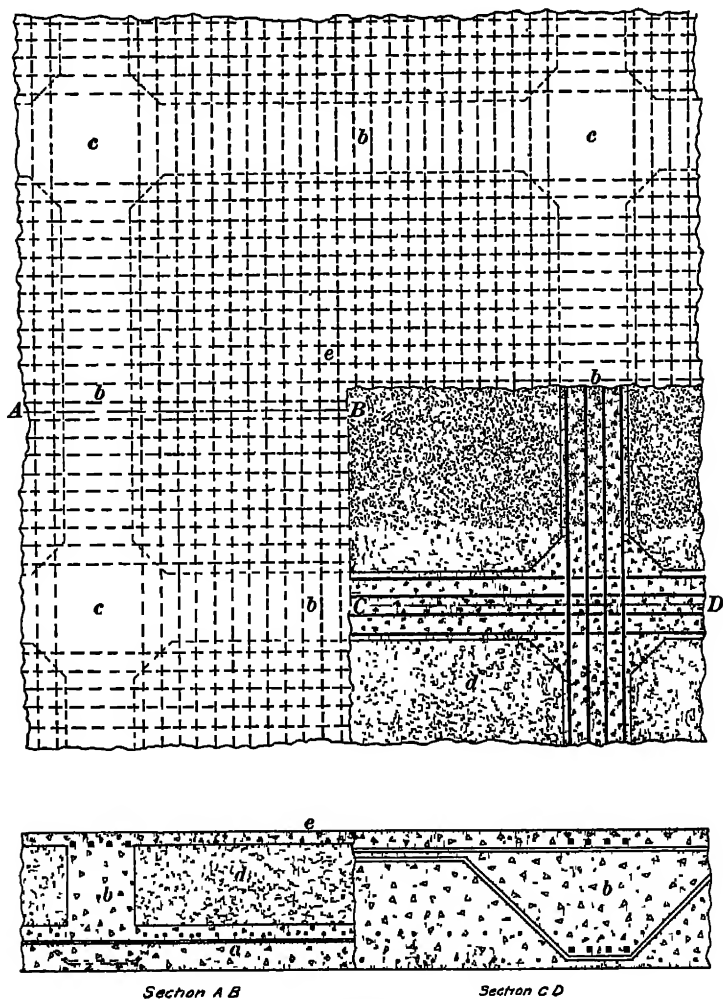
FIG. 26

**49. Structural-Steel Footings.**—Footings consisting wholly or even mainly of steel and iron are not often con-

structed. There is, however, a much used type of footings known as **I-beam grillage,** in which steel I beams are utilized in connection with concrete in such manner that the **I** beams alone are depended upon for strength, the function of the con-
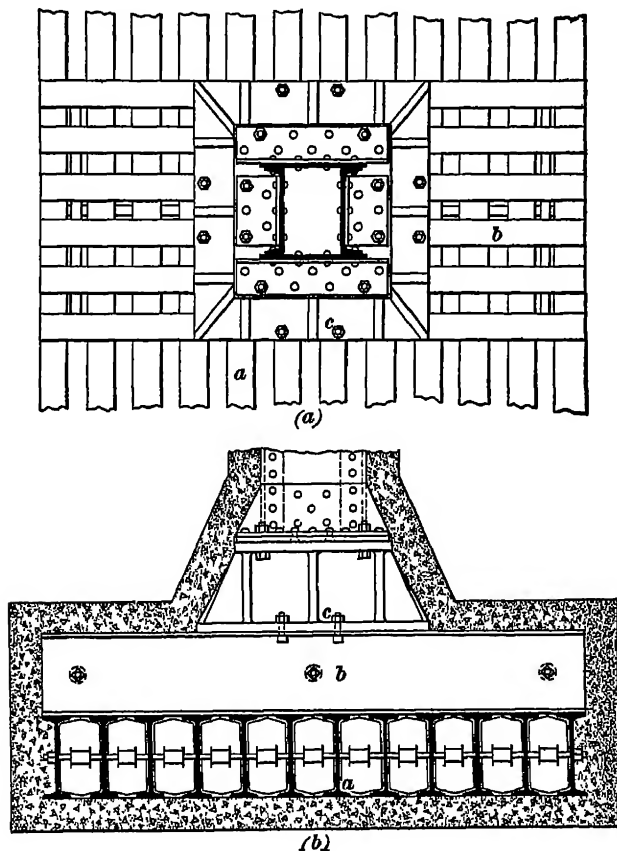


Fig. 27

crete being merely that of a protecting covering or envelope, in distinction from reinforced-concrete footings, where both concrete and steel are depended upon for strength. Footings of the grillage type are extensively used for tall buildings of steel-skeleton construction, especially where it is desired to

construct a footing that occupies a minimum of height, because the I beams require less space in a vertical direction than is required by any other form of construction. As illustrated in Fig. 27, an I-beam grillage contains at least two layers of I beams. Each layer consists of a number of I beams side by side; in the illustration, the bottom layer contains 12 I beams $a$, and the top layer 6. The central portion of the top layer $b$ supports a cast-iron chair $c$ intended to support a column. The I beams are not set closely side by side, but a space of at least 3 or 4 inches is always left between beams so that concrete can be filled in between them. In order to maintain uniformly the desired distance between beams, separators, often of cast iron, are placed between the beams to keep them apart. The separators as well as the beams have holes through which long bolts are passed for the purpose of keeping the beams together. Some engineers object to the use of the cast-iron separators because these are so large that they interrupt the continuity of the concrete; to avoid this difficulty short lengths of gas pipe may then be used as separators, the pipes being slipped sleeve-fashion over the bolts between I beams. Grillage footings are very expensive, especially when steel is high in price, and the previously described reinforced-concrete footings are far more economical.

# STATICS

## INTRODUCTION

**1. Mechanics** is that science which treats of the action of forces on bodies and the effects that they produce. By a *body*, as here used, is meant any material thing; it may be a beam or a column, a brick, or any object that occupies space and has weight.

**2.** A body is **at rest** when it constantly occupies the same position with respect to its surroundings.

**3.** A body is **in motion** when the body or its parts occupy successively different positions with respect to its surroundings.

**4.** A **force** is any cause that changes or tends to change a body's state of rest or motion.

**5.** A body is said to be in **equilibrium** when all the external forces acting on it are balanced and will not, therefore, move the body or any part of it. All bodies that are at rest are in equilibrium and the forces acting on them are balanced. This is an important point for the structural engineer to observe. He deals with beams and columns that are at rest and therefore in equilibrium. Thus, the laws of equilibrium apply to the problems he is required to solve.

**6. Statics** is that division of the science of mechanics which treats of the relation between the forces acting on a body at rest.

This subject of statics is the basis for the entire theory of structural engineering. An engineer practicing this profession must design economically the parts of a structure so that it will never move, and he is concerned with nothing else.

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### NEWTON'S LAWS OF MOTION

**7.** Although this Section is devoted to statics, that is, to the action of forces on a body at rest, yet since forces are primarily compared by the motion they may create, it is necessary to consider the action of forces in making bodies move. In the study of bodies in motion, there are three laws enunciated by Sir Isaac Newton and proved by experiment that are of primary importance. They are known as **Newton's laws of motion,** and may be stated as follows:

**Law I.**—*Every body remains in its state of rest or of uniform motion in a straight line, except in so far as it may be compelled by a force to change that state.*

**Law II.**—*Change of motion is proportional to the force applied, and takes place in the direction of the straight line in which the force acts.*

**Law III.**—*The actions of two bodies on each other are always equal and directly opposite.*

**8.** The first law means that whenever a body that is at rest moves, or if in motion its motion is changed either in direction or in amount, this movement or change can always be traced to some external force. A body cannot move itself, but must be moved by some force external to itself.

**9.** The second law of motion is the one of importance to the engineer in compounding and resolving forces. Since it states that the motion is proportional to the force applied, the magnitude of two forces may be compared by the motion they can impart to a body, and their directions can be ascertained by the directions of the imparted motion.

The deduction from the second law is that, if two or more forces act on a body, their final effect on that body will

be in proportion to their magnitude and to the directions in which they act. Thus, if the wind is blowing due west with a velocity of 50 miles per hour, and a ball is thrown due north with the same velocity, or 50 miles per hour, the wind will carry the ball just as far west as the force of the throw carries it north, and the combined effect will be to cause it to move northwest. The amount of departure from due north will be proportional to the force of the wind, and independent of the velocity due to the force of the throw. The full meaning of this second law will be more fully understood when the force polygon is explained

**10.** The third law of motion means simply that if a certain weight, say one of 100 pounds, rests on a column, it of course presses down on the column with a force of 100 pounds. The column therefore presses up against the weight with the same force of 100 pounds. When the wind blows against a building with a force of 50 pounds per square foot, the building presses back against the wind with the same pressure of 50 pounds. If it did not, the building would move. This law is of importance because it enables the engineer to find with what force the supports of a beam press up against the beam itself. These forces of the supports pressing up, called the *reactions*, together with the loads, are the forces acting on the beam.

# COMPOSITION AND RESOLUTION
# OF FORCES

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### VECTOR QUANTITIES

**11.** A vector quantity is a quantity to give a complete description of which it is necessary to give a *direction*. Thus, when it is said that a ship is moving at the rate of 10 miles an hour, its velocity is not completely described. To do this, it is necessary to add that the ship is moving northeast or southeast or in a circle, as the case may be. Therefore, velocity is a vector quantity.

To saw a certain stick of wood with a hand saw requires 2 minutes of time. Now the 2 minutes is not a vector quantity because it has no direction. When it is said that for a certain event to happen it takes 2 minutes, the time is completely described and nothing more need be added.

**12. Force.**—The quantity of most importance to the structural engineer is force, which is a vector quantity because it acts in a certain specific direction.

**13. Effect of a Force on a Body.**—The effect of a force on a body may be compared with the effect of another force when the three conditions that follow are fulfilled with regard to both forces:

1. *The point of application, or point at which the force acts on the body, must be known.*

2. *The direction of the force, or, what is the same thing, the straight line along which the force tends to move the point of application, must be known.*

3. *The magnitude of the force, when compared with a given standard, must be known.*

In this and the succeeding Sections, the unit of force will always be taken as 1 pound, and the magnitudes of all forces will be expressed in pounds.

**14. Representation of a Force.**—A force may be represented by a line.   Thus, in Fig. 1, let $A$ be the *point of application* of the force, let the length of the line $AB$ represent its

$A$———→———$B$

Fig 1

*magnitude* and *line of action* to any convenient selected scale, as for instance, 1 inch equals 10 pounds, and let the arrowhead indicate the *direction* in which the force acts.   Then the line $AB$ fulfils the three required conditions in regard to point of application, direction, and intensity, and the force is fully represented.

The force, represented graphically in Fig. 1, is termed a *vector.*   $AB$ might in some other interpretation also indicate to some other scale a velocity.   A **vector** is therefore a graphical representation of a vector quantity.

**15. Changing the Point of Application.**—There is one property of a force that is yet to be mentioned.   The
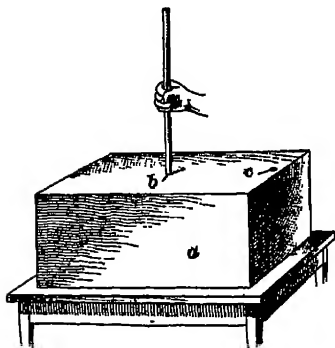


Fig 2
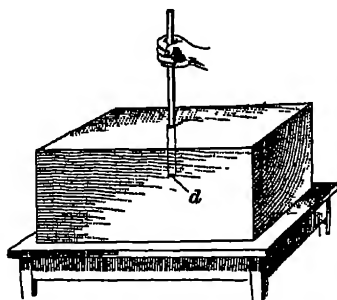


Fig. 3

point of application of a force may be changed in the direction of its line of action without changing the external effect of that force.   Thus, in Fig. 2, let a rod press in the direction of its length on the block $a$ at the point $b$.   The force exerted by this rod presses the block down on the table with a

certain definite pressure on each square inch of the table that is under the block. Now, imagine that the rod is moved and presses on the block at the point $c$. It can be seen that in this case the pressure will change on each square inch of the table and that one edge of the block will bear much more heavily than the other. Now, suppose that at the point $b$ a hole is bored directly down into the block to a point $d$, as shown in Fig. 3, and that the rod is inserted in this hole and pressed down. Then, the rod pressing at $d$ will have precisely the same effect on the pressure between the block and the table as the rod pressing at $b$, Fig. 2, because the point of application is moved in the direction of the line of action of the force; but while pressing at the point $c$ the point of application is not located in the direction of the line of action.

## COMPOSITION OF FORCES

**16.** If several forces act on a body, and if they are replaced by a single force that has the same effect in moving the body through space as the several forces combined, the single force is called the **resultant** of the several forces; and, conversely, the several forces are called the **components**



of the single force. The process of finding the resultant when the various components are known is called the **composition of forces.**
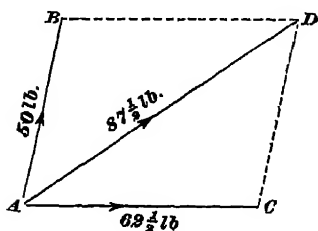
FIG 4

**17. Parallelogram of Forces.**—When two forces act on a body at the same time, but at different angles, their final effect may be obtained as follows:

In Fig. 4, let $A$ be the common *point of application* of the two forces, and let $A B$ and $A C$ represent the *magnitude* and *direction* of the forces. The final effect of these two forces will be the same whether they act singly or together. Let, for instance, the line $A B$ represent the distance that the force $A B$ would cause the body to move in a certain

length of time; similarly, let $A\,C$ represent the distance
that the force $A\,C$ would cause the body to move in the same
length of time, when both forces were acting separately.
The second law of motion states that the motion is propor-
tional to the force applied, and, therefore, while $A\,B$ and $A\,C$
represent the magnitude of the forces to some scale, they
are also proportional to the distances these forces would
move the same body in the same length of time. The
force $A\,B$, acting alone, would carry the body to $B$. If
the force $A\,C$ were now to act on the body, it would carry it
along the line $B\,D$, parallel to $A\,C$, to a point $D$, at a dis-
tance from $B$ equal to $A\,C$. Join $C$ and $D$, then $C\,D$ is
parallel to $A\,B$ and $A\,B\,D\,C$ is a parallelogram. Draw the
diagonal $A\,D$. The body will stop at $D$, whether the forces
act separately or together, but if they act together, the path
of the body will be along $A\,D$, the diagonal of the parallelo-
gram. Moreover, the length of the line $A\,D$ represents the
magnitude of a force, which, acting at $A$ in the direction $A\,D$,
would cause the body to move from $A$ to $D$; in other words,
$A\,D$, measured to the same scale as $A\,B$ and $A\,C$, represents
the magnitude and direction of the combined effect of the
two forces $A\,B$ and $A\,C$.

The force represented by the line $A\,D$ is the resultant of
the forces $A\,B$ and $A\,C$. Suppose that the scale used was
50 pounds to the inch; then, if $A\,B = 50$ pounds and $A\,C$
$= 62\frac{1}{2}$ pounds, the length of $A\,B$ would be $50 \div 50 = 1$ inch,
and the length of $A\,C$ would be $62.5 \div 50 = 1\frac{1}{4}$ inches. If the
line $A\,D$ measures $1\frac{3}{4}$ inches, the magnitude of the resultant,
which it represents, would be $1\frac{3}{4} \times 50 = 87\frac{1}{2}$ pounds.

Therefore, a force of $87\frac{1}{2}$ pounds, acting on a body at $A$,
in the direction $A\,D$, will produce the same result as the
combined effects of a force of 50 pounds acting in the direc-
tion $A\,B$ and a force of $62\frac{1}{2}$ pounds acting in the direction $A\,C$.

18. The foregoing method of finding the resulting action
of two forces acting on a body when their lines of action inter-
sect, is correct, whatever may be their direction and magni-
tudes. Hence, to find the resultant of two forces when the

point of intersection of their lines of action, their direction, and magnitudes are known, the following rule may be applied:

**Rule.**—*Through the point of intersection draw two lines parallel with the direction of the two forces. With any scale, measure from the point of intersection, in the direction of the forces, distances corresponding to the magnitudes of the respective forces, and from the points thus obtained complete the parallelogram. Draw the diagonal of the parallelogram from the point of intersection of the two forces; this diagram will represent the resultant, and its direction will be away from the point of intersection of the two forces. To find the magnitude of the resultant, the diagonal must be measured with the same scale that is used to lay off the two forces.*

EXAMPLE.—If two forces act on a body at a common point, and the angle between them is 80°, what is the value of the resultant, the magnitude of the two forces being 60 pounds and 90 pounds, respectively?

SOLUTION.—Draw two indefinite lines having an angle of 80° between them (Fig. 5). With any convenient scale, say 10 lb. to the in., measure off

$$A B = 60 \div 10 = 6 \text{ in.}$$
and $$A C = 90 \div 10 = 9 \text{ in.}$$

Through $B$ draw $B D$ parallel to $A C$, and through $C$ draw $C D$ parallel to $A B$, intersecting at $D$. Then draw $A D$, and $A D$ will be the resultant, its direction is toward the point $D$, as shown by the arrow. By measurement, the length of $A D$ is found to be 11.7 in. Hence, the magnitude of the resultant is

$$11.7 \times 10 = 117 \text{ lb.} \quad \text{Ans.}$$

The preceding problem, instead of being actually laid out on paper, could be solved by the rules of trigonometry, but the graphic method as just described will usually be found more simple.

**19. Triangle of Forces.**—Let Fig. 5 represent a parallelogram of forces. $A B$ and $A C$ are the two component

forces and $AD$ is the resultant.   Now, to find this resultant $AD$ without drawing the entire parallelogram, first lay off $AB$, and then from the point $B$ lay off $BD$ equal to $AC$ and parallel to its line of action; then, draw $AD$, which completes the triangle $ABD$ and makes it unnecessary to draw $AC$ and $CD$. Or, the force $AC$ could be drawn first, and then from $C$, the line $CD$, thus completing



FIG 5

the triangle $ACD$ and not drawing $AB$ and $BD$.   In either case, it is seen that, if desired, the resultant of two forces can be found by drawing a triangle instead of a parallelogram.
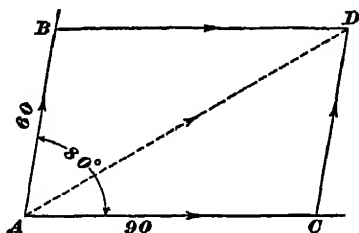
**Rule.**—*Through any point draw a line to represent either of the forces in magnitude and direction.   At the extremity of this line draw a second line parallel to the line of action of the second force and representing this force in magnitude and direction. Join the extremities of the two lines by a straight line; then this joining line will represent the resultant, and its direction will be in the same general direction as that of the two forces.*

**20.   Resultant of Several Forces.**—When three or more forces act on a body at a given point, their resultant may be found by the following rule:

**Rule.**—*Find the resultant of any two forces; treat this resultant as a single force, and combine it with a third force to find a second resultant.   Combine this second resultant with a fourth force, to find a third resultant, etc.   After all the forces have been thus combined, the last resultant will be the resultant of all the forces, both in magnitude and direction.*

EXAMPLE —Find the resultant of all the forces acting on the point $O$, Fig 6, the length of the lines being proportional to the magnitude of the forces.

SOLUTION —Draw $OE$ parallel and equal to $AO$, and $EF$ parallel and equal to $BO$, then $OF$ is the resultant of these two forces, and its direction is from $O$ to $F$, as explained in Art. **19.**   Consider $OF$ as

replacing $OE$ and $EF$, and draw $FG$ parallel and equal to $CO$; $OG$ will be the resultant of $OF$ and $FG$; but $OF$ is the resultant of $OE$ and $EF$; hence, $OG$ is the resultant of $OE$, $EF$, and $FG$, and likewise of $AO$, $BO$, and $CO$. The line $FG$, parallel to $CO$, could not be drawn from the point $F$ to the right of $OF$, for in that case it would be opposed in direction to $CO$; but $FG$ must have the same direction as $CO$, as stated in Art. **19**.

For the same reason, draw $GL$ parallel and equal to $DO$. Join $O$ and $L$, and $OL$ will be the resultant of all the forces $AO$, $BO$, $CO$ and $DO$ (both in magnitude and direction) acting at the point $O$. If $L'O$ is drawn parallel and equal to $OL$, and having the same direction, it will represent the effect produced on the body by the combined action of the forces $AO$, $BO$, $CO$, and $DO$. For brevity, the terms forces $AO$, $BO$, etc. and resultants $OF$, $OG$, and $OL$ have been used
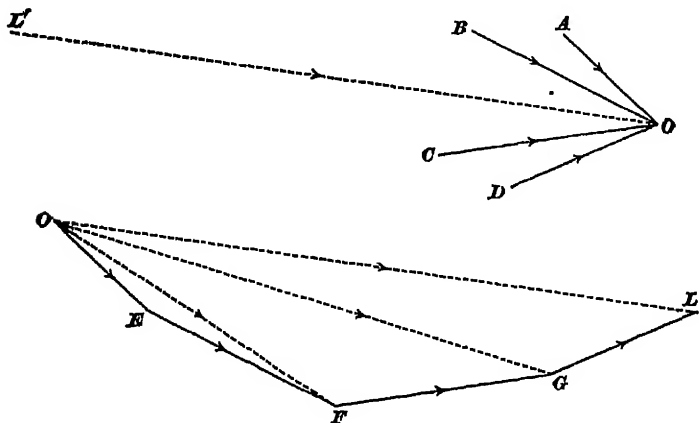


Fig 6

in this solution. It should be remembered, however, that these are merely lines that represent the forces in magnitude and direction.

**21.** It will be noted that in Fig. 6 the forces $OE$, $EF$, etc., all point in the same direction; that is, a body at $O$ acted on by these forces in succession would move from $O$ to $L$. The resultant therefore acts from $O$ to $L$, and not from $L$ to $O$. This line of reasoning will give the direction of the resultant in a force diagram. In Fig. 6, the forces were taken in the order $AO$, $BO$, $CO$, and $DO$. However, the magnitude and direction of the resultant $OL$ would be the same, no matter in what order the forces were taken.

**22. Method of the Polygon of Forces.**—In Fig. 6, it will be noticed that $OE$, $EF$, $FG$, $GL$, and $LO$ are sides of a polygon $OEFGL$, in which $OL$, the resultant, is the closing side, and that the direction of the resultant is such that it would carry a body to the same place as would all the other forces acting in succession. This fact is made use of in what is called the **method of the polygon of forces.**

To find the resultant of several forces acting on a body at the same point:

**Rule.**—*Let the several forces be represented in direction and magnitude by lines, as explained in Art. 14. Through any*
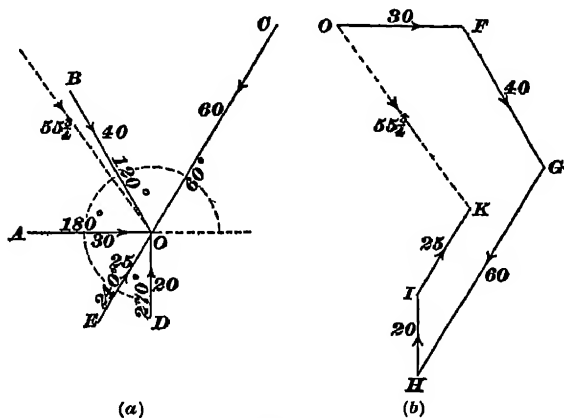


Fig. 7

*point draw a line parallel to any one of the forces, and having the same direction and magnitude. At the end of this line draw another line, parallel to a second force, and having the same direction and magnitude as this second force; at the end of the second line draw a line parallel and equal in magnitude and direction to a third force. Thus continue until lines have been drawn equal in magnitude, and having the same directions, respectively, as the lines representing the several forces.*

*The straight line joining the free ends of the first and last lines will be the closing side of the polygon. Mark its direction as moving a body the same as the individual forces, and it will represent in magnitude and direction the resultant of all the forces.*

EXAMPLE —Assuming that five forces act on a body at angles of 60°, 120°, 180°, 240°, and 270° toward the same point, and that their respective magnitudes are 60, 40, 30, 25, and 20 pounds, find the magnitude and direction of their resultant by the method of polygon of forces.*

SOLUTION.—From a common point $O$, Fig. 7 $(a)$, draw the lines of action of the forces, making the given angles with a horizontal line through $O$, and mark them as acting toward $O$, by means of arrowheads, as shown. Now, choose some convenient scale, such that the whole figure may be drawn in a space of the required size on the drawing. Choose any one of the forces, as $A\,O$, and draw $O\,F$, Fig. 7 $(b)$, parallel to it, and equal in length to 30 lb. on the scale; it must also act in the same direction as $A\,O$  From $F$, draw $F\,G$ parallel to $B\,O$, and make its length equal to 40 lb. on the assumed scale. In a similar manner, draw $G\,H$, $H\,I$, and $I\,K$ parallel to $C\,O$, $D\,O$, and $E\,O$, and equal on the scale to 60, 20, and 25 lb., respectively. Join $O$ and $K$ by $O\,K$, then $O\,K$ will represent in magnitude and line of action the resultant of the combined action of the five forces. The direction of the resultant is from $O$ to $K$ and its magnitude is $55\frac{3}{4}$ lb.   Ans.

**23.** If the resultant $O\,K$, Fig. 7, were to act alone on the body in the direction shown by the arrowhead with a force of $55\frac{3}{4}$ pounds, it would produce exactly the same effect as the combined action of the five forces. If $O\,F$, $F\,G$, $G\,H$, $H\,I$, and $I\,K$ represent the distances and directions that the forces would move the body, if acting separately, $O\,K$ is the direction and distance of movement of the body when all the forces act together.

From what has been said, it is evident that any number of forces acting on a body at the same point, or having their lines of action pass through the same point, can be replaced by a *single force* (resultant) whose line of action shall pass through that point.

Heretofore, it has been assumed that the forces acted on a single point on the surface of the body, but it will make no difference in finding the resultant where they act, so long as the lines of action of all the forces intersect somewhere at a common point.

*All the angles in the figure are measured from a horizontal line in a direction opposite to the movement of the hands of a watch, from 1° up to 360°.

**24.** When the lines of action of all the forces acting on a body do not meet at one point, the forces may either be parallel or not. The method of finding the resultant of parallel forces will be explained later. When the forces are not parallel the *magnitude* and the *direction* of the resultant may be determined in the same manner as when the forces intersect in one point, that is, as the closing line of the polygon of forces drawn exactly as explained in the foregoing articles. But the process



Fig. 8
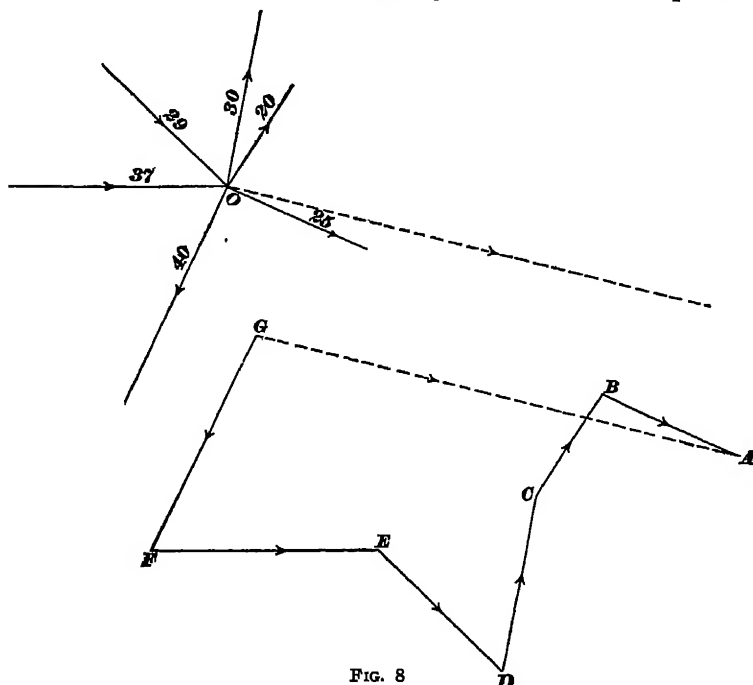
of determining the *line of action* of this resultant is not so simple. This subject will be treated in another Section.

If two forces act on a body in the same straight line and in the same direction, their resultant is the *sum* of the two forces; but if they act in opposite directions, their resultant is the *difference* of the two forces, and its direction is the same as that of the greater force. If they are equal and opposite, the resultant is zero, for one force just balances the other.

EXAMPLE.—Find the resultant of the forces shown in Fig 8.

SOLUTION.—Take any convenient point $G$, and draw a line $GF$ parallel to one of the forces, say the one marked *40*, making it equal in length to 40 lb. on the scale, and indicate its direction by the arrow-head.    Take some other force—the one marked *37* will be convenient; the line $FE$ represents this force.    From the point $E$ draw a line parallel to some other force, say the one marked *29*, and make it equal in magnitude and direction to that force.    So continue with the other forces, taking care that the general direction around the polygon is not changed    The last force drawn in the figure is $BA$, representing the force marked *25*    Join the points $G$ and $A$, then, $GA$ represents the resultant of all the forces shown in the figure.    Its direction is from $G$ to $A$.    It does not matter in what order the different forces are taken, the resultant will be the same in magnitude and direction if the work is done correctly.

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### RESOLUTION OF FORCES

**25.**    Since two forces can be combined to form a single resultant force, a single force may also be treated as if it were the resultant of two forces whose joint action on a body will be the same as that of a single force.    Thus, in Fig. 9, the force $OA$ may be resolved into two forces, $OB'$ and $B'A$.

It will be noted in the illustration that one resultant force may have an innumerable number of combinations of com-
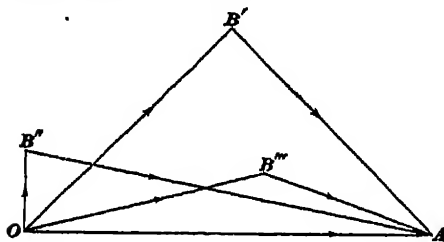


FIG. 9

ponents.    Instead of $OB'$ and $B'A$, Fig. 9, $OB''$ or $B''A$ or $OB'''$ and $B'''A$ may be taken as components.    In fact, wherever the point $B$ of a triangle $OBA$ is located, the sides $OB$ and $BA$ will always represent in magnitude and direction two components that will together make the resultant force $OA$.    It is customary, however, for reasons that will appear later, to make $OB'$ and $B'A$ perpendicular to each other, as shown in the illustration.    That is to say, it is customary to make the angle at $B'$ a right angle.

**26.** It frequently happens that the position, magnitude, and direction of a certain force are known, and that it is desired to know the effect of the force in some direction other than that in which it acts. Thus, in Fig. 10, suppose that $OA$ represents, to some scale, the magnitude, direction, and line of action of a force acting on a body at $A$, and that it is desired to know what effect $OA$ produces in the direction $BA$. From $A$ draw a line $AB$ in the required direction; from $O$ draw a line perpendicular to $AB$. Then $BA$ is the component required. It is necessary, of course, that $OB$ be at right angles to $BA$, so that all the effect of $OA$ in the required direction may be represented by $BA$ and none of it by $OB$. Thus, $OB'$ and $B'A$, although components of $OA$, and although one of them is in the required direction, would not be a
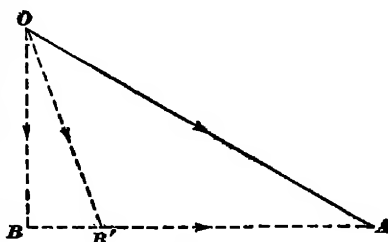


FIG. 10

correct solution of the problem because, besides $B'A$, $OA$ exerts some more effect in the line $BA$, namely, a part of $OB'$ is in that direction with an amount $BB'$. To find the value of the component of $OA$, which acts in the direction $BA$, the following rule is employed:

**Rule.**—*From one extremity of the line representing the given force draw a line parallel to the direction in which it is desired that the component shall act; from the other extremity of the given force draw a line perpendicular to the component first drawn and intersecting it. The length of the component, measured from the point of intersection to the intersection of the component with the given force, will give the magnitude of the effect produced by the given force in the required direction.*

Thus, suppose that $OA$, Fig. 10, represents a force acting on a body resting on a horizontal plane, and that it is desired to know what vertical pressure $OA$ produces on the body. Here the desired direction is vertical; hence, from ore extremity, as $O$, draw $OB$ parallel to the desired direc-

tion (vertical in this case), and from the other extremity draw $AB$ perpendicular to $OB$ and intersecting $OB$ at $B$. Then $OB$, when measured to the same scale as $OA$, will give the magnitude of the vertical pressure produced by $OA$.

EXAMPLE.—If a body weighing 200 pounds rests on an inclined plane whose angle of inclination to the horizontal is 18°, what force does it exert perpendicular to the plane, and what force does it exert parallel to the plane, tending to slide it downwards?

SOLUTION.—Let $ABC$, Fig. 11, be the plane, the angle $A$ being 18°, and let $W$ be the weight. Draw a vertical line $FD$ = 200 lb., to repre-



FIG. 11

sent the magnitude of the weight. Through $F$ draw $FE$ parallel to $AB$, and through $D$ draw $DE$ perpendicular to $EF$, the two lines intersecting at $E$. $FD$ is now resolved into two components, one $FE$ tending to pull the weight down the incline, and the other $ED$ acting as a perpendicular pressure on the plane. On measuring $FE$ with the same scale by which the weight $FD$ was laid off, it is found to be about 61 8 lb , and the perpendicular pressure $ED$ on the plane is found to measure 190 2 lb Ans.

The preceding problem can be done by trigonometry as follows: According to the principles of geometry, the angle $EDF$ is equal to the angle $BAC$, namely, 18°. $ED$, the component pressure perpendicular to the plane, is equal to $FD \cos EDF = 200 \cos 18 = 200 \times .951 = 190.2$ pounds. Likewise, the force $EF$ that will cause the weight to slide down the plane is



FIG. 12

$FD \sin EDF = 200 \sin 18 = 200 \times .309 = 61.8$ pounds

**27.** As it is often necessary to resolve a force into two components at right angles to each other, a simple method
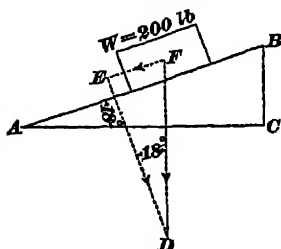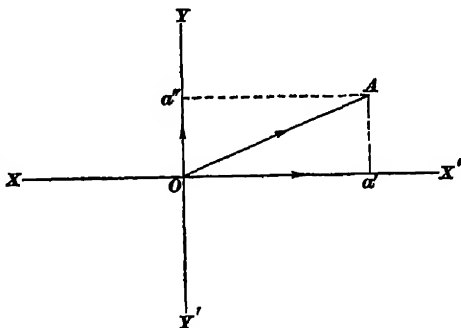
of solution is employed.  In Fig. 12, let $OA$ be the force it
is desired to resolve into two components at right angles
to each other.   Through $O$ as a center draw an indefinite
line $XX'$ in the required direction of one of the components,
and through the same center $O$ draw the indefinite line $YY'$
in the direction of the other component.   Then $XX'$ and
$YY'$ are at right angles to each other, since the components
to be found are to be at right angles to each other.   The



Fig. 12
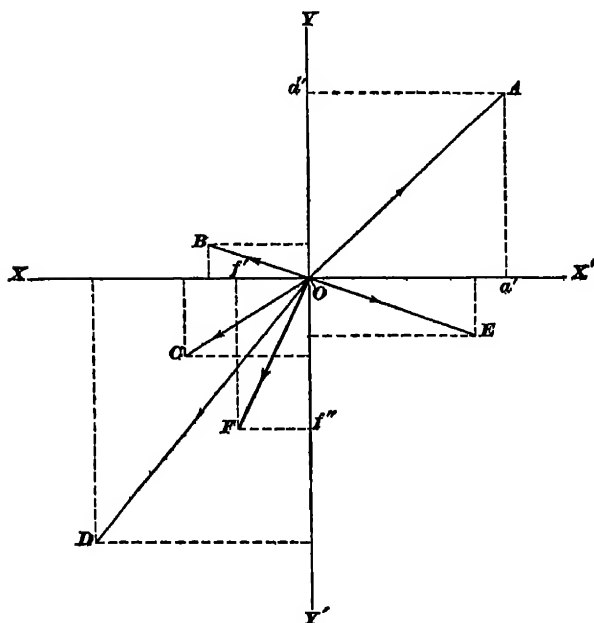
line $XX'$ is commonly called the $XX'$ *axis*, or simply the
$X$ *axis*, and the line $YY'$, the $YY'$ *axis* or the $Y axis$.   From
$A$ draw a line perpendicular to the $X$ axis, as $A a'$, and also
a line perpendicular to the $Y$ axis, as $A a''$.   Then $O a'$ is the
component force in the direction of $XX'$ and $O a''$ is the other
component in the direction of $YY'$.

    When the components $O a'$ and $O a''$ are horizontal and
vertical, as in the present case, they are called, respectively,

the *horizontal component* and the *vertical component* of the force $OA$.

**28.** The preceding article gives another and useful method of finding the resultant of two or more forces acting at one point. In Fig. 13 let the forces $OA$, $OB$, $OC$, $OD$, and $OE$ act on a body at the point $O$. Through $O$ draw two axes at right angles to each other—say one horizontal and one vertical, as $XX'$ and $YY'$. Find the horizontal and vertical components of each of the five forces; thus, $OA$ has a horizontal component of $Oa'$ and a vertical component of $Oa''$. In a similar manner, each of the other forces may be divided into horizontal and vertical components along the $XX'$ and $YY'$ axes. Now, the horizontal components of $OA$ and $OE$ act toward the right, and the horizontal components of $OB$, $OC$, and $OD$ act in the opposite direction. Hence, the difference of the sum of the horizontal components of $OA$ and $OE$ and the sum of the horizontal components of $OB$, $OC$, and $OD$ will give the resultant horizontal component, which is $Of'$. Likewise, the difference of the sum of the vertical components of $OA$ and $OB$ and the sum of the vertical components of $OC$, $OD$, and $OE$ gives $Of''$, the resultant vertical component. Now, by completing the parallelogram, of which $Of'$ and $Of''$ are two sides, it will be found that $OF$ is the diagonal. This is the resultant of all the forces $OA$, $OB$, $OC$, $OD$, and $OE$.

**29.** In the previous discussion it has been assumed that the lines of action of forces intersect. This, however, is not always the case. They may not be in the same plane or they may be parallel. The study of forces not in the same plane is of no concern here, but the study of forces parallel is. The method of finding the point of application of the resultant of parallel forces involves the principle of moments and will be described later. Here will simply be mentioned the method of finding the magnitude of the resultant. The condition of equilibrium of parallel forces, so far as motion through space is concerned, is simply this: The sum of the forces acting one way must equal the sum of

those acting the other way. Great care must be taken to note that parallel forces may be in equilibrium, so far as moving the body from one place to another is concerned, but that the body itself on which they act is not in equilibrium. That is to say, the forces may not tend to move the body through space along a straight path, but they may tend to rotate it. This tendency to rotate will be discussed later.

Let Fig. 14 represent the forces acting on any body, as, for instance, the one shown. Now, there are four forces acting upwards and their sum is $10 + 30 + 80 + 5 = 125$ pounds. There are two forces acting downwards and their sum is
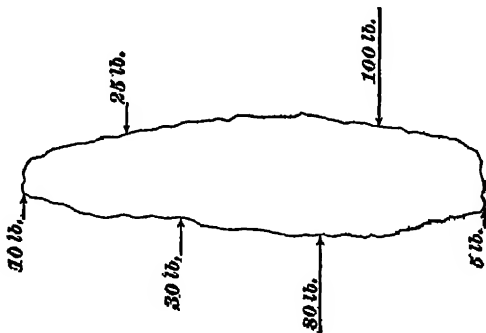


FIG. 14

$25 + 100 = 125$ pounds. Therefore, there is the same amount of force acting one way as there is acting the other, and the body will move neither up nor down. Suppose, however, that the 25-pound force acting downwards is taken away. There will then be a predominance of force acting upwards and the body will move upwards.

**30.** It will be noted that if to any system of forces a force is added equal in magnitude, but acting in the opposite direction to the resultant of the forces in the system, this new force will put the system in equilibrium so far as moving the body as a whole is concerned; that is, the body on which they act will not tend to move through space. This

fact is self-evident. By referring to Fig. 13 the forces $OA$, $OB$, $OC$, $OD$, and $OE$ may be replaced by $OF$. Now, if to this system is added a force $FO$ acting from $F$ to $O$, it will just balance $OF$, which is the same thing as all the forces in the system.

## MOMENTS OF FORCES

### DEFINITION AND MEASUREMENTS

**31.** In Fig. 15, $W$ is a weight that tends to fall—that is, to act downwards—with a force of 10,000 pounds. It is well known that if some fixed point, as $a$, not in the line along which the weight $W$ acts, is connected with the line of action of $W$ by a rigid arm, so that $W$ pulls on one end of this arm while the other end is firmly held at $a$, the pull of $W$ will tend to turn, or rotate, the arm about the point $a$.

It is also known that the tendency to rotation is directly proportional to the magnitude of the force, provided the arm remains of the same length, and directly proportional to the length of the arm, if the force remains constant. In general, therefore, the rotative effect is proportional to the product of the magnitude of the force and the length of the lever arm. This product is called the moment of the force with respect to the point in question. Thus, in Fig. 15, the moment of the force $W$ with respect to the point $a$, is the product obtained, by multiplying the magnitude, 10,000 pounds, by the perpendicular distance, 10 feet, from the point $a$ to the line of action of $W$.
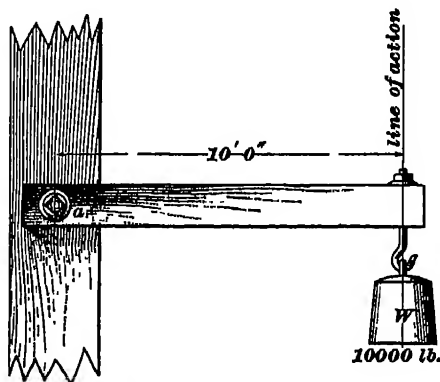


FIG. 15

**32.　Center of Moments.**—The point *a* that is assumed as the center around which there is a tendency to rotate, is called the **center, or origin, of moments.**

**33.　Lever Arm.**—The perpendicular distance from the center of moments to the line along which the force acts, is the **lever arm** of the force, or the **leverage** of the force.

**34.　Foot-Pound.**—Since the unit of force is the pound, and the ordinary unit of length is the foot, the unit of moment will be a derived unit, the **foot-pound,** and
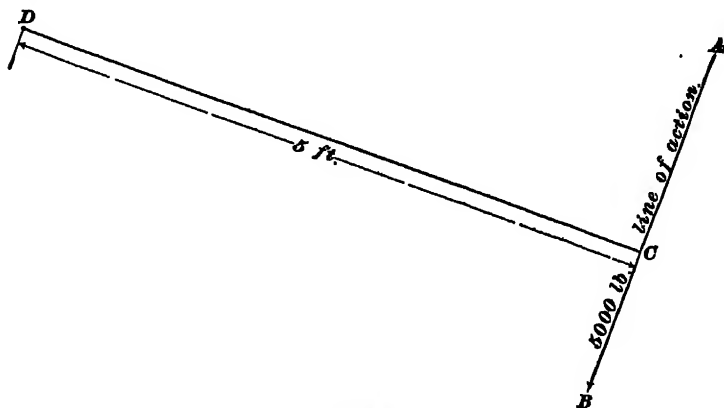


Fig. 16

moments will usually be expressed in foot-pounds. In Fig. 15, for example, the moment of the force *W* with respect to the point *a* is 10,000 × 10 = 100,000 foot-pounds.

EXAMPLE.—What is the moment of the force of 5,000 pounds whose line of action is *A B*, Fig. 16, the center of the moments being at *D*?

SOLUTION.—The perpendicular distance *D C* from the center of moments to the line of action of the force being 5 ft., and the magnitude of the force 5,000 lb., the moment is

5,000 × 5 = 25,000 ft.-lb.　Ans.

**35.**　In Fig. 17 the line of action of the force of 1,000 pounds passes through the point *D*; consequently, the perpendicular distance from the point *D* to the line of action is zero, and there is no tendency to rotate around that point; that is, the moment of the force about the point *D* is zero.

**36.** The moment of a force may be expressed in *inch-pounds*, *foot-pounds*, or *foot-tons*, depending on the unit of measurement used to designate the magnitude of the force and the length of its lever arm. For instance, if the magnitude of a force is measured in pounds, and the lever arm through which it acts in inches, the moment will be in inch-pounds; again, if a force of 10 tons acts through a lever arm of 20 feet, the moment of the force is $10 \times 20 = 200$ foot-tons.

EXAMPLE.—What is the moment, in inch-pounds, of a force of 8,000 pounds, if the length of the lever arm is 13 feet?

SOLUTION —Since the moment is to be in inch-pounds, the length of the lever arm must be in inches. Therefore, 13 ft. $= 13 \times 12 = 156$ in. and the moment is

$$8,000 \times 156 = 1,248,000 \text{ in.-lb. Ans.}$$

**37.** Positive and Negative Moments.—In order to distinguish between the directions in which there is a tendency to produce rotation, the signs $+$ and $-$ may be used.

$D$———————————————————————— $\underline{1000\ lb.}$

<center>FIG. 17</center>

Thus, if a force tends to produce right-hand rotation, that is, rotation in the same direction as the hands of a clock, it is called *positive*, and its moment takes the sign $+$, while a force that tends to produce rotation in the opposite direction is called *negative*, and its moment takes the sign $-$. The selection of one direction for positive and another for negative is merely an arbitrary distinction to show that the directions are opposite. It has, however, been adopted by engineers, and should always be used as given.

**38.** Resultant Moments.—In Fig. 18 is shown a lever composed of two arms at right angles to each other, and free to turn about the center $C$. A force $A$ acts on the horizontal arm in such a manner that it tends to produce left-hand rotation, its moment being $10 \times 5 = 50$ foot-pounds, which, since it tends to produce left-hand rotation, shall be called negative. Another force $B$, whose moment with respect to the center $C$ is $12 \times 3 = 36$ foot-pounds, tends to

produce right-hand rotation.   Therefore, $-50 + 36 = -14$ foot-pounds, which is the **resultant moment,** and has the same turning effect as the two moments already given.

If, instead of the two forces just considered, there is a body that is acted on by any number of forces whose moments
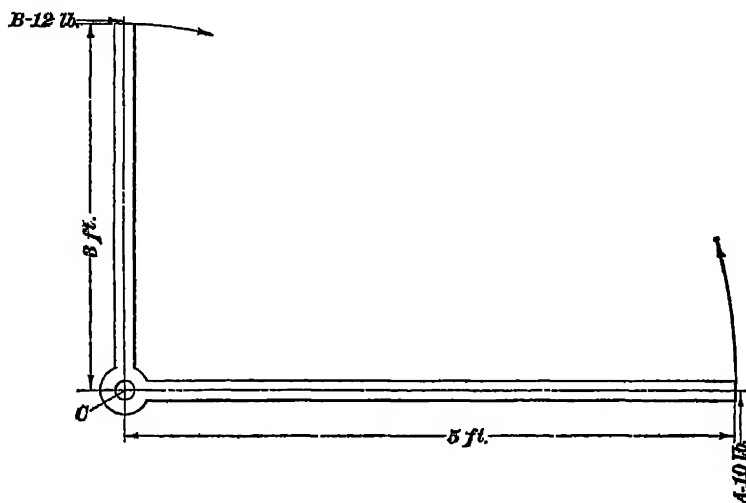


FIG 18

about a given center are known, the resultant moment of these forces will be the algebraic sum of the moments of the given forces.

**39.   Equilibrium of Moments.**—Let Fig. 19 represent a rod 15 feet long with a weight on each end, one weighing
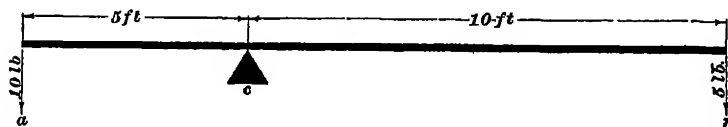


FIG. 19

10 pounds and one 5 pounds.   It will be found that if the rod is supported 5 feet from the 10-pound end, the two weights will just balance each other.   Any one that has played "seesaw," or "teeter," in childhood, knows that the lighter

playmate was always given the longer end of the board.
This is exactly the same principle with the two weights in
Fig. 19.   When they balance at 5 feet from the 10-pound end,
the positive moment about the point of support is 10 feet
× 5 pounds = 50 foot-pounds.   The negative moment is
5 feet × 10 pounds = 50 foot-pounds.   It is seen that the
moments balance each other.   The general rule can be
stated that when the values of the moments of two forces
are equal at the point around which they tend to turn and
when they are in opposite directions, they neutralize each
other.   The moments are then said to be in equilibrium.

This subject of the equilibrium of moments will be treated
more fully in another Section.
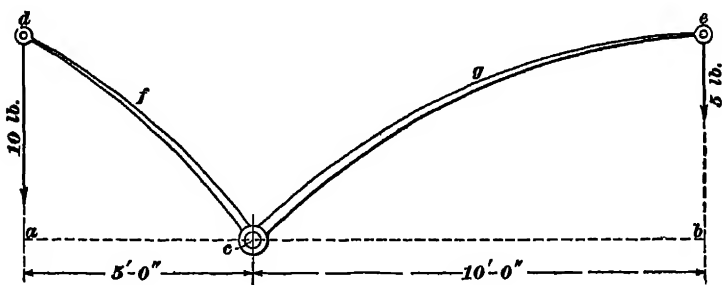


FIG. 20

**40.**   It is not necessary that the lever arm of the moment
coincide with the rod, as in Fig. 19.   The true length of the
lever arm is in every case the perpendicular distance from the
center of moments to the line of action of the force.   For
instance, in the lever shown in Fig. 20, it makes no differ-
ence what forms the arms *f* or *g* may assume.   Leaving out
of consideration the weight of the arms themselves, the per-
pendicular distances *a c* or *b c* from the center *c* to the line of
actions *a d* or *b e* are what determine the lengths of the lever
arms.   As the loads and lever arms of Fig. 20 are identical
with those of Fig. 19, the latter lever may be replaced by the
former.

**41.**   The principles involved in the theory of moments are
among the most simple in mechanics, and at the same time

they are of the greatest practical importance in the solution of problems relating to the strength of beams, girders, and trusses.

EXAMPLE.—In Fig. 21, the lower tie-member in the roof truss has been raised to get a vaulted-ceiling effect in the upper story of the building which the truss covers. The weight transmitted through this member to the pier wall is 30,000 pounds; there is, consequently, an equal upward force due to the reaction of the wall. This force of 30,000 pounds tends to break the truss by producing rotation about the point *b*. What is its moment around the point *b*?

SOLUTION.—Since the perpendicular distance from the line of action of the force is 3 ft., the moment of the force *a* around the point *b* is

30,000 × 3 = 90,000 ft.-lb.

Ans.

### THE LEVER

**42.** A lever is a bar capable of being turned about a pin, pivot, or point, as in Figs. 22, 23, and 24.

The object *W* to be lifted is called the **weight**, the force *P* used is called the **power**, and the point, or pivot, *F* is called the **fulcrum**.

That part of the lever between the weight and the fulcrum, or *Fb*, is called



Fig. 21

the **weight arm**, and the part between the power and the fulcrum, or *Fc*, is called the **force arm**.

Take the fulcrum, or point *F*, as the center of moments; then, in order that the lever shall be in equilibrium, the moment of *P* about *F*, or *P* × *Fc*, must be equal to the
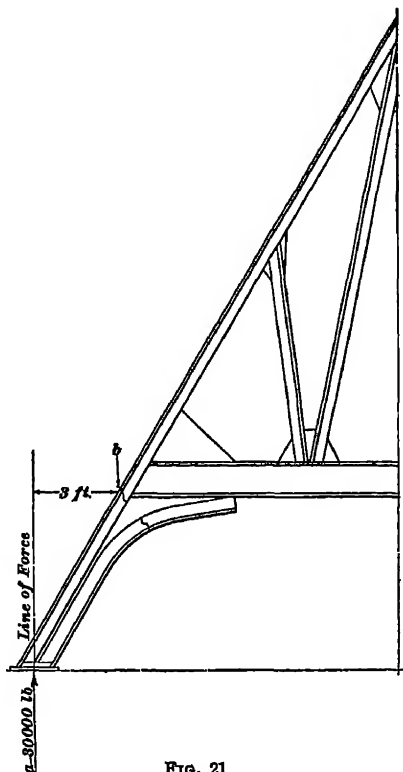
moment of $W$ about $F$, or $W \times Fb$. That is, $P \times Fc = W \times Fb$, or, in other words, *the product of the force and the force arm is equal to the product of the weight and the weight arm.*
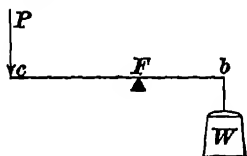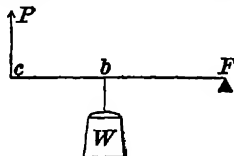


FIG 22        FIG 23

If $F$ is taken as a center, and arcs are described through $b$ and $c$, it will be seen that, if the weight arm is moved



FIG. 24

through a certain angle, the force arm will move through the same angle. From this, it is seen that the power arm is proportional to the distance through which the force moves, and the weight arm is proportional to the distance through which the weight moves.

Hence, instead of writing $P \times Fc = W \times Fb$, $P \times$ distance through which $P$ moves $= W \times$ distance which $W$ moves may be written. This is the general law of all machines, and can be applied to any mechanism, from the simple lever up to the most complicated arrangement. Stated in the form of a rule, the law of machines is as follows:
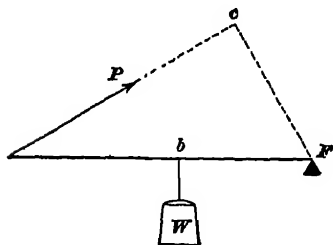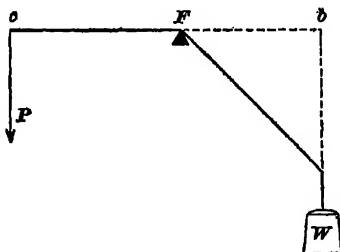


FIG. 25        FIG 26

**Rule.**—*The force multiplied by the distance through which it moves is equal to the weight multiplied by the distance through which it moves.*

EXAMPLE 1.—If the weight arm of a lever is 6 inches long and the power arm is 4 feet long, how great a weight can be raised by a force of 20 pounds at the end of the power arm?

SOLUTION.— 4 ft. = 48 in. Hence, $20 \times 48 = W \times 6$, or $W = 160$ lb.
                                               Ans.

EXAMPLE 2.—(a) What is the ratio between the power and the weight in the last example? (b) If $P$ moves 24 inches, how far does $W$ move? (c) What is the ratio between the two distances?

SOLUTION.—(a) $20 : 160 = 1 : 8$; that is, the weight moved is 8 times the force. Ans.

(b) $20 \times 24 = 160 \times x$. $x = 480 \div 160 = 3$ in , the distance that $W$ moves. Ans.

(c) $3 : 24 = 1 : 8$, or the ratio is $1 : 8$. Ans.

The law that governs the straight lever also governs the bent lever; but care must be taken to determine the true
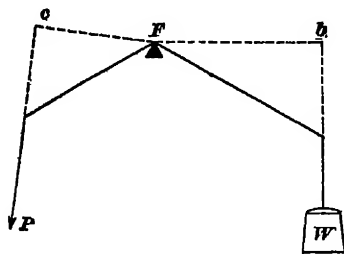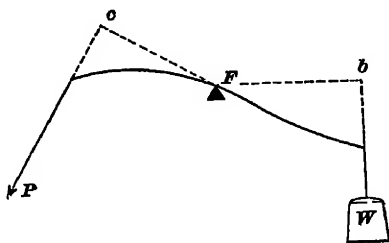


FIG. 27                              FIG. 28

lengths of the lever arms, which are, in every case, *the perpendicular distances from the fulcrum to the line of direction of the weight or force.*

Thus, in Figs. 25, 26, 27, and 28, $Fc$ in each case represents the power arm, and $Fb$, the weight arm.

---

### EXAMPLES FOR PRACTICE

1. A lever arm has a length of 10 feet; the load acting on the end of the lever is 6,000 pounds, what is the moment of this load in inch-pounds?                          Ans. 720,000

2. A pole 20 feet long is balanced at a point 8 feet from one end; the load at one end being 9,000 pounds, what is the load at the other end?                         Ans. 6,000 lb.

3. A beam extending 6 feet outside of the center of a building wall and 8 feet inside is required to support a load of 4,000 pounds on the outside end; what load on the inner end will keep the beam from tilting?                                                         Ans 8,000 lb.

<hr>

### CENTER OF GRAVITY

**43.** The center of gravity of a body, or of a system of bodies, or forces, is that point at which the body or system may be balanced, or it is the point at which the whole weight of the body or bodies may be considered as concentrated. If the body or system were suspended from any other point than the center of gravity, and in such a manner as to be free to turn about the point of suspension, it would rotate until the center of gravity reached a position directly under the point of suspension.
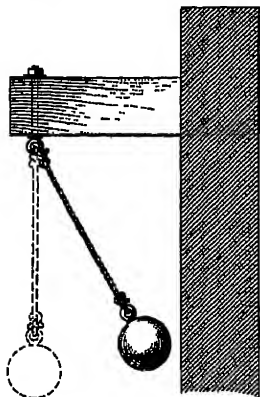
In Fig. 29 is shown a weight suspended by a rope. Experience teaches that the weight will not remain in the position shown in the figure, but will take the position shown dotted. The center of gravity



FIG 29

of the weight is at the center of the sphere, and in moving to the position shown dotted, it obeys the law that the center of gravity takes its place directly below the point of suspension.

**44.** An experimental method of finding the center of gravity of a body is illustrated in Fig. 30. Suspend a body as shown in (*a*). It is evident that the center of gravity of the body, if free to move, is directly below the point of support. That is, in Fig. 30 (*a*) the center of gravity is directly below *a* on the line *a b*. The body is next suspended from another point *c* on its surface, as shown in (*b*). The center of gravity is then below the new point of suspension or on the line *c d*. Being located both on line *a b* and line *c d*, it

follows that their point of intersection $e$ must also be the center of gravity.
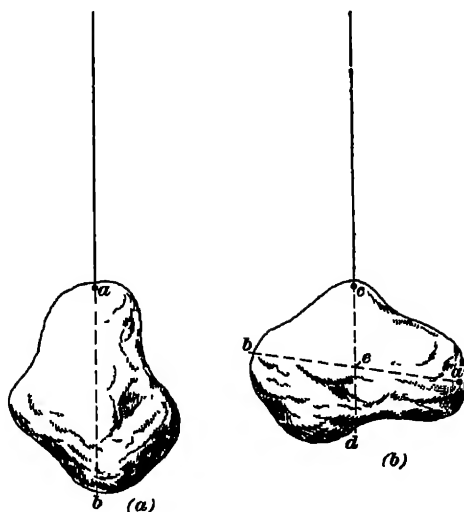


FIG. 80

While the preceding method is theoretically correct, it is usually difficult to apply because the point $e$ is generally inside of the body. Fortunately, the engineer rarely has to find the center of gravity of such irregular figures. The simpler problem of finding the center of gravity of a plane figure is the one that he more frequently has to solve.



FIG. 81

45. If the plane figure has an *axis of symmetry*, this axis passes through its center of gravity. If the figure has *two* axes of symmetry, its center of gravity is at their point of intersection. Thus, in Fig. 31 ($a$), the center
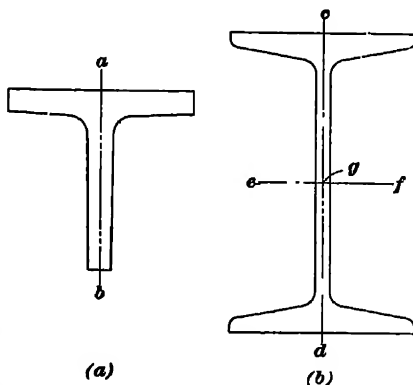
of gravity is on the line *a b*, which is the axis of symmetry, while in (*b*) the center of gravity is on the line *c d* and also the line *e f*, both of which are axes of symmetry. It must therefore be at their point of intersection, or *g*.

**46.** A simple approximate method of locating the center of gravity of a flat figure is shown in Fig. 32. Draw the outline of the figure, either full size or to some convenient scale, on a piece of heavy cardboard. Cut out the figure and balance it carefully over a knife edge, as shown in the illustration; the line along which it rests on the edge of the knife is a line passing through the center of gravity, and by
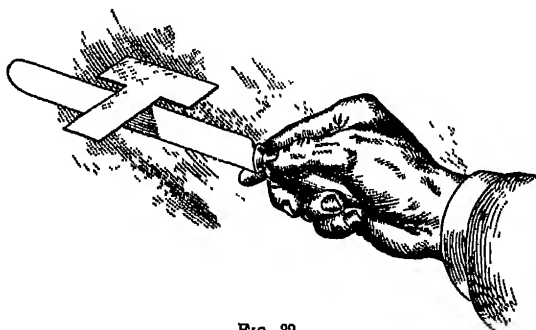


Fig. 32

locating two such lines in different directions, the center of gravity will be found at their point of intersection.

**47. Center of Gravity of Plane Figures.**—The center of gravity of a *triangle* lies on a line drawn from a vertex to the middle point of the opposite side, and at a distance from that side equal to one-third of the length of the line; or it is at the intersection of lines drawn from the vertexes to the middle of the opposite sides. The perpendicular distance of the center of gravity of a triangle from the base is equal to one-third of the altitude.

The center of gravity of a *parallelogram* is at the intersection of its two diagonals; consequently, it is midway between its sides.

The center of gravity of an *irregular four-sided figure* may be found by first dividing it, by a diagonal, into two triangles and joining the centers of gravity of the triangles by a straight line; then, by means of the other diagonal, divide the figure into two other triangles, and join their centers of gravity by another straight line; the center of gravity of the figure is at the intersection of the lines joining the centers of gravity of the two sets of triangles.

Another method by which to locate the center of gravity of an irregular four-sided figure is illustrated in Fig. 33. Draw the diagonals *a c* and *b d*, and from their intersection *e*, measure the distance to any vertex, as *a e*. From the opposite vertex, lay off this distance, as at *c f*. Then from *f*, draw a line to one of the other vertexes, as *f d*, and bisect this line as at *g*. Connect *g* and *b* and lay off one-third of its length from *g* at the point *h*. This point is the center of gravity of the figure.



FIG. 33

The distance of the center of gravity of the *surface of a half circle* from the center is equal to the radius multiplied by .424.

**48. Neutral Axis.**—The **neutral axis** of a flat or plane figure is any line passing through the center of gravity. Thus in Fig. 31 (*a*) the line *a b* is a neutral axis, and in Fig. 31 (*b*) both the lines *c d* and *e f*, and in fact any line through *g*, is a neutral axis. This definition of neutral axis is correct for use in the mechanics of ordinary materials. However, in speaking of the neutral axis of reinforced-concrete beams, something else is meant. This will be explained in connection with that subject. The location of the horizontal neutral axis, that is, a horizontal line through the

center of gravity of a plane figure, is of great importance in engineering. Of course, if the center of gravity is located, the horizontal neutral axis can be located easily; but, as a rule, the horizontal neutral axis is located without first locating the center of gravity.

**49. Locating the Neutral Axis by Means of the Principle of Moments.**—A convenient method of locating the neutral axis is based on the principle that the moment of any plane figure, with respect to a given line as an axis or origin of moments, is equal to the product of its area by the perpendicular distance from the center of gravity of the figure to the given axis.

Let   $M$ = moment;

      $A$ = area of the figure or section;

      $c$ = perpendicular distance from center of gravity of figure to given axis.

Then,          $M = A \times c$      **(1)**

and            $c = \dfrac{M}{A}$      **(2)**

If necessary, the figure may be subdivided; then the moment of the figure is equal to the sum of the moments of its separate parts with respect to the same axis. Designating
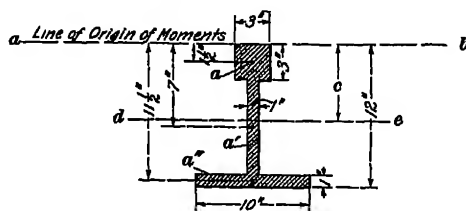


Fig. 34

the areas of the subdivisions by the letters $a$, $a'$, $a''$, etc., and their moments by $m$, $m'$, $m''$, etc., then formula 2 becomes

$$c = \frac{m + m' + m'', \text{ etc.}}{a + a' + a'', \text{ etc.}} \qquad \textbf{(3)}$$

Thus, in Fig. 34, take the line $ab$ as the line of origin of moments. Divide the figure into the three rectangles $a$, $a'$, $a''$.

In accordance with the principles previously stated, the center of gravity of each of these rectangles is midway between its edges; the distances of the respective centers from the axis are, therefore, $1\frac{1}{2}$, 7, and $11\frac{1}{2}$ inches. The areas of the figures are, respectively, $3 \times 3$ or 9 square inches, $8 \times 1$ or 8 square inches, and $10 \times 1$ or 10 square inches. The moments of these areas about the axis $a\,b$ are:

Section $a$,    $9 \times 1\frac{1}{2}$   $=$    1 3.5
Section $a'$,    $8 \times 7$   $=$    5 6.0
Section $a''$, $10 \times 11\frac{1}{2}$ $=$ 1 1 5.0
         Total,    1 8 4.5

The area of the whole section is equal to the sum of the areas of the rectangles, or $9 + 8 + 10 = 27$ square inches. Then, from formula **3**,

$$c = \frac{184.5}{27} = 6.83, \text{ or nearly } 6\frac{27}{32}, \text{ inches}$$

It is not necessary that the line of the origin of moments should coincide with an edge of the figure, as in Fig. 34, since any other line parallel with the direction of the required neutral axis gives the same results. In most cases, however, it will be found more convenient to take the axis about which the moments are calculated on one of the extreme edges of the section.

Since the section shown in Fig. 34 is symmetrical with respect to an axis perpendicular to the neutral axis, it is evident that the center of gravity is on their intersection. If, however, there were no axis of symmetry, the center of gravity could be located by taking a second line perpendicular to $a\,b$ as an origin of moments and finding the neutral axis parallel to it. The intersection of this neutral axis with the one first found is the center of gravity of the section. In accordance with the principles illustrated in this example, the following rule has been deduced:

**Rule.**—*To find the neutral axis of any section, first divide it into a number of simple parts, each of whose areas and centers of gravity can be readily found, then find the sum of the*

*moments of the areas of each of these parts with respect to an axis parallel to the required neutral axis; finally, divide this sum by the sum of the areas of the parts of the section. The result will be the perpendicular distance from the axis of the origin of moments to the required neutral axis.*

**50. Application of the Rule to a Built-Up Section.** Fig. 35 shows a section of the rafter member of a large roof truss formed of a $\frac{3}{8}'' \times 16''$ web-plate and a $\frac{3}{8}'' \times 12''$ flange plate, the two joined by two $4'' \times 4'' \times \frac{1}{2}''$ angles. It is desired to know the distance from the neutral axis of the
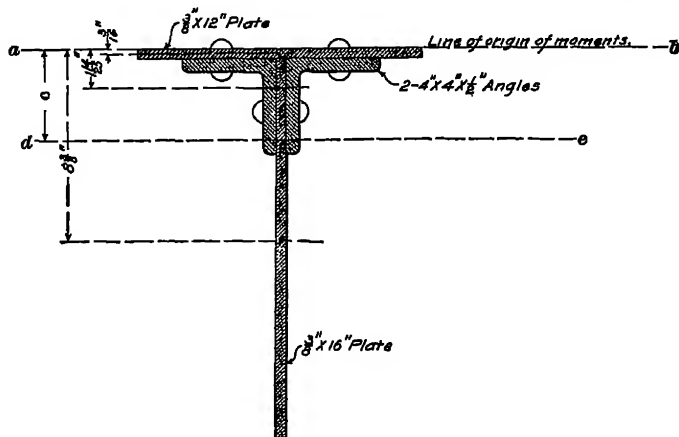


Fig. 35

section to the top edge of the flange plate. By means of the principles given, the centers of gravity of the two rectangular plates are easily located as shown. The centers of gravity of the angles might also be located by applying the rule given in the preceding article; this, however, is unnecessary, since the position of the center of gravity can be obtained directly by referring to the tables of the properties of rolled sections that are furnished by the various steel manufacturers. By referring to any of these tables, the center of gravity of a $4'' \times 4'' \times \frac{1}{2}''$ angle is found to be 1.18 inches from the back of a flange, thus giving the distance $1.18 + .375 = 1.555$, or about $1\frac{14}{25}$, inches from the

top edge of the flange plate to the axis through the centers of gravity of the angles. From the same tables it is also found that the area of the section of a $4'' \times 4'' \times \frac{1}{2}''$ angle is 3.75 square inches.

The area of the section of the flange plate is $\frac{3}{8} \times 12$ $= 4.5$ square inches, and of the web-plate, $\frac{3}{8} \times 16 = 6$ square inches; the area of the whole section is therefore $2 \times 3.75$ $+ 4.5 + 6 = 18$ square inches.

The moments of the areas of the separate sections, with respect to the line $a\,b$, are as follows:

Flange plate,        $4.5 \times \frac{3}{16} =$        .8 4
Two angles, $2 \times 3.75 \times 1\frac{14}{24} = $  1 1.7 0
Web-plate,         $6 \times 8\frac{3}{8} \;= 5$ 0.2 5

                Total,        6 2.7 9

The distance $c$ from the top edge of the flange plate to the neutral axis $d\,e$ of the section is therefore, from formula **3**, Art. **49**, $62.79 \div 18 = 3.48$ inches.

---

**LOCATION OF RESULTANT OF PARALLEL FORCES**

**51.** In Art. **29**, the method of finding the resultant of parallel forces was described with reference to the motion of a body along a straight path, but the location of the resultant was not considered. When parallel forces act in opposite directions, the resultant cannot always be represented by one force; but when all the parallel forces act in the same direction, the magnitude, direction, and point of application of the resultant may be found. The magnitude is the sum of the component forces, and the direction is the same as the direction of the component forces, as was indicated in Art. **29**. The point of application is at the center of gravity of all the component forces. This may conveniently be found by the method given in Art. **49** for finding the neutral axis.

**52.** The method of locating the resultant of parallel forces may perhaps best be illustrated by an example. Thus, find

the resultant of all the forces acting on the stick of timber shown in Fig. 36. From Art. **29,** the magnitude of the resultant is $50 + 75 + 40 + 100 = 265$ pounds. Its direction is the same as that of the several component forces, namely, downwards. To find the point of application of the resultant, take the sum of the moments of the component forces
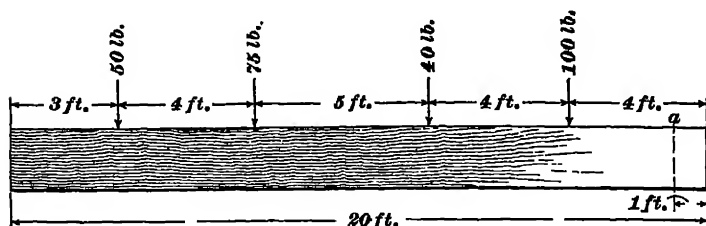


FIG 36

about any convenient point, for example, the left-hand end of the timber. They are as follows:

$$
\begin{array}{lcr}
 & & \text{FOOT-POUNDS} \\
50 \times 3 & = & 1\,5\,0 \\
75 \times (3+4) & = & 5\,2\,5 \\
40 \times (3+4+5) & = & 4\,8\,0 \\
100 \times (3+4+5+4) & = & 1\,6\,0\,0 \\
\hline
 & & 2\,7\,5\,5
\end{array}
$$

According to Art. **49,** this value, 2,755 foot-pounds, is also the moment of the resultant about the left-hand end of the timber. The value of the resultant is 265 pounds. Therefore, the moment arm is, from formula **3,** Art. **49,** $c = 2,755 \div 265 = 10.396$ feet, and this is the distance of the line of action of the resultant from the left-hand end of the timber.

**53.** To prove the assertion that it is not necessary to use any particular point for the origin of moments, the location will be again calculated, taking the origin of moments as, say, 1 foot from the right-hand end of the timber, at the point marked $a$. The moments of the various forces about this point are as follows:

FOOT-POUNDS

$$100 \times 3 \qquad\qquad\quad = \quad 3\ 0\ 0$$
$$40 \times (4 + 3) \qquad\quad = \quad 2\ 8\ 0$$
$$75 \times (5 + 4 + 3) \quad\ \ = \quad 9\ 0\ 0$$
$$50 \times (4 + 5 + 4 + 3) = \quad \underline{8\ 0\ 0}$$
$$2\ 2\ 8\ 0$$

The moment arm of the resultant from the point $a$ is therefore $2,280 \div 265 = 8.604$ feet, which gives the point of application of the resultant the same location as was found in Art. **52.**

# FORCES ACTING ON BEAMS

## INTRODUCTION

### STYLES OF BEAMS

**1.** In a broad sense, a **beam** is a body that is supported in a horizontal position at one or more points. In engineering, beams are usually made of wood, steel, cast iron, stone, concrete, or a combination of these materials. Both the floor joists in an ordinary floor and bridge-plate girders are examples of beams.

Columns or piers, or whatever holds up a beam, are known as the **supports**, and the place where a support touches a beam, is called the **point of support.**

**2. Simple Beams.**—A beam that has two points of support, one at each end, as shown in Fig. 1, is known as a **simple beam.** The points of support of a simple beam are usually considered to be directly at the



FIG 1

places where the supports first touch the beam, as at *a*, although some engineers consider these points to be at the middle of the supports, as at *b*.

The distance between the points of support is called the **span.** The extra length of the beam *a c* at each end is usually neglected in calculating the forces on the beam, and the

length of the latter is therefore considered as equal to
the span.

3. **Continuous Beams.**—A beam that has more than
two supports, as shown in Fig. 2, is known as a continuous



FIG 2

beam. The point of
support of the middle
support is considered to
be at its center, as at $a$.
Continuous beams,
except in reinforced-concrete work, have not found much
favor among engineers in the last few years.
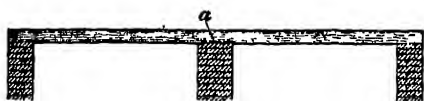
4. **Cantilever Beams.**—A cantilever beam is one
with only one support, which is at the middle, or any part of
a beam that projects out
beyond its support.    A
common form of canti-
lever is shown in Fig. 3.
This beam projects from
a wall or some other
solid structure and has no support at its outer end.



FIG. 3

5. **Restrained Beam.**—When a beam is rigidly held,
or fixed, at both ends, as shown in Fig. 4, it is called a



FIG. 4

restrained beam,
or, more commonly,
a beam fixed at both
ends.    This type of
beam must be care-
fully distinguished
from the type shown in Fig. 1, which is a simple beam and
is merely supported at the ends.

Between these two types of support there are many
gradations in which the beams are held more or less rigidly,
and it is often difficult to determine whether a beam is a
simple beam or a beam with fixed ends.    However, unless
the ends are absolutely rigidly fixed, it is always safe to
consider  the  beam as a simple one, because such a beam is

not so strong as one with fixed ends.   Therefore, by design-
ing a beam as a simple beam, the engineer errs on the side
of safety.

### LOADS ON BEAMS

**6.**   The forces due to the weights that a beam supports
are known as **loads**.   When the whole load is applied at one
point, or practically one point, it is called a **concentrated
load**; when it extends over a portion of the beam, it is
called a **distributed load**; and when the load is equally
distributed over the beam, so that each unit of length has
the same load, it is called a **uniform load**.

There are certain methods by which such loads may be
represented graphically.   These methods may best be illus-
trated by referring to Fig. 5, which shows a simple beam.
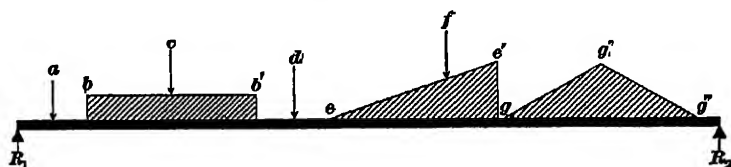


FIG. 5

Starting at the left, the point of the arrow shown at $R_1$ is the
point of support.   The arrow $a$ represents a concentrated
load that might be caused by either a column or a floor joist
resting on the beam.   At $b\,b'$ is shown a uniformly dis-
tributed load, of a certain number of pounds per foot of beam,
that extends from $b$ to $b'$.   At $c$, $d$, and $f$ are shown other
concentrated loads similar to the one at $a$.   From $e$ to $e'$ is a
distributed load, represented by the shaded triangle under $e\,e'$,
that starts from nothing at $e$ and increases to a given number
of pounds per foot of beam at $e'$.   At $g\,g''$ is a distributed
load, starting from nothing at $g$ and increasing to a maximum
at $g'$, and then decreasing to nothing at $g''$.   At $R_2$ is shown
the right-hand support.

# METHODS OF ANALYSIS

## REACTIONS

**7.** A beam with any loads it may carry is held up by the supports; that is, the beam presses on its supports at the ends. The supports resist this pressure and prevent the beam from falling. This upward force exerted by each support is known as the **reaction**.

**8.** It has been stated in *Statics* that all bodies at rest are in equilibrium. A beam is at rest, and must therefore be in equilibrium. Now, there are two facts that are true when a body is in equilibrium: first, the resultant of all the forces acting on a body or beam must be zero, and second, the resultant moment of all the forces about any point must be zero. This first condition of equilibrium has already been briefly discussed in *Statics*. The second condition, however, has not been mentioned, but it is of equal importance when compared with the first. The first condition is the one that prevents the beam from moving off through space, and the second is the one that prevents it from revolving.

**9.** The forces acting on a beam are the forces due to loads on the beam and the weight of the beam itself, if that is considered, and then there are the reactions of the supports on the beam. Now, all the loads act vertically downwards and the reactions act vertically upwards. They are therefore parallel, and it follows from the principles given in *Statics* that as their resultant must be zero, and since they are opposite, the sum of the loads must equal the sum of the reactions.

A beam with two supports will be considered. The sum of the reactions is known from the loads, but the value of each reaction is not known. To find this, it is necessary to

resort to the second condition of equilibrium, namely, that
the resultant moment of all the forces about any point must
be zero.   Any point can be assumed, but it is found con-
venient to choose the reaction of one support as the point
about which to take moments.   The moment of the reaction
at the point is zero, since the arm of the moment is zero.   It
is therefore necessary that the sum of the moments of the
loads about one point of support and the moment of the
other reaction about the same point shall equal zero.   In this
way, by solving this equation, one reaction is found, and by
subtracting this reaction from the sum of the loads, the
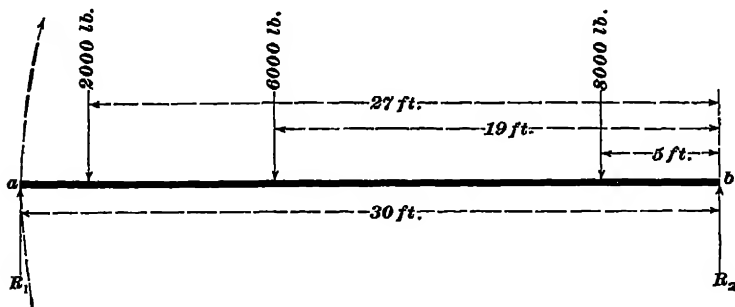


Fig. 6

other reaction is found.   There is also a graphical method
by which reactions may be found, but it will not be necessary
to give this method here.

**10.**   A practical example will now be considered.   In
Fig. 6 let it be required to find the reactions $R_1$ and $R_2$ at the
points of support $a$ and $b$.   (In all the subjoined problems,
$R_1$ and $R_2$ represent the reactions.)   The center of moments
may be taken at either $R_1$ or $R_2$.   Let the point $b$ be taken in
this case.   The three loads are forces acting in a downward
direction; the sum of their moments, in foot-pounds, with
respect to the assumed center, may be computed as follows:

$$8,000 \times 5 = 4\,0\,0\,0\,0$$
$$6,000 \times 19 = 1\,1\,4\,0\,0\,0$$
$$2,000 \times 27 = 5\,4\,0\,0\,0$$

Total,    $2\,0\,8\,0\,0\,0$

This is the total negative moment about the point $b$. The positive moment about this point is of course $R_1 \times 30$. Since the resultant moment, according to the law of equi-
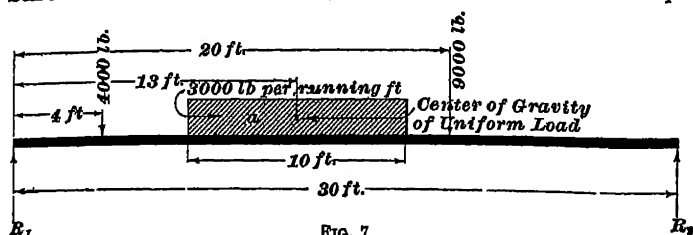


FIG. 7

librium, is zero, the positive moment must equal the negative moment, or $208,000 = 30\,R_1$ or $R_1 = 208,000 \div 30 = 6,933\frac{1}{3}$ pounds. The sum of all the loads is $2,000 + 6,000 + 8,000 = 16,000$ pounds. This is also the sum of the reactions. Therefore, $R_2 = 16,000 - 6,933\frac{1}{3} = 9,066\frac{2}{3}$ pounds. In this problem, as in many problems of a similar nature, the weight of the beam itself is neglected.

EXAMPLE 1.—What is the reaction at $R_2$ in Fig. 7?

SOLUTION.—In computing the moment due to a uniform, or evenly distributed, load, as at $a$, the lever arm is always considered as the distance from the center of moments to the center of gravity of the load. The amount of the uniform load $a$ is $3,000 \times 10 = 30,000$ lb., and the distance of its center of gravity from $R_1$ is 13 ft. Therefore, the moments of the loads on this beam about $R_1$, in foot-pounds, are as follows:

$$30,000 \times 13 = 3\ 9\ 0\ 0\ 0\ 0$$
$$4,000 \times 4 = \phantom{0}1\ 6\ 0\ 0\ 0$$
$$9,000 \times 20 = 1\ 8\ 0\ 0\ 0\ 0$$
$$\text{Total,} \quad \overline{5\ 8\ 6\ 0\ 0\ 0}$$
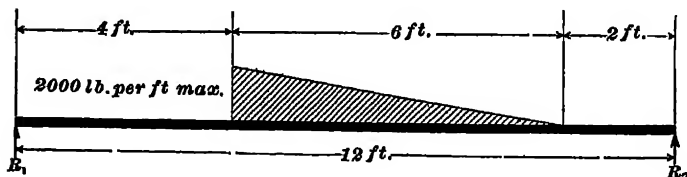


FIG. 8

This is the sum of the moments of all the loads about $R_1$, as a center. The leverage of the reaction $R_2$ is 30 ft. Hence, the reaction at $R_2$ is
$$586,000 \div 30 = 19,533\frac{1}{3} \text{ lb.} \quad \text{Ans.}$$

EXAMPLE 2.—What is the reaction at $R_2$ in Fig. 8?

SOLUTION —Taking moments about $R_1$, the center of gravity of the load is $4 + \frac{6}{3} = 6$ ft. from $R_1$   The total load is $\frac{2000}{2} \times 6 = 6,000$ lb. The positive moment of the load is $6,000 \times 6 = 36,000$ ft.-lb. Therefore, $R_2 \times 12 = 36,000$ ft.-lb., or

$$R_2 = \frac{36000}{12} = 3,000 \text{ lb}\quad \text{Ans.}$$
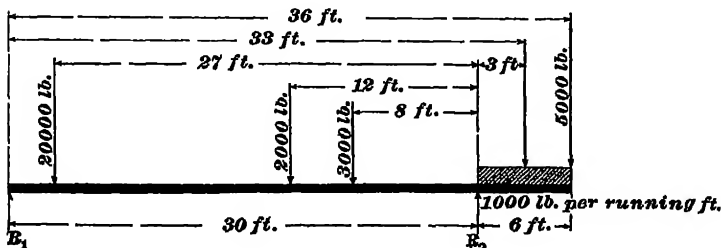


FIG 9

EXAMPLE 3 —A beam is loaded as shown in Fig 9   Compute the reactions $R_1$ and $R_2$.

SOLUTION —Consider, say $R_2$, as the center of moments   Then, the negative moments of the loads about $R_2$, in foot-pounds, are

$$
\begin{aligned}
20,000 \times 27 &= 540000 \\
2,000 \times 12 &= 24000 \\
3,000 \times 8 &= 24000 \\
\text{Total,} \quad & \overline{588000}
\end{aligned}
$$

Now, the positive moments of the loads about $R_2$, in foot-pounds, are

$$
\begin{aligned}
1,000 \times 6 \times 3 &= 18000 \\
5,000 \times 6 \quad &= 30000 \\
\text{Total,} \quad & \overline{48000}
\end{aligned}
$$

The resultant moment of the loads about $R_2$ is negative and is $-588,000 + 48,000 = -540,000$ ft.-lb ; $540,000 - 30 = 18,000$ lb , which is the value of $R_1$

The sum of the loads is

$$
\begin{aligned}
& 20000 \\
& 2000 \\
& 3000 \\
6 \times 1,000 = \;& 6000 \\
& 5000 \\
& \overline{36000}
\end{aligned}
$$

$$R_2 = 36,000 - 18,000 = 18,000 \text{ lb}\quad \text{Ans.}$$

EXAMPLE 4.—Compute the reactions at the supports $R_1$ and $R_2$ in a beam loaded as shown in Fig. 10.

SOLUTION.—Letting $R_1$ be the center of moments, the moments of the loads, in foot-pounds, are

$$5,000 \times 10 = \phantom{1\ 2\ 0\ }5\ 0\ 0\ 0\ 0$$
$$10,000 \times 20 = \phantom{1\ }2\ 0\ 0\ 0\ 0$$
$$30,000 \times 40 = 1\ 2\ 0\ 0\ 0\ 0$$

$$\overline{\text{Total,}\quad 1\ 4\ 5\ 0\ 0\ 0\ 0}$$

Now, 1,450,000 ÷ 30, the distance between the supports, = 48,333⅓ lb., which is the required reaction of $R_2$. The sum of the loads is 5,000 + 10,000 + 30,000 = 45,000 lb.; therefore, the reaction $R_2$ is greater than the sum of the loads. This shows that the force at $R_1$ must act in a downward direction in order that the sum of the downward forces may equal the upward force at $R_2$. Since this is opposite to the usual direction, the reaction at $R_1$ is called negative, or minus. In other words, instead of an upward reaction at $R_1$, there must be a downward force at this point in order to prevent the beam from rotating around
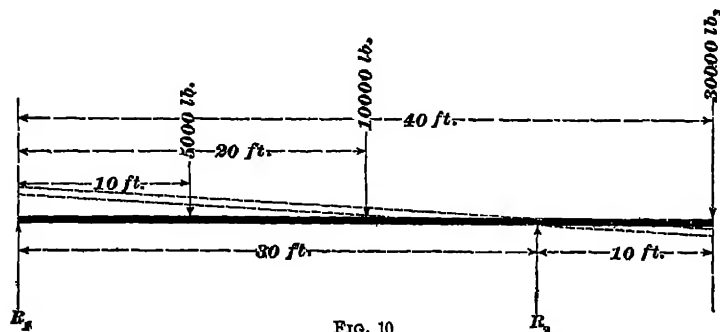


FIG. 10

the support $R_2$, as indicated by the dotted lines. The magnitude of this downward force $R_1$ is the difference between the upward reaction at $R_2$ and the sum of the downward pressures due to the loads, that is, 48,333⅓ − 45,000 = 3,333⅓ lb.

As a check, the reaction at $R_1$ will now be computed by taking the center of moments at $R_2$, and finding the magnitude and direction of action of the force at $R_1$ whose moment is equal and opposite to the resultant of the moments of the loads on the beam. The load of 30,000 lb. tends to produce right-hand rotation around the center $R_2$, hence, its moment, 30,000 × 10 = 300,000 ft.-lb , is positive  The 10,000-lb. load is 10 ft. to the left of $R_2$, and its tendency is to produce left-hand rotation about $R_2$, consequently, its moment is negative and equal to 10,000 × 10 = 100,000 ft.-lb. In a similar manner, the moment of

the 5,000-lb. load is found to be negative and equal to 5,000 × 20 = 100,000 ft.-lb.  These results may be collected as follows:

Positive moment:
      30,000 × 10 =          +3 0 0 0 0 0 ft.-lb.
Negative moments:
      10,000 × 10 = 1 0 0 0 0 0
       5,000 × 20 = 1 0 0 0 0 0
                          −2 0 0 0 0 0 ft.-lb.

        Algebraic sum,   +1 0 0 0 0 0 ft.-lb.

This is the resultant of the moments of the three loads.  Since the positive moment is greater than the sum of the negative moments, it is evident that to produce equilibrium the force at $R_1$ must tend to produce left-hand rotation; that is, it must act downwards, its lever arm being 30 ft. long, its magnitude must be 100,000 ÷ 30 = 3,333⅓ lb., which is the same result as was obtained before.

---

### EXAMPLES FOR PRACTICE

1. The span of a simple beam is 25 feet   At distances of 9, 16, and 18 feet from the left-hand end are placed concentrated loads of 8,000, 4,000, and 16,000 pounds, respectively.  What is the magnitude of the left-hand reaction?                          Ans. 11,040 lb.

2. The two reactions supporting a beam are 2,500 and 3,000 pounds. What is the single concentrated load necessary to produce these reactions?                          Ans. 5,500 lb.

3. The length of a beam is 30 feet, and it overhangs the right-hand support 6 feet, leaving a clear span of 24 feet.  At the overhanging end, there is a weight of 6,000 pounds; 10, 12, and 18 feet from the left-hand support are loads of 8,000, 6,200, and 7,800 pounds, respectively.  What is the magnitude of the right-hand reaction?
Ans. 19,783⅓ lb

4. If for a distance of 10 feet from the left-hand end of a beam is distributed a load of 1,000 pounds per running foot, and at the center of the beam is located a concentrated load of 16,500 pounds, what is the magnitude of the left-hand reaction, provided the beam is supported at both ends and is 30 feet long?          Ans. 16,583⅓ lb.

## VERTICAL SHEAR

### METHODS OF CALCULATION

**11.** In any beam, as for instance the one shown in Fig. 11, there are forces—either loads or reactions—acting both upwards and downwards. In this figure, the left-hand reaction $R_1$ acts upwards and tends to push the end of the beam up. On account of the strength of the
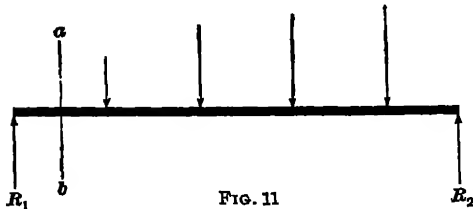


FIG. 11

beam, however, the end remains stationary. If the beam were suddenly cut on the line $a\,b$, the left-hand portion of the beam would move up in relation to the right-hand portion. This action of the forces on a beam in tending to make the surfaces at any imaginary section slide past each other, from its similarity to a shearing action, is called **shear**.

**12.** Consider now the beam shown in Fig. 12. Since the loads are symmetrically applied, each reaction is equal to 40 pounds, or one-half of the total load on the beam. Beginning at the left reaction $R_1$, there is an upward force of 40 pounds acting on the beam. Since the forces are in equilibrium, this upward force is balanced by an equal
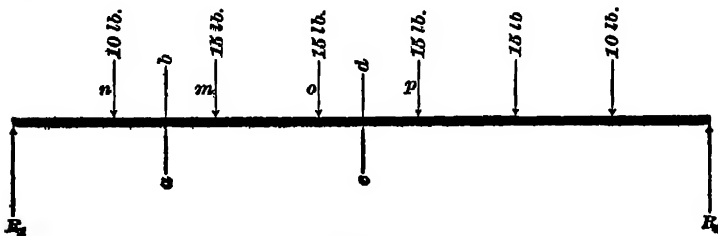


FIG. 12

downward force, which is the vertical resultant of the loads and the reaction $R_2$. Considering, therefore, any transverse

section of the beam between $R_1$ and the point of application of the load $n$, it is evident that the part of the beam at the left of this transverse section is subjected to an upward thrust of 40 pounds, while the part at the right is subjected to an equal downward thrust. The result is a shearing action at this place, the magnitude of which is equal to the reaction $R_1$. This shearing action is resisted by the strength of the fibers of the beam at the section under consideration. If these fibers were severed, it would take a vertical force of 40 pounds to prevent the beam from moving.

When the point of application of $n$ is reached, the effect of the upward force $R_1$ is partly balanced by the downward force of 10 pounds due to the load $n$. Considering, therefore, any section of the beam, as $a\,b$, between the points of application of the loads $n$ and $m$, it will be seen that the part of the beam at the left is acted on by the vertical resultant of the reaction $R_1$ and the load $n$, that is, by an upward force of $40 - 10 = 30$ pounds, while the part at the right is acted on by an equal downward force, the vertical resultant of the remaining loads and the reaction $R_2$. Any section between the points of application of $n$ and $m$ is therefore subject to a shearing stress equal to the difference between the reaction $R_1$ and the load $n$, or $40 - 10 = 30$ pounds. In the same way, it follows that the shearing stress for any section between $m$ and $o$ is $40 - (10 + 15) = 15$ pounds. For any section, as $c\,d$, between the points of application of $o$ and $p$, the shearing stress is $40 - (10 + 15 + 15) = 0$. In other words, on each side of this section the downward forces and the reactions are equal, and their resultant is zero; it is therefore a section in which there is no shear. If the beam were suddenly cut at this section, there would be no tendency for one half to rise above or to descend below the other half.

**13.** For convenience, it is customary to call the reactions, or forces, acting in an upward direction, *positive*, and the loads, or downward forces, *negative*. Since the difference between the sums of the positive and negative numbers

representing a given set of values is called their algebraic sum, it follows that *the shear for any section of a beam is equal to the algebraic sum of either reaction and the loads between this reaction and the transverse section under consideration.* In speaking of the shear at a certain section of a beam as being positive or negative, it is simply meant that the resultant of the forces acting on the portion to the *left* of the section under consideration is either positive or negative.

In nearly all cases, the external forces—loads and reactions—act on a beam along vertical lines; the shearing action just considered, being the resultant of these forces along a section formed by an imaginary vertical cutting plane, is often called the **vertical shear.**

From what has been said, it is evident that the shear in any simple or any cantilever beam is always greatest between the reactions and the nearest loads, and that in any case the maximum shear is equal to the greater reaction.

**14. Positive and Negative Shear.**—If a transverse section of a simple beam is taken near the left reaction and the forces acting on the part of the beam at the left are considered, it will be seen that their resultant acts upwards. The shear at this section is therefore called **positive shear.** If, however, a section near the right reaction is taken, the
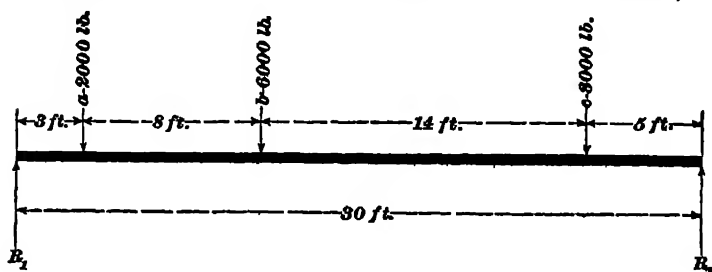


Fig. 13

resultant of the forces at the left of this section is found to act downwards, and in consequence the shear is called **negative.** It is also evident that there is a section between the two where the resultant of the forces changes from positive to negative. At such a section the shear is said to **change sign.**

It can readily be seen that in a cantilever beam the conditions are somewhat reversed; that is, in a beam having one support at the middle, the shear in the part to the left of the section taken to the left of the support is negative, while if the section is taken to the right of the support, it is positive. Like in a simple beam, however, there is an intermediate point where the shear changes sign.

EXAMPLE 1.—(a) What is the maximum shear on the beam shown in Fig. 13? (b) What is the shear at a point 9 feet from the right support? (c) What is the shear at a point 18 feet from the right support?
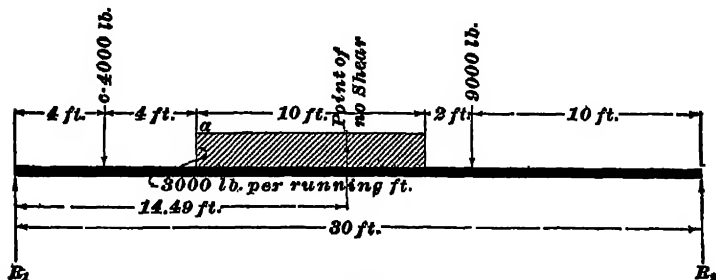


FIG. 14

SOLUTION.—(a) First estimate the reactions as follows: Taking the center of moments at the left support, the moments of the loads, in foot-pounds, are

$$2{,}000 \times 3 = \phantom{0}6\ 0\ 0\ 0$$
$$6{,}000 \times 11 = \phantom{0}6\ 6\ 0\ 0\ 0$$
$$8{,}000 \times 25 = 2\ 0\ 0\ 0\ 0\ 0$$
$$\text{Total,} \quad \overline{2\ 7\ 2\ 0\ 0\ 0}$$

The reaction at $R_2$ is 272,000 ÷ 30 = $9{,}066\frac{2}{3}$ lb The sum of the loads equals 2,000 + 6,000 + 8,000 = 16,000 lb.; 16,000 − $9{,}066\frac{2}{3}$ = $6{,}933\frac{1}{3}$ lb., which is the reaction at $R_1$. The maximum shear is therefore $9{,}066\frac{2}{3}$ lb. Ans.

(b) As the reaction $R_2$ at the right support is equal to $9{,}066\frac{2}{3}$ lb., and as there is only the one load $c$ of 8,000 lb. between $R_2$ and a point 9 ft. away, the shear at this point must equal

$$9{,}066\frac{2}{3} - 8{,}000 = 1{,}066\frac{2}{3} \text{ lb} \quad \text{Ans.}$$

(c) The shear at 18 ft. from the reaction $R_2$ is also $1{,}066\frac{2}{3}$ lb., because there is no other weight occurring between this point and $R_2$.
Ans.

EXAMPLE 2.—At what point in the beam loaded as shown in Fig. 14 does the shear change sign?

SOLUTION.—Compute the reaction $R_1$ as follows:  With the center of moments at $R_2$, the moments of the loads, in foot-pounds, are

$$
\begin{array}{rcl}
9,000 \times 10 & = & 9\ 0\ 0\ 0\ 0 \\
4,000 \times 26 & = & 1\ 0\ 4\ 0\ 0\ 0 \\
3,000 \times 10 \times 17 & = & 5\ 1\ 0\ 0\ 0\ 0 \\
\hline
\text{Total,} & & 7\ 0\ 4\ 0\ 0\ 0
\end{array}
$$

The reaction at $R_1$ is therefore $704,000 \div 30 = 23,466\frac{2}{3}$ lb.  Proceeding from $R_1$, the first load that occurs is $c$ of 4,000 lb.  Then, $23,466\frac{2}{3} - 4,000 = 19,466\frac{2}{3}$ lb.  The next load that occurs on the beam is the uniform load of 3,000 lb. per running ft.  There being altogether 30,000 lb. in this load, it is evident that the load will more than counteract the remaining amount of the reaction $R_1$; the point where the change of sign occurs must consequently be somewhere in that part of the beam covered by the uniform load.  The load being 3,000 lb. per running ft., if the remaining part of the reaction, $19,466\frac{2}{3}$ lb., is divided by the 3,000 lb., the result will be the number of feet of the uniform load required to counteract the remaining part of the reaction, and this will give the distance of the section, beyond which the resultant of the forces at the left becomes negative, from the edge of the uniform load at $a$; thus, $19,466\frac{2}{3} \div 3,000 = 6\ 49$ ft.  The distance from $R_1$ to the edge of the uniform load is 8 ft.  The entire distance to the section of change of sign of the shear is, therefore,

$$8 + 6.49 = 14.49 \text{ ft. from } R_1. \quad \text{Ans.}$$

### EXAMPLES FOR PRACTICE

1.  The uniformly distributed load on a beam supported at both ends is 40,000 pounds.  What is the maximum shear on the beam?

Ans. 20,000 lb.

2.  A beam is loaded with three concentrated loads: A of 2,000 pounds, B of 6,000 pounds, and C of 8,000 pounds; they are located 10 feet, 12 feet, and 18 feet, respectively, from the left-hand end of the beam, the span of which is 40 feet.  What is the shear between the loads C and B?

Ans. 2,100 lb.

3.  The span of a beam is 20 feet, and there is a uniformly distributed load on three-quarters of this distance from the left-hand support of 9,000 pounds.  At distances of 8 feet and 12 feet from the right-hand support are located concentrated loads of 5,000 pounds and 6,000 pounds, respectively.  At what distance from the left-hand end of the beam does the shear change sign?

Ans. 8 ft. $8\frac{1}{2}$ in.

## SHEAR DIAGRAMS

**15.** It is interesting as well as instructive to plot the shear of the forces acting on a beam in a diagram known as the **shear diagram**. In order to illustrate how this may be done the beam shown in Fig. 15 (*a*), the span of which is 12 feet, will be considered. Since the beam is symmetrically loaded, each reaction is equal to one-half the sum of the loads; that is, each reaction is equal to $\dfrac{400 + 1,200 + 400}{2} = 1,000$ pounds. The shear at $R_1$ is equal to the reaction, or 1,000 pounds. To plot the diagram proceed as follows: Draw a line *a b* to any convenient scale to represent the length of the beam. This line will also represent the *base*, or *datum, line* of the shearing forces, positive values being laid off a b o v e and negative values below the base line. The shear at $R_1$ being positive and equal to 1,000 pounds, draw from *a*
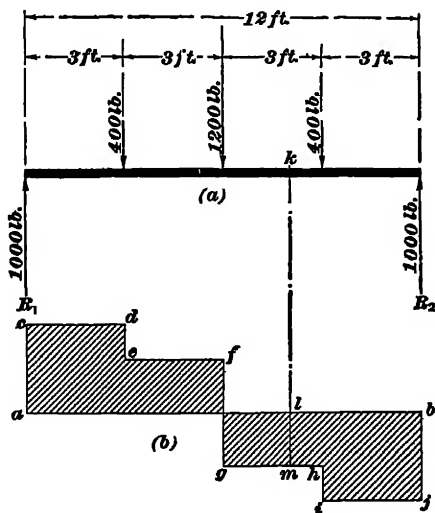


Fig. 15

upwards a vertical line *a c* equal to 1,000 pounds, according to any convenient scale. From *c* draw to the same scale as *a b* a horizontal line *c d* equal to 3 feet. Ordinates drawn from any point on the line *a b* to *c d* will be of equal lengths, showing that the shear retains the value of 1,000 pounds from the reaction $R_1$ to the first load of 400 pounds. At this point the shear is reduced by the load of 400 pounds; therefore, from *d*, draw downwards a vertical line *d e* of a length corresponding to 400 pounds, as shown in the diagram.

The shear is then uniform until the central load is reached,

and may be represented by the distance of the line *e f* from the line *a b*, namely, 600 pounds. At this point the shear is reduced by 1,200 pounds, shown on the diagram by the line *f g*, and becomes negative. At the third load, counting from the left, the shear is still further reduced by 400 pounds, and is represented by the line *h i* in the diagram. From this load on to the right-hand reaction the shear is negative in sign and equal to 1,000 pounds. This right-hand reaction drawn from *j* to *b* closes the diagram.

It may now be seen that to find the shear at any point in a beam all that is necessary is to draw an ordinate from the corresponding point on the line *a b* and to measure the length of the part included in the shaded diagram with the correct scale. For instance, if it is desired to ascertain the shear and its sign at the point *k*, draw a perpendicular *k m* crossing the shear diagram. The length *l m* measured by the scale selected for the shear will show that the shear at this point is 600 pounds, and, as *l m* is located below the base line *a b*, the shear is negative.

**16.** Fig. 16 shows a beam that is uniformly loaded. Since the load is uniform, the two reactions are equal to
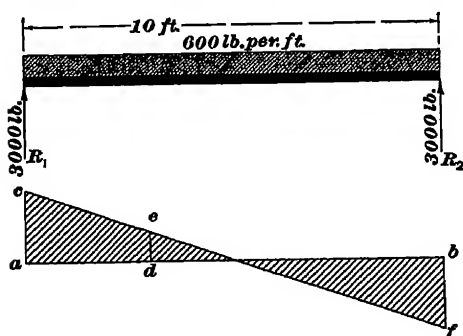


Fig. 16

each other and each equal to one-half the total load; that is, $6,000 \div 2 = 3,000$ pounds. To draw the shear diagram, lay off the line *a b* equal to the length of the beam in the illustration. From *a* draw a vertical line *a c*, and lay off from *a* to *c* to any convenient scale, the reaction $R_1$, or 3,000 pounds. This is the shear at the left-hand reaction. Now, take any other point on the beam, for instance, the point *d* located 3 feet from the left-hand reaction. The shear here is equal

to $+3{,}000 - 3 \times 600 = +1{,}200$ pounds. Lay this shear off from $d$ vertically upwards, since it is positive, and make $d e$ equal to 1,200 pounds on the assumed scale. Now, between the points $a$ and $d$, the shear diminishes uniformly; therefore, the height to a straight line drawn from $c$ to $e$ will represent the shear between $a$ and $d$. But since there is a uniform load over the entire beam, this same diminution of
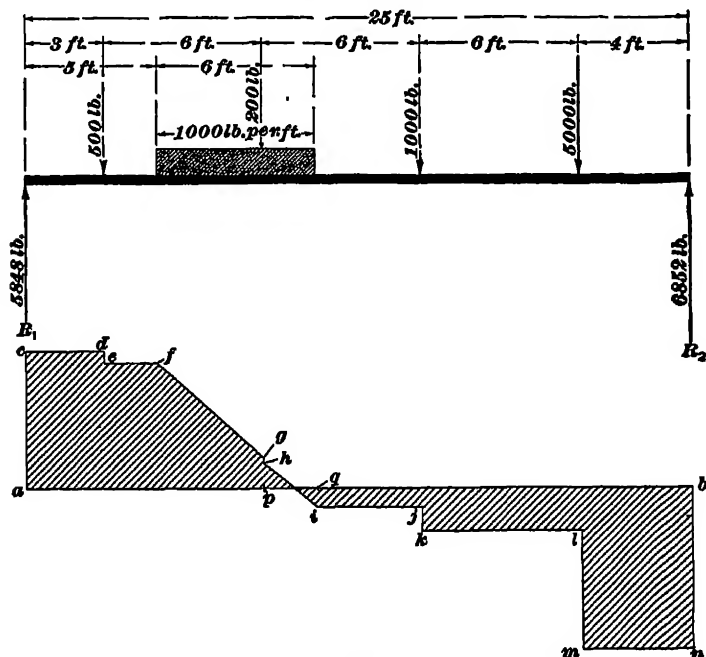


Fig. 17

shear will be maintained throughout its entire length; that is, the line $c e$ should be continued until it intersects a vertical $b f$ drawn from the right-hand end of the beam. If the diagram is drawn correctly, it will be found that the shear at the center of the beam is zero and that the shear at $f$ is negative and equal to the reaction $R_1$, namely, 3,000 pounds.

**17.** As a general example, the shear diagram of the beam shown in Fig. 17 may be plotted. As the load here is not symmetrical, it is first necessary to calculate the

reactions $R_1$ and $R_2$. The positive moments about the left-hand end of the beam, in foot-pounds, are:

$$
\begin{aligned}
500 \times 3 &= 1\,5\,0\,0 \\
(1{,}000 \times 6) \times 8 &= 4\,8\,0\,0\,0 \\
200 \times 9 &= 1\,8\,0\,0 \\
1{,}000 \times 15 &= 1\,5\,0\,0\,0 \\
5{,}000 \times 21 &= 1\,0\,5\,0\,0\,0 \\
\text{Total,} \quad &\overline{1\,7\,1\,3\,0\,0}
\end{aligned}
$$

The total negative moment is $R_2 \times 25$. Therefore $R_2$ = 171,300 ÷ 25 = 6,852 pounds. The sum of the loads is 500 + (1,000 × 6) + 200 + 1,000 + 5,000 = 12,700 pounds. $R_1$ is therefore equal to 12,700 − 6,852 = 5,848 pounds.

The plotting of the shear diagram shown in Fig. 17 may now be started. Draw the line $a\,b$ equal to the length of the beam; $a\,c$ equal to $R_1$, or 5,848 pounds; $c\,d$, horizontally, equal to 3 feet; $d\,e$, vertically downwards, equal to 500 pounds; and $e\,f$, horizontally, equal to 2 feet. From $a$ on $a\,b$ lay off 9 feet, as at $p$. The shear at this point, just to the left of the concentrated load, is +5,848 − 500 − (1,000 × 4) = +1,348 pounds. This value laid off above $a\,b$ gives the point $g$. Draw $f\,g$. From $g$ lay off vertically downwards 200 pounds to the point $h$. From $a$, on the line $a\,b$, lay off 11 feet, as at $q$. The shear at $q$ is +5,848 − 500 − (1,000 × 6) − 200 = −852 pounds. From $q$ lay off, vertically, 852 pounds to $i$, downwards in this case because the shear is negative. Join $h$ and $i$. If correctly drawn, $h\,i$ should be parallel to $f\,g$. From $i$ draw $i\,j$ horizontal and equal to 4 feet; from $j$ draw $j\,k$ vertically downwards, equal to 1,000 pounds; from $k$ draw $k\,l$ horizontally, equal to 6 feet; from $l$ draw $l\,m$ vertically downwards, equal to 5,000 pounds; and from $m$ draw $m\,n$ horizontally, equal to 4 feet. Then, if the diagram is correctly constructed, $b\,n$ should be vertical and equal to $R_2$, or 6,852 pounds. Then $a\,c\,d\,e \ldots m\,n\,b$ is the shear diagram, and ordinates drawn from any point on $a\,b$ across the diagram will, when measured by the proper scale, indicate the shear at that section, which is positive if above the line $a\,b$ and negative if below it.

**18.** As a final example in drawing a shear diagram, the cantilever beam shown in Fig. 18 may be taken. This beam is loaded with a uniform load and two concentrated loads, as shown. It will be noted that all the shear on the beam is negative. Only the exposed part of the beam is shown in the diagram, as the conditions of the beam in the wall are not known. How-
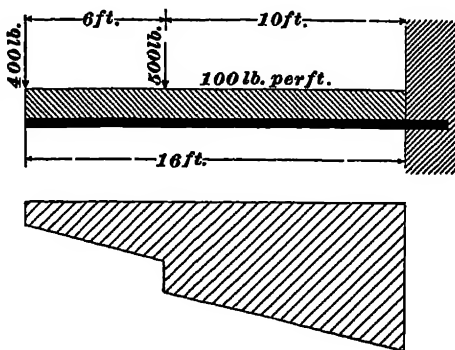


FIG. 18

ever, it would be safe to assume that the shear changes sign directly at the wall and becomes positive, only to become zero again at the end of the beam. The example is shown in the figure and is drawn in the same manner as the previous examples.

## BENDING MOMENTS

### METHODS OF CALCULATION

**19.** In order to illustrate the method of calculating the **bending moment,** or the moment of a force that tends to bend a beam, the beam shown in Fig. 19 will be considered.
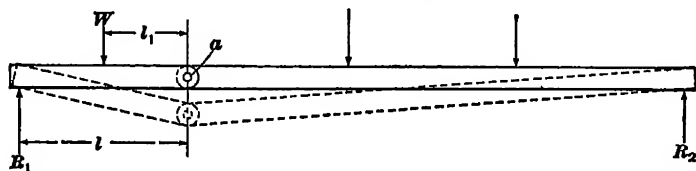


FIG. 19

At the point $a$ is a joint, or hinge. It is evident that when loads are applied, as shown by the arrows, the beam will bend at the joint and take the position indicated by the

dotted lines; or, to be more exact, will bend still further until it falls off the supports entirely.

The loads and reactions (and weight of the beam itself, if this is considered) are what cause the beam to bend. Considering the portion of the beam to the left of the joint, this portion moves clockwise, and the moment that moves it is therefore positive. The magnitude of the moment tending to move the left-hand part of the beam, according to *Statics*, is $R_1 l - W l_1$, since the load $W$ acts in the opposite direction to $R_1$.

It will thus be seen that no matter in what part of the beam the joint $a$ is placed, the beam will collapse. Therefore, it is evident that in any beam carrying loads, these loads and the reactions exert a moment at any transverse section that tends to bend the beam. It is only on account of its own strength that a beam does not break. A beam is therefore designed to withstand the bending moment of the
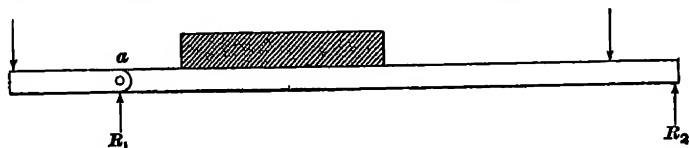
forces acting on it, and for this reason it is always necessary for the engineer to be able to find this moment.

This Section considers the method of finding the bending moment that the forces produce, while a future Section will explain how to design a beam to withstand this moment.

As was stated, the moment about the joint $a$ on the portion to the left, which is the part always considered, is positive. It will be seen also that no matter where the joint is located in a simple beam, the moment will always be positive. In the beam shown in Fig. 20 it will be seen that the moment acting on the part to the left of the joint, that is, the cantilever part, is negative. It can thus be said that with a simple beam the bending moment (considering as usual the part to the left of the section) is always positive, while with cantilever beams it is negative.

**20.** As an illustration, the bending moment around various sections of the beam shown in Fig. 21 will be considered. It is of course first necessary to find the reactions, so that the moments may be calculated. To find $R_2$, take moments about $R_1$. The positive moments, in foot-pounds, are

$$
\begin{aligned}
1{,}200 \times 3 &= 3\ 6\ 0\ 0 \\
(800 \times 9) \times 10\tfrac{1}{2} &= 7\ 5\ 6\ 0\ 0 \\
2{,}000 \times 18 &= \underline{3\ 6\ 0\ 0\ 0} \\
\text{Total,} &\quad 1\ 1\ 5\ 2\ 0\ 0
\end{aligned}
$$

The span is 24 feet. Therefore, $R_2$ = 115,200 ÷ 24 = 4,800 pounds. The sum of the loads is 1,200 + (800 × 9) + 2,000 = 10,400 pounds. Therefore, $R_1$ = 10,400 − 4,800 = 5,600 pounds.

The bending moment at any section of the beam may now be found. For example, find the bending moment
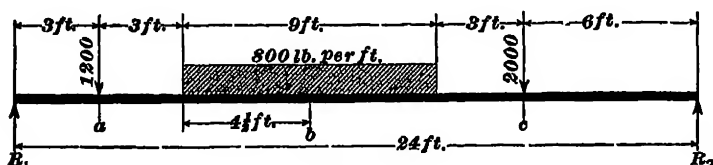


FIG. 21

around point $a$ directly under the first load. The moment here, on the left-hand part of the beam, is $R_1 \times 3 = 5{,}600 \times 3$ = 16,800 foot-pounds.

Then find the moment on the left-hand portion of the beam around point $b$. Here the positive moment is $R_1 \times 10\tfrac{1}{2}$, or $5{,}600 \times 10\tfrac{1}{2}$ = 58,800 foot-pounds. There are, however, two negative moments acting, and these must be subtracted from the positive moment in order to get the resultant moment. One of these moments is that due to the concentrated load of 1,200 pounds, and the other is that due to the part of the distributed load to the left of the point $b$. The negative moment of the concentrated load is therefore 1,200 × 7½ = 9,000 foot-pounds. The portion of the distributed load considered is 800 × 4½ = 3,600 pounds. Its moment

arm may be considered to extend from the point $b$ to the center of the portion under consideration, or $4\frac{1}{2} \div 2 = 2\frac{1}{4}$ feet. Its moment is therefore $3,600 \times 2\frac{1}{4} = -8,100$ foot-pounds. The resultant moment of the left-hand section about $b$ is therefore $58,800 - 9,000 - 8,100 = +41,700$ foot-pounds.

The bending moment about $c$ is $5,600 \times 18 - 1,200 \times 15 - (800 \times 9) \times 7\frac{1}{2} = 100,800 - 18,000 - 54,000 = +28,800$ foot-pounds.

·21. Consider now the moment of the forces acting on the left-hand portion of the beam about the point $c$. Their moment has been found to be $+28,800$ foot-pounds.

One of the fundamental principles of the equilibrium of beams, as stated in the beginning of this Section, is that the algebraic sum of the moment of *all* the forces acting on the beam about *any* point is zero. It has been customary, when finding reactions, to take moments about one point of support, because it was more convenient; nevertheless, the proposition is true as stated, namely, that the algebraic sum of the moments about any point is zero.

The sum of the moments of all the forces about $c$ and to the left of $c$ is equal to $+28,800$ foot-pounds. The moment of all the forces to the right about the same point must therefore be $-28,800$ foot-pounds. There is in this case only one force to the right, namely, the right-hand reaction $R_1$. The force of 2,000 pounds goes directly through the point $c$, and can therefore exert no moment about $c$. The moment of $R_2$ about $c$ is $-4,800 \times 6 = -28,800$ foot-pounds, which is correct. The sign is minus, since it tends to turn the right-hand end of the beam counter-clockwise.

22. From the foregoing, it is seen that the bending moment of the forces acting on a beam, tending to break it at any section, may be calculated from either end of the beam. It is customary to call the bending moment positive if it tends to turn the left-hand part in the direction of the hands of a clock, and negative if it tends to turn it in the opposite direction. However, as shown, the moment may be calculated from either end, remembering that if it

is calculated from the right-hand end the bending moment acts in an opposite direction, and will therefore receive a sign opposite to that given the left-hand end of the beam. If nothing is said to the contrary, it is always understood that when the designation positive or negative is applied to the bending moment of a beam, the portion at the left of the center of moments is referred to.

### MAXIMUM MOMENT

**23.**  It has been stated that the reason a beam does not break is because its strength at any transverse section is sufficient to resist the moment of the forces.about that section.  It has been shown that the bending moment is different at different points along a beam.   Theoretically, therefore, a beam should be of different strengths in different parts of its length.   In reinforced-concrete and plate-girder construction, this is more or less true, but in wood or steel beams, it is cheaper to make the beam of uniform size throughout, rather than vary the sectional area of the beam in accordance with the exact load it is to carry.   Even in plate-girder and reinforced-concrete work, the section at the strongest part is designed first.   It is therefore of prime importance to find out around what point the forces acting on a beam exert their maximum moment, and then, by the method already given, to find this maximum moment.   When this is done, the engineer may design the beam to withstand this moment by methods explained in a future Section, and he will then be positive that the beam is more than strong enough at other points in its length.

The method of finding the point around which maximum moment occurs is very simple and always obeys the following rule:

**Rule.**—*The point where the shear on a beam changes sign, that is, changes from positive to negative, or from negative to positive, is the point around which the maximum bending moment occurs.*

This proposition can be proved both by mathematics and by experiment. Its proof, however, is rather long and complicated, and will therefore not be given here.

**24.** The method by which to find the maximum bending moment may be explained as follows:

Assume that it is desired to find the maximum bending moment of the forces acting on the beam shown in Fig. 22. The moments about $R_1$ are as follows:

$$1,600 \times 1 = 1\,6\,0\,0$$
$$(300 \times 4) \times 4 = \underline{4\,8\,0\,0}$$
$$\text{Total,} \quad 6\,4\,0\,0$$

$R_2 \times 8 = 6,400$ foot-pounds; $R_2 = 6,400 \div 8 = 800$ pounds; and $R_1 = 1,600 + (300 \times 4) - 800 = 2,000$ pounds.

Starting from $R_1$, the shear here is positive and equal to 2,000 pounds. At the concentrated load, the shear is reduced to $2,000 - 1,600 = 400$ pounds. This value of the



300 lb. per ft

1600 lb.

8 ft.

1 ft.    1 ft.    4 ft.    2 ft.

$R_1$    $R_2$
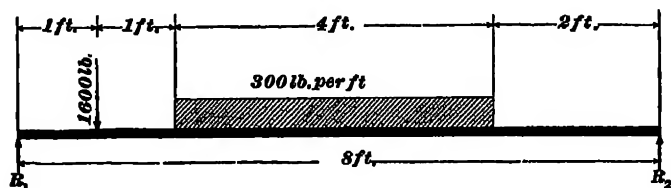
FIG. 22

shear continues up to the distributed load. At a distance of 1 foot in the distributed load, or 3 feet from the left-hand end of the beam, the shear is $400 - 300 = 100$ pounds. The length of distributed load equal to 100 pounds is $\frac{100}{300} = \frac{1}{3}$ foot. Hence, the point where the shear changes sign is $3 + \frac{1}{3} = 3\frac{1}{3}$ feet from the left-hand end of the beam. From this point on to the right-hand end of the beam the shear is continually negative. The point around which the bending moment is maximum is therefore $3\frac{1}{3}$ feet from the left-hand reaction. This maximum bending moment is equal to the sum of the following moments: $2,000 \times 3\frac{1}{3} - 1,600 \times 2\frac{1}{3} - (300 \times 1\frac{1}{3}) \times \frac{1}{3} = 2,666\frac{2}{3}$ foot-pounds. This is the moment that the beam must be designed to withstand.

**25.   Beams With Two Maximum Bending Moments.**
Sometimes, the shear will change from positive to negative
or vice versa two or more times on a beam, and each sec-
tion, when the shear is zero, must therefore be investigated
to determine around which point the maximum moment
occurs.   Fig. 23 shows an example of this kind.   To solve
this example, it is first necessary to find the reactions.   The
moments of the loads about $R_1$, in foot-pounds, are as
follows:

$$500 \times \phantom{0}2 = 1\,0\,0\,0$$
$$800 \times \phantom{0}5 = 4\,0\,0\,0$$
$$300 \times 14 = \underline{4\,2\,0\,0}$$
$$\text{Total,} \quad 9\,2\,0\,0$$

$R_2 \times 10 = 9{,}200$ foot-pounds; $R_2 = 9{,}200 \div 10 = 920$
pounds; and $R_1 = 500 + 800 + 300 - 920 = 680$ pounds.
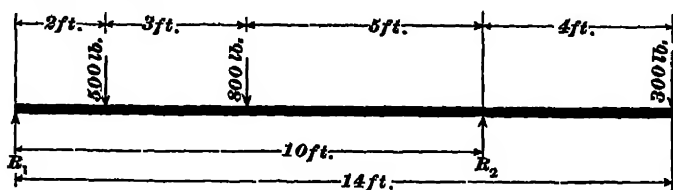


FIG. 23

At $R_1$, the shear is $+680$ pounds, and at 2 feet from the
left-hand end the shear changes from $+680$ to $680 - 500$
$= +180$ pounds.   At 5 feet from the left-hand end, the
shear changes from $+180$ to $180 - 800 = -620$ pounds.
This, therefore, is one place where the shear changes sign.
Under the reaction $R_2$, the shear changes from $-620$ to
$920 - 620 = +300$ pounds, and this is therefore another
point where the shear changes sign.

There are therefore two places to be investigated for
maximum bending moment, one 5 feet from the left-hand
end and the other 10 feet from the left-hand end.   The
bending moment about the point 5 feet from the left-hand
end is $680 \times 5 - 500 \times 3 = +1{,}900$ foot-pounds, and that
about the point 10 feet from the left-hand end is
$680 \times 10 - 500 \times 8 - 800 \times 5 = -1{,}200$ foot-pounds.   It

is thus seen that the greater of the two maximum bending moments occurs about the point 5 feet from the left-hand end.

**26. Cantilever Beams Fixed at One End.**—The beams so far discussed have been beams supported at two points—some simple beams and some beams with cantilever ends. The same method of finding the maximum moment that the forces exert on a simple beam may also be used for a cantilever beam that is fixed firmly in a wall at one end and is free and unsupported at the other. At first it would seem difficult to find the point where the shear changes sign, as part of the beam is buried in the wall, but this point is always in the same place, namely, directly at the face of the wall.

**Rule.**—*The point where the shear on a cantilever beam changes sign, that is, the point around which the moment is maximum, is always at the point of support.*

Therefore, in a cantilever beam, it will not be necessary to calculate the point where the shear is zero to find the maximum moment, because that point is already known.

EXAMPLE.—A cantilever beam projects from a wall 13 feet and is loaded with a concentrated load of 8,000 pounds placed 1 foot from the free end. Find the maximum bending moment, in foot-pounds.

SOLUTION.—The point where the shear changes sign, that is, the point of maximum bending moment, occurs at the wall and is 13 ft. from the free end of the beam. As the load is concentrated 1 ft. from the end, this end, if the weight of the beam itself is neglected, is of no importance in solving the problem and may therefore be disregarded. The distance from the load to the point around which the maximum bending moment occurs is therefore $13 - 1 = 12$ ft. The maximum bending moment, therefore, is

$$12 \times 8,000 = 96,000 \text{ ft.-lb. Ans.}$$

**27. Moments Due to Weight of Beams.**—In all the examples that have been given it is assumed that the weight of the beam does not enter into consideration. The weight of the beam itself will of course exert a certain amount of shear and bending moment, but this moment is so slight

that it may usually be neglected in comparison to the bend-
ing moment exerted by the superimposed loads. Besides,
in most tables giving the strength of beams, corrections are
made for the weight of the beam itself. However, it is
sometimes necessary in calculating the maximum moment
to include in the problem the weight of the beam. In such
cases, the weight of the beam is treated simply as a uni-
formly distributed load.

EXAMPLE 1.—Find the reactions, the point where the shear
changes sign, and the maximum bending moment of a beam on a
14-foot span. The beam weighs 40 pounds per foot. It carries one
concentrated load, as shown in Fig. 24, of 8,100 pounds.
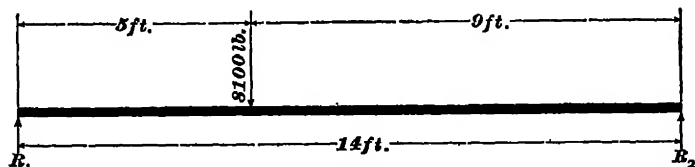


FIG. 24

SOLUTION.—The total weight of the beam is $14 \times 40 = 560$ lb.
Taking moments about $R_1$, the positive moment is $560 \times 7 + 8,100$
$\times 5 = 44,420$ ft.-lb. Therefore, $R_2 = \frac{44,420}{14} = 3,173$ lb , approxi-
mately The sum of the loads is $560 + 8,100 = 8,660$ lb. Therefore,
$R_1 = 8,660 - 3,173 = 5,487$ lb. The shear immediately to the left of
the concentrated load is $+5,487 - (5 \times 40) = +5,287$ lb. Immediately
to the right of the concentrated load the shear is $+5,287 - 8,100$
$= -2,813$ lb. Therefore, the shear changes sign at this concentrated
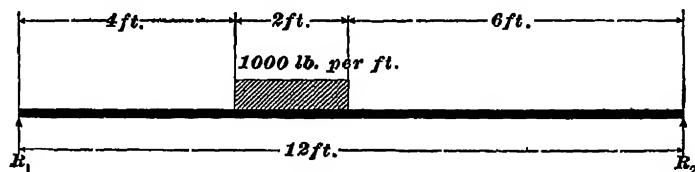


FIG 25

load, and about this point the maximum moment occurs. This
moment is

$$5,487 \times 5 - 200 \times 2\tfrac{1}{2} = 26,935 \text{ ft.-lb.  Ans.}$$

EXAMPLE 2.—Find the maximum bending moment of a beam
weighing 100 pounds per foot and carrying a uniformly distributed
load over part of its length of 1,000 pounds per foot. The beam and
load is located as shown in Fig. 25.

SOLUTION.—Take the moments around $R_2$. The negative moments, in foot-pounds, are

$$(1,000 \times 2) \times 7 = 1\,4\,0\,0\,0$$
$$(100 \times 12) \times 6 = \underline{\phantom{0}7\,2\,0\,0}$$
$$\text{Total,}\quad \overline{2\,1\,2\,0\,0}$$

Therefore, $R_1 = 21,200 \div 12 = 1,766.67$ lb. The shear 4 ft. from the left-hand end is therefore $+1,766\;67 - 4 \times 100 = +1,366\;67$ lb. From here on there are two loads, one of 1,000 lb. per ft. and one of 100 lb. per ft , or a total of 1,100 lb. per ft. The length of beam beneath the distributed load of 2,000 lb required to bring the shear to zero is $1,366.67 - 1,100 = 1.2424$ ft. The point at which the shear changes sign is therefore $4 + 1.2424 = 5.2424$ ft from the left-hand end. The positive moment about this point is $1,766.67 \times 5.2424$ $= 9,261.6$ ft.-lb., and the negative moment, in foot-pounds, is

$$(100 \times 5.2424) \times \frac{5.2424}{2} = 1\,3\,7\,4$$
$$(1,000 \times 1.2424) \times \frac{1.2424}{2} = \underline{\phantom{0}7\,7\,2}$$
$$\text{Total,}\quad \overline{2\,1\,4\,6}$$

The total maximum moment is therefore

$$9,261.6 - 2,146 = 7,115.6 \text{ ft.-lb.}\quad \text{Ans.}$$

---

### EXAMPLES FOR PRACTICE

1. What is the maximum bending moment in a cantilever carrying a uniformly distributed load of 170 pounds per foot, the length of the beam being 20 feet and its weight 10 pounds per foot?

Ans. 36,000 ft.-lb.
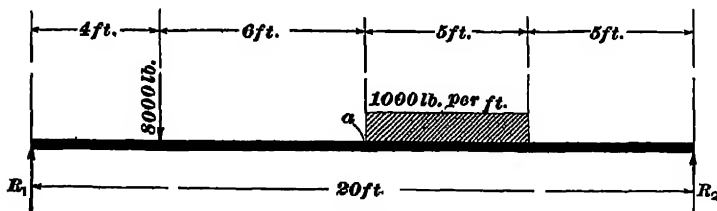


FIG. 26

2. A simple beam 24 feet long carries a concentrated load of 15,000 pounds at a distance of 8 feet from the left-hand reaction. Neglecting the weight of the beam itself, what is the value of: ($x$) the left-hand reaction?  ($b$) the maximum bending moment?

Ans $\begin{cases} (a) & 10,000 \text{ lb.} \\ (b) & 80,000 \text{ ft.-lb.} \end{cases}$

3. A beam 20 feet long and weighing 90 pounds per foot carries a single concentrated load of 8,000 pounds and a load of 5,000 pounds uniformly distributed over a length of 5 feet. The loads are situated as shown in Fig. 26. Calculate the point around which the maximum bending moment occurs.

Ans. 252 ft. to right of point $a$

4. What is the maximum bending moment in example 3?

Ans. 39,284.7 ft.-lb.

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**28. Bending-Moment Diagrams.**—The bending moments that act at various points of a loaded beam may be
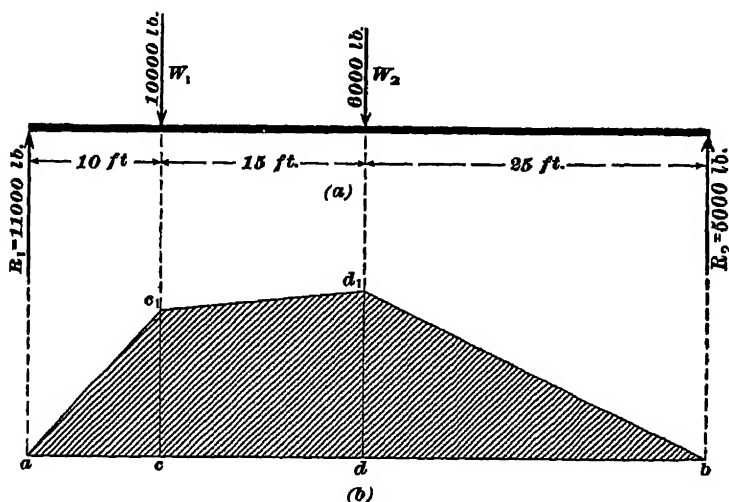


Fig. 27

represented graphically in the same manner in which the shearing forces were shown. For example, a beam 50 feet long supports two concentrated loads of the magnitudes and in the positions indicated in Fig. 27. Now, assuming that Fig. 27 $(a)$ is drawn to scale, draw the horizontal line $a\,b$ and produce the lines indicating the reactions and concentrated loads until they intersect line $a\,b$ at $a$, $c$, $d$, and $b$. By calculating the moments about the left-hand reaction, the moment for the load $W_1$ is found to be $10,000 \times 10 = 100,000$

foot-pounds and that for $W_2$, $6,000 \times 25 = 150,000$ foot-pounds. The reaction $R_2$ is therefore $\dfrac{100,000 + 150,000}{50}$ = 5,000 pounds, and reaction $R_1$ is $10,000 + 6,000 - 5,000$ = 11,000 pounds.

The bending moment at $W_1$ is $11,000 \times 10 = 110,000$ foot-pounds, and that at $W_2$ is $(11,000 \times 25) - (10,000 \times 15)$ $= 125,000$ foot-pounds.

Lay off the line $c\,c_1$ to any convenient scale to represent the bending moment at $W_1$, and at $d\,d_1$ to represent that at $W_2$. Connect points $a$, $c_1$, $d_1$, and $b$, as shown, and the diagram is complete.
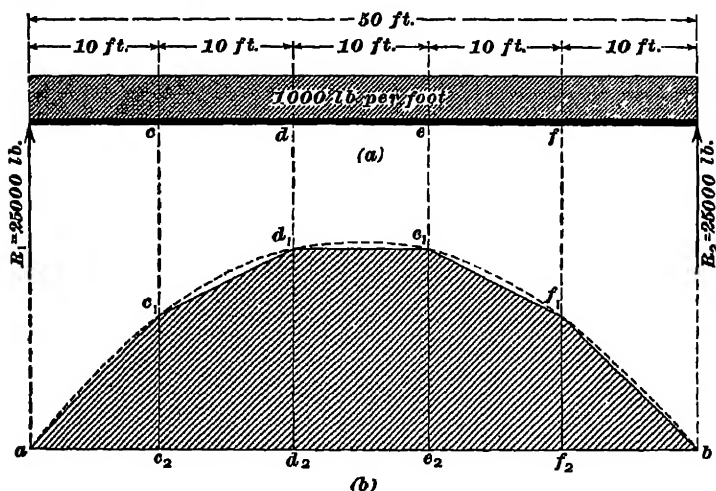


FIG. 28

**29.** As another illustration, assume that it is desired to plot the bending-moment diagram of the beam shown in Fig. 28. This beam is 50 feet long and has a uniform load of 1,000 pounds per running foot.

First, divide the beam into any number of smaller parts, each, for instance, 10 feet long, and then find the bending moment for each of these parts. In this instance, the reactions $R_1$ and $R_2$ are each equal to one-half the total load, or $50,000 \div 2 = 25,000$ pounds. The bending moment at $c$

is $(25,000 \times 10) - (1,000 \times 10 \times 5) = 200,000$ foot-pounds; at $d$ it is $(25,000 \times 20) - (1,000 \times 20 \times 10) = 300,000$ foot-pounds. In a similar manner, it will be found that the bending moments at points $e$ and $f$ are 300,000 and 200,000 foot-pounds, respectively.

On laying off these bending moments at $c_1 c_2$, $d_1 d_2$, etc. and connecting points $a$, $c_1 \ldots b$, the diagram is complete. The outline of this diagram, however, is only approximately correct. To find its true form, the beam should be divided into an infinite number of parts, in which case the lines $a c_1$, $c_1 d_1$, $d_1 e_1$, $e_1 f_1$, and $f_1 b$ would be parts of a curve called a *parabola*, shown dotted, instead of a series of short, broken lines.

EXAMPLE.—A cantilever, Fig. 29, is 50 feet long and has a concentrated load $W$ of 1,000 pounds at the free end. Show the bending-moment diagram.

SOLUTION.—As the point of maximum bending moment is at $a$, the line $a c$ is laid off to represent $1,000 \times 50$ or 50,000 ft.-lb. The bending moment at some other point as at $d$, is



FIG. 29

found to be $30 \times 1,000 = 30,000$ ft.-lb.; the line $e f$ is then laid off to represent the latter moment. By connecting points $c$ and $e$ by the line $c e$ and producing the latter, it is found to intersect line $a b$ at $b$. The triangle $a b c$ is then the required diagram. Ans.
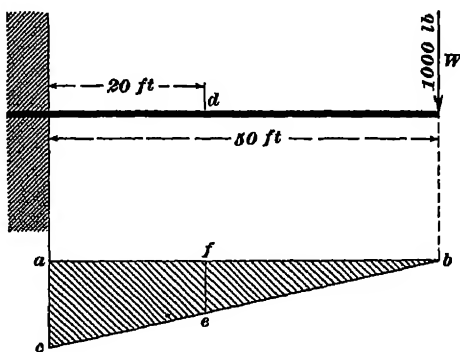
### MOMENT TABLES

**30.** In the preceding pages, the methods of finding the shear and maximum moment occurring in simple and in cantilever beams have been explained. As was said, in these calculations the weight of the beam itself may usually be omitted. The various different and unusual loadings were also considered, and the method of finding the maximum

# TABLE I

## FORMULAS FOR MAXIMUM SHEAR AND BENDING MOMENTS ON BEAMS
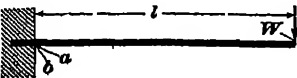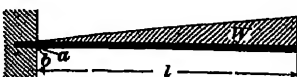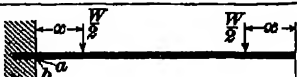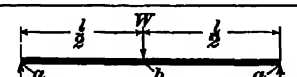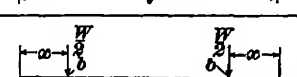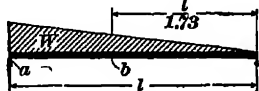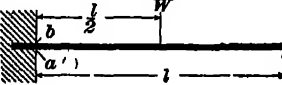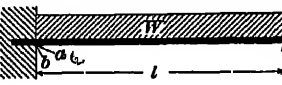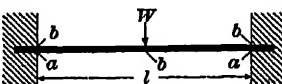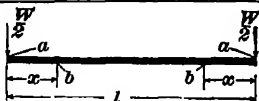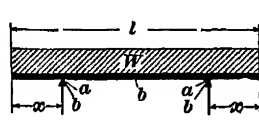
| Case | Method of Loading | Maximum Shear | Maximum Moment |
|------|-------------------|---------------|----------------|
| I | | $W$ | $Wl$ |
| II | | $W$ | $\dfrac{Wl}{2}$ |
| III | | $W$ | $\dfrac{Wl}{3}$ |
| IV | | $W$ | $\dfrac{2\,Wl}{3}$ |
| V | | $W$ | $\dfrac{Wl}{2}$ |
| VI | | $W$ | $\dfrac{Wl}{2}$ |
| VII | | $W$ | $\dfrac{Wl}{2}$ |
| VIII | | $\dfrac{W}{2}$ | $\dfrac{Wl}{4}$ |
| IX | | $\dfrac{Wy}{l}$ or $\dfrac{Wx}{l}$ | $\dfrac{Wxy}{l}$ |
| X | | $\dfrac{W}{2}$ | $\dfrac{Wx}{2}$ |

TABLE I—(*Continued*)

| Case | Method of Loading | Maximum Shear | Maximum Moment |
|------|-------------------|---------------|----------------|
| XI | | $\dfrac{W}{2}$ | $\dfrac{W\,l}{8}$ |
| XII | | $\dfrac{W}{2}$ | $\dfrac{W\,l}{12}$ |
| XIII | | $\dfrac{W}{2}$ | $\dfrac{W\,l}{6}$ |
| XIV | | $\dfrac{2\,W}{3}$ | $\dfrac{52\,W\,l}{405}$ |
| XV | | $\tfrac{11}{16}\,W$ | $\tfrac{3}{16}\,W\,l$ |
| XVI | | $\tfrac{5}{8}\,W$ | $\tfrac{1}{8}\,W\,l$ |
| XVII | | $\dfrac{W}{2}$ | $\dfrac{W\,l}{8}$ |
| XVIII | | $\dfrac{W}{2}$ | $\dfrac{W\,l}{12}$ |
| XIX | | $\dfrac{W}{2}$ | $\dfrac{W\,x}{2}$ |
| XX | | $\dfrac{W\,x}{l}$ or $W\left(\dfrac{l-2\,x}{2\,l}\right)$ | $\dfrac{W\,x^2}{2\,l}$ or $\dfrac{W}{2}\left(\dfrac{l}{4}-x\right)$ |

bending moment was explained. These methods will be found useful if such cases ever arise. However, in actual practice the loads on the beams are often more uniform and regular, and as the weight of the beam itself is usually neglected, the formulas given in Table I have been devised.

Beams with fixed ends or even with one end fixed and one simply supported have not been mentioned before, because they present certain difficulties in solving. The solutions, however, are given in Table I.

**31. Notation.**—In each case in Table I the total load on the beam, in pounds, is denoted by $W$. If there are two separate and equal loads on the beam, each one is called $\dfrac{W}{2}$, so as to make the total load $W$. If the load is uniformly distributed over the entire length of the beam, the load per foot will be $W \div$ length of beam, in feet. The length of the beam is denoted by $l$. If $l$ is taken in inches, the moment will be in inch-pounds; but if taken in feet, the moment will be in foot-pounds.

**32.** The usual method of indicating loading is adopted. A simple support under a beam is shown by the conventional arrow, while a cantilever or a beam fixed at the ends is shown as in Case I or XVII, respectively. Cases XV and XVI indicate a beam fixed at one end and supported at the other.

Case XI is the one most used, as it applies to a beam uniformly loaded, such as floor joists, girders, etc. Case XIII will be found convenient in calculating the bending moment on lintels supporting brickwork or masonry over openings.

It will be observed that if the beam supporting a concentrated load at the center is firmly fixed, or fastened, at both ends, as in Case XVII, instead of being simply supported, as designated in Case VIII, the bending moment under the same conditions will be only half as much. Also, in Case XVIII, where the ends of the beam are firmly fixed, the uniformly distributed load that may be supported is one and one-half times as great as where the ends merely rest

on supports, as in Case XI.   As previously stated, it is
seldom advisable, in ordinary building practice, to consider
the ends of a beam as fixed, it being good practice to
assume the ends of the beam as simply bearing on their
supports.

The maximum shear on a cantilever or on a beam fixed
at the end depends somewhat on the method of holding it in
the wall and the length that extends into the wall.   How-
ever, the maximum shears given in the table are correct
for usual cases.   In Case XX, two values are given for
both shear and bending moment, and the maximum of the
two values obtained must be the one used.   The sign of
the shears or bending moments has not been put in Table I,
but it can be told by inspection whether they are positive or
negative, as every-day experience should enable the engi-
neer to determine whether the loads will bend the beam up
or down.

The point of maximum shear is marked on the beam at
the point $a$, while maximum bending moment is at the
point $b$.   In some cases, either of these values may reach
its maximum at two or more places, in which case it is so
marked.   It should not be difficult to tell whether these
maximum values are at only one point or are continuous for
some distance along the beam.

In Case XX, the first maximum moment given applies to
the supports, while the second maximum moment applies to the
center of the beam.   Both values for the maximum shear occur
at the reactions.   In Case IX, the first shear given applies at
the left-hand reaction and the second value applies at the
right-hand reaction.

**33.   Application of Table of Moments.**—The following
examples are given to show the use of Table I:

EXAMPLE 1.—A simple beam on a span of 13 feet $2\frac{1}{2}$ inches carries
a uniformly distributed load of 85 pounds per foot   What is the maxi-
mum shear and the maximum bending moment developed?

SOLUTION.—The length of the beam reduced to feet is 13 2083   The
total load $W$ is therefore 85 × 13 2083 = 1,122.7055, say 1,123, lb.

Referring to Case XI, Table I, the maximum shear $= \dfrac{W}{2} = \dfrac{1,123}{2}$ $= 561.5$ lb. Likewise, the maximum bending moment is

$$\frac{Wl}{8} = \frac{1,123 \times 13.2083}{8} = 1,854.115, \text{ say } 1,854, \text{ ft.-lb Ans.}$$

EXAMPLE 2.—A cantilever beam is 11 feet 7 inches long. It is loaded with a total load of 22,000 pounds. This load is distributed over the beam so that it reaches a maximum at the outer end of the beam and is equal to zero at the wall. Find the maximum shear and the maximum bending moment.

SOLUTION.—The length of the beam, in inches, is $(11 \times 12) + 7$ $= 139$ This problem is similar to Case IV, Table I. The maximum shear then is equal to $W = 22,000$ lb. The maximum bending moment, therefore, is

$\frac{2}{3} Wl = \frac{2}{3} \times 22,000 \times 139 = 2,038,666\frac{2}{3}$ in.-lb , or nearly, 169,889 ft.-lb. Ans.

NOTE.—It should be remembered that when a uniform load is given in pounds per foot, it is necessary to use the length of the beam in feet to calculate $W$ After this $l$ may be taken in either feet or inches for use in the bending-moment formulas to find the maximum bending moment in either foot-pounds or inch-pounds, as described.

EXAMPLE 3 —A beam fixed at both ends is 13 feet long. It carries a load uniformly distributed whose total value is 12,000 pounds. What is the maximum bending moment produced?

SOLUTION.—From Case XVIII, Table I, the maximum bending moment is

$$\frac{Wl}{12} = \frac{12,000 \times 13}{12} = 13,000 \text{ ft.-lb. Ans.}$$

---

### EXAMPLES FOR PRACTICE

1. A beam fixed at both ends is on a span of 16 feet. It has a load concentrated at the center of 25,000 pounds. What is the maximum bending moment developed?      Ans. 50,000 ft.-lb.

2. A beam fixed at one end and supported at the other carries a uniformly distributed load whose total value is 40,000 pounds. What is the maximum shear developed?      Ans. 25,000 lb.

3. A beam supported at both ends is 7 feet 6 inches long. It carries a load whose total value is 8,000 pounds. This load is distributed unevenly over the entire length, being a maximum at the center and uniformly diminishing to nothing at both ends. What is the maximum bending moment developed?      Ans 10,000 ft.-lb.

4. A beam on a 22-foot span is uniformly loaded with a load of 90 pounds per foot. What is the maximum bending moment developed in inch-pounds?                                    Ans 65,340 in.-lb.

5. A cantilever beam 18 feet long has a load of 300 pounds at the outer end. What is the maximum bending moment produced?
                                                     Ans. 5,400 ft.-lb.

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### CONTINUOUS BEAMS

**34.** When a single beam extends over three or more supports it is said to be continuous (see Art. 3). The bending moments produced are very different from those in an ordinary beam. For this reason, the treatment of this class of beams, which thus far has only been defined, must be considered separately.

Fig. 30 shows a continuous beam with two end supports
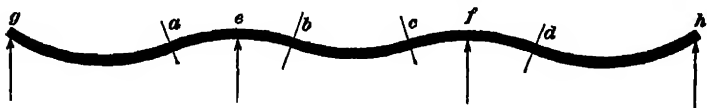


Fɪɢ 30

and two intermediate supports. The beam is uniformly loaded, which is the most general condition of loading for continuous beams. The beam is shown bent down by the imposed load. In actual practice, the sag in the beam would ordinarily be impossible to detect, but in the illustration the sag, or deflection, is shown very much exaggerated so as to illustrate the nature of the bending moment produced.

The points $a$, $b$, $c$, and $d$, where the direction of the curve changes and, consequently, where the sign of the bending moment changes, are called the points of **contraflexure**. The portions $a e$, $e b$, $c f$, and $f d$ act as cantilevers, and the bending moment acting about any point in these sections is negative. The portions $g a$, $b c$, and $d h$ act as simple beams and the moment of the forces about any point in these sections is positive.

**35.** The calculations in regard to continuous girders are very complicated and consequently will not be discussed here. The values in Tables II and III, however, will give

all the information required for finding the maximum shears and bending moments of uniform loads and of spans that are equal for homogeneous beams.

### TABLE II

**REACTIONS FOR CONTINUOUS BEAMS OVER EQUAL SPANS**

*(Coefficients of W)*

| Number of Spans | Number of Each Support | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | 1st | 2d | 3d | 4th | 5th | 6th | 7th | 8th | 9th | 10th |
| 2 | $\frac{3}{8}$ | $\frac{10}{8}$ | $\frac{3}{8}$ | | | | | | | |
| 3 | $\frac{4}{10}$ | $\frac{11}{10}$ | $\frac{11}{10}$ | $\frac{4}{10}$ | | | | | | |
| 4 | $\frac{11}{28}$ | $\frac{32}{28}$ | $\frac{26}{28}$ | $\frac{32}{28}$ | $\frac{11}{28}$ | | | | | |
| 5 | $\frac{15}{38}$ | $\frac{43}{38}$ | $\frac{37}{38}$ | $\frac{37}{38}$ | $\frac{43}{38}$ | $\frac{15}{38}$ | | | | |
| 6 | $\frac{41}{104}$ | $\frac{118}{104}$ | $\frac{100}{104}$ | $\frac{100}{104}$ | $\frac{100}{104}$ | $\frac{118}{104}$ | $\frac{41}{104}$ | | | |
| 7 | $\frac{56}{142}$ | $\frac{151}{142}$ | $\frac{137}{142}$ | $\frac{143}{142}$ | $\frac{143}{142}$ | $\frac{137}{142}$ | $\frac{151}{142}$ | $\frac{56}{142}$ | | |
| 8 | $\frac{153}{388}$ | $\frac{440}{388}$ | $\frac{374}{388}$ | $\frac{392}{388}$ | $\frac{386}{388}$ | $\frac{392}{388}$ | $\frac{374}{388}$ | $\frac{440}{388}$ | $\frac{153}{388}$ | |
| 9 | $\frac{209}{530}$ | $\frac{601}{530}$ | $\frac{511}{530}$ | $\frac{535}{530}$ | $\frac{529}{530}$ | $\frac{529}{530}$ | $\frac{535}{530}$ | $\frac{511}{530}$ | $\frac{601}{530}$ | $\frac{209}{530}$ |

### TABLE III

**BENDING MOMENTS FOR CONTINUOUS BEAMS OVER EQUAL SPANS**

*(Coefficients of W l)*

| Number of Spans | Number of Each Support | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | 1st | 2d | 3d | 4th | 5th | 6th | 7th | 8th | 9th | 10th |
| 2 | 0 | $\frac{1}{8}$ | 0 | | | | | | | |
| 3 | 0 | $\frac{1}{10}$ | $\frac{1}{10}$ | 0 | | | | | | |
| 4 | 0 | $\frac{3}{28}$ | $\frac{2}{28}$ | $\frac{3}{28}$ | 0 | | | | | |
| 5 | 0 | $\frac{4}{38}$ | $\frac{3}{38}$ | $\frac{3}{38}$ | $\frac{4}{38}$ | 0 | | | | |
| 6 | 0 | $\frac{11}{104}$ | $\frac{8}{104}$ | $\frac{9}{104}$ | $\frac{8}{104}$ | $\frac{11}{104}$ | 0 | | | |
| 7 | 0 | $\frac{15}{142}$ | $\frac{11}{142}$ | $\frac{12}{142}$ | $\frac{12}{142}$ | $\frac{11}{142}$ | $\frac{15}{142}$ | 0 | | |
| 8 | 0 | $\frac{41}{388}$ | $\frac{30}{388}$ | $\frac{33}{388}$ | $\frac{32}{388}$ | $\frac{33}{388}$ | $\frac{30}{388}$ | $\frac{41}{388}$ | 0 | |
| 9 | 0 | $\frac{58}{530}$ | $\frac{41}{530}$ | $\frac{45}{530}$ | $\frac{44}{530}$ | $\frac{44}{530}$ | $\frac{45}{530}$ | $\frac{41}{530}$ | $\frac{58}{530}$ | 0 |

Table II gives the coefficients for the reactions of the beam at its supports. The value given in the table, multiplied by $W$, which represents the load per span, will give the reaction. By knowing the reactions and the loads per foot, the shear at any section may be found.

The coefficient from Table III multiplied by $Wl$, which represents the load per span and the length of span, gives the negative bending moment over each support. As the maximum moment occurs over a support, it is seen, by referring to Table III, where this moment will be. If the coefficient given in this table is multiplied by the load in pounds and the span in feet the result will be in foot-pounds; but if the span is taken in inches, the result will be in inch-pounds.

To illustrate the method of using Tables II and III, the following example will be assumed:

EXAMPLE —A beam of three spans is supported by four reactions. Each span is 11 feet   The beam carries a load of 2,000 pounds per foot. (a) What will be the amount of the end reactions?  (b) What is the greatest bending moment produced?

SOLUTION —(a) The load $W$ supported by each span is $2,000 \times 11$ = 22,000 lb. From Table II, the reaction at each end support is equal to $\frac{4}{10} W$. Substituting these values, the reaction at each end is equal to

$$\frac{4}{10} \times 22,000 = 8,800 \text{ lb. Ans.}$$

(b) The greatest bending moment occurs at the intermediate supports; from Table III, this moment is equal to

$$\frac{1}{10} Wl, \text{ or } \frac{1}{10} \times 22,000 \times 11 = 24,200 \text{ ft -lb. Ans.}$$

---

### EXAMPLES FOR PRACTICE

1   What is the ratio between the maximum bending moment developed in a simple uniformly loaded beam on a span of 30 feet and in a continuous beam having three supports with each span equal to 15 feet, or the entire length equal to 30 feet?   The load per foot of span in each case is 400 pounds.                    Ans  4 to 1

2.  What will be the reaction on the central support in example 1 if the load is 2,000 pounds per foot?                    Ans. 37,500 lb.

3   A girder carrying a uniformly distributed load of 1,500 pounds per foot is supported by four supports spaced 10 feet apart   What is the maximum reaction developed?                    Ans. 16,500 lb.

# STRESSES AND STRAINS

## DEFINITIONS

### STRESS

**1.** It is evident that the weight of the materials composing a building and its contents produces forces that must be resisted by the different members of the structure. The action of these forces has a tendency to change the relative position of the particles composing the members, and this tendency is, in turn, resisted by the cohesive force in the materials, which acts to hold the particles together.

The internal resistance with which the force of cohesion opposes the tendency of an external force to change the relative position of the particles of any body subjected to a load is called a **stress**; or, stress may be defined as the load that produces an alteration in the form of a body, and this alteration of form is called the *strain*.

**2.** **Kinds of Stress.**—In accordance with the direction in which the forces act with reference to a body, the stress produced may be either *tensile, compressive*, or *shearing*.

**3.** **Tensile stress** is the effect produced when the external forces act in such a direction that they tend to stretch a body; that is, to pull the particles of a body away from each other. A rope by which a weight is suspended is an example of a body subjected to a tensile stress.

**4.** **Compressive stress** is the effect produced when . the tendency of the forces is to compress the body or to

§ 45

push its particles closer together. A post or the column of a building is an example of a body subjected to a compressive stress.

Shearing Stress is the effect produced when the ~t as in a shear, so as to produce a tendency for the section of a body to slide over the particles :ction. When a steel plate is acted on by a .ves of a shear, or where a load acts on a beam, as shown in Fig. 1, the plate or the beam is subjected to a shearing stress, as at the sections $a\,b$ and $c\,d$, where the beam is shown to have already sheared or failed.



Weight

Beam

Support

Support
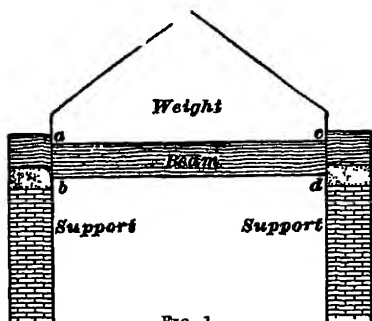
FIG. 1

6. Bending Stress. When a beam is loaded in such a manner that there is in it a tendency to bend, as , it is subjected to a bending stress. In ~ombination of the three stresses already compression, and shear) in different The stresses in a beam are more nple tensile, compressive, and shearing :refore be explained in *Theory of Beams.*



Weight

Beam

pport

Support
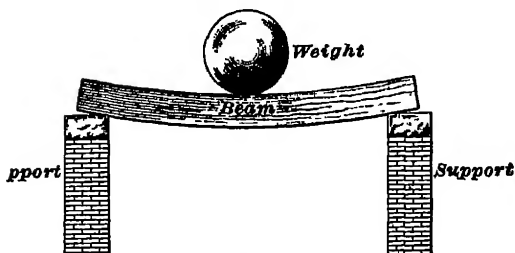
FIG. 2

7. Unit Stress.—The unit stress (called, also, the intensity of stress) is the name given to the stress per unit of area; or, it is the total stress in a tie-rod, column, or the

like divided by the area of the cross-section. Thus, if a weight of 1,000 pounds is supported by an iron rod whose area in cross-section is 4 square inches, the unit stress is $1,000 \div 4 = 250$ pounds per square inch. If, with the same load, the area is $\frac{1}{2}$ square inch, the unit stress is $1,000 \div .5 = 2,000$ pounds per square inch.

Let    $P =$ total stress, in pounds;

       $A =$ area of cross-section, in square inches;

       $s =$ unit stress, in pounds per square inch.

Then, .                $s = \dfrac{P}{A}$, or $P = A s$

That is, *the total stress is equal to the area of the section multiplied by the unit stress.*

EXAMPLE.—An iron rod 2 inches in diameter sustains a load of 90,000 pounds. What is the unit stress?

SOLUTION.—Using the formula,

$s = \dfrac{P}{A} = \dfrac{90,000}{2^2 \times .7854} = 28,647.8$ lb. per sq. in.   Ans.

---

### STRAIN

**8.** When a body is stretched, shortened, or in any way deformed through the action of a force, the deformation is called a **strain**. Thus, if a rod were elongated $\frac{1}{10}$ inch by a load of 1,000 pounds, the strain would be $\frac{1}{10}$ inch. Within certain limits, to be given hereafter, strains are proportional to the stresses producing them.

**9. Unit Strain.**—The **unit strain** is the strain per unit of length or of area, but is usually taken per unit of length and called for tension the *elongation* per unit of length. If the unit of length is taken as 1 inch, the unit strain is equal to the total strain divided by the length of the body in inches.

Let         $l =$ length of body, in inches;

          $e =$ deformation, in inches;

          $q =$ unit strain.

Then,               $q = \dfrac{e}{l}$, or $e = l q$

Example —A steel rod 12 feet 7 inches long elongates .053 inch. What is the unit strain produced?

SOLUTION.—The length of the rod is $12 \times 12 + 7 = 151$ in. Using the formula,

$$q = \frac{e}{l} = \frac{.053}{151} = .000351 \text{ in., nearly. Ans.}$$

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### EXAMPLES FOR PRACTICE

1. A wrought-iron tension member in a roof truss has a load on it of 27,000 pounds. If it has been designed to sustain a unit stress of 6,000 pounds, what will be its diameter?    Ans. $2\frac{1}{2}$ in., nearly

2. The sectional dimensions of a wooden compression member are 8 in. $\times$ 10 in. If the load on it is 64,000 pounds, what is the unit stress on the material?    Ans. 800 lb.

3. What is the total load on a hollow cast-iron column 10 inches outside diameter, the thickness of the metal being 1 inch and the unit stress on the column 20,000 pounds?    Ans. 565,486 lb.

4. Steel eyebars form the tension member in a roof truss; they are subjected to excessive stress and are stretched $\frac{1}{4}$ inch. What is the unit strain in them if they are 10 feet long?    Ans. .00208 in.

5. A certain post is designed to carry a load of 60,000 pounds with an allowable unit stress of 1,200 pounds per square inch. What is the area of the post?    Ans. 50 sq. in.

6. A column made of cast iron is in compression. The load compresses it .098 inch. The column is 11 feet 6 inches long. What is the unit strain produced?    Ans. .0007101 in.

7. In example 6, if the total load is 96,000 pounds and the area of the column is 32 square inches, what is the unit stress produced?
Ans. 3,000 lb. per sq. in.

8. A certain steel rod is stretched until the unit strain developed is .0012 inch. How long must the rod be to have a total stretch of $\frac{3}{8}$ inch?    Ans. 26 ft. $\frac{1}{4}$ in.

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### ELASTIC PROPERTIES

**10. Hooke's Law.**—It can be proved by experiment that when a certain unit stress is created in a substance, a certain definite unit strain is developed. If the unit stress is doubled, it will be found that the unit strain also has doubled; that is to say, the alteration of shape or the strain in a body

is proportional to the force applied to that body. This experimental fact is known as **Hooke's law.**

**11. Elastic Limit.**—When a certain stress is created in a body a certain strain is produced. When the stress is removed, the body returns to its original shape, provided the unit stress has not been too great. For each substance, however, there is a certain maximum unit stress that the substance will stand and still return to its original shape after the external forces are removed. This unit stress is called the **elastic limit** of the material. If a body is strained beyond the elastic limit, it will maintain a permanent distortion, or *set*, even after the strain forces are removed. Hooke's law, which is quite exact for most materials below the elastic limit, does not hold good for these materials above the elastic limit, as the strain increases much more rapidly than the stress. Thus, if the unit stress is doubled beyond the elastic limit, the unit strain will be more than doubled. The unit stress that is so great that the strain increases greatly with very little increment of stress is called the *yield point*. For all practical purposes, with many materials the yield point commences at the elastic limit.

**12. Modulus of Elasticity.**—To restate Hooke's law, the ratio of the unit stress to the unit strain for any substance is constant below the elastic limit. This ratio of unit stress to unit strain is called the **modulus of elasticity, or coefficient of elasticity,** which will be represented by the symbol $E$. It is

$$E = \frac{s}{q}$$

From this equation, when $s$ and $E$ are known, $q$ may be found; that is to say, if the modulus of elasticity of a certain substance and the unit stress are known, the unit strain can readily be found.



Fig. 3

EXAMPLE 1.—The unit stress at the elastic limit of medium structural steel is 33,000 pounds. The modulus of elasticity is 29,000,000.
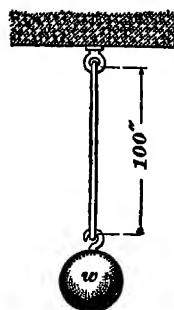
How much would a bar 100 inches long stretch when loaded by a weight $w$ to its elastic limit, as shown in Fig. 3?

SOLUTION.— $s = 33,000$ and $E = 29,000,000$   Therefore,

$$q = \frac{s}{E} = \frac{33,000}{29,000,000},$$

which is the unit strain or the elongation for 1 in. of length. Now, the bar in question is 100 in long. Therefore, the total elongation is

$$100 \times \frac{33,000}{29,000,000} = .1138 \text{ in.   Ans.}$$

EXAMPLE 2.—A steel rod of 2 square inches cross-section is loaded with 58,000 pounds. How long will the rod have to be to stretch $\frac{11}{100}$ inch?

SOLUTION —Here  $E = 29,000,000$   and  $s = 58,000 \div 2 = 29,000$. Therefore,
$$q = \frac{29,000}{29,000,000} = \tfrac{1}{1000},$$

which is the stretch for 1 in. of the rod. Now, the length of the rod to stretch $\frac{11}{100}$ in. is as many inches long as $\frac{1}{1000}$ will go into $\frac{11}{100}$, or

$$\tfrac{11}{100} - \tfrac{1}{1000} = 110 \text{ in.   Ans.}$$

---

### EXAMPLES FOR PRACTICE

1. A structural-steel bar is stressed with a unit stress of 18,000 pounds per square inch   The modulus of elasticity of the material is 29,000,000. What is the unit strain produced?        Ans   000621 in.

2. The modulus of elasticity of white oak is 1,500,000 pounds. An oak post 10 in. × 10 in. is 11 feet long, and it carries a load of 400,000 pounds. How much will it be compressed?        Ans. .352 in.

3. An annealed-copper wire has a cross-section of $\frac{1}{6}$ square inch. It is 250 feet long, carries a load of 1,000 pounds, and stretches 1 inch. What is its modulus of elasticity?        Ans. 15,000,000

4. The modulus of elasticity of cast iron is 12,000,000. What is the unit stress required to compress a column that is 10 feet long, $\frac{1}{10}$ inch?        Ans. 10,000 lb. per sq. in.

# STRENGTH OF BUILDING MATERIALS

## TESTING OF MATERIALS

**13.** The **ultimate strength** of any material is that unit stress which is just sufficient to break it.

**14.** The **ultimate elongation** is the total elongation produced in a unit of length of the material by a unit stress equal to the ultimate strength of the material.

**15.** The most important duty of the testing engineer of today is to determine what the ultimate strength of various materials may be. The ultimate elongation, while interesting, is not so useful. By knowing the ultimate strength, it is possible to tell what load will cause a structure to break, or fail. To this end, various materials have been tested in testing machines by compression, tension, or the like, and the stress developed when the material broke, as measured by the instrument, has been observed. By repeating this experiment a great many times, it was possible to obtain average values for the ultimate strengths. These values, however, are only approximate, because in an actual building there are many considerations that affect the ultimate strength of the members in some slight degree, as for example, the temperature, the time taken to apply the load, etc. Nevertheless, testing engineers have obtained quite accurate values for the ultimate strength of building materials that serve for the design of a building.

**16.** The strengths of these different materials will be given in various Sections, but in order to make the terms clear, the average stresses, in pounds per square inch, for medium structural steel are given here. They are as follows:

| | |
|---|---:|
| Ultimate compression . . . . . . . . . | 64,000 |
| Ultimate tension . . . . . . . . . | 64,000 |
| Elastic limit . . . . . . . . . . . | 33,000 |
| Ultimate shearing . . . . . . . . . | 50,000 |
| Modulus of rupture . . . . . . . | 60,000 |
| Modulus of elasticity . . . . . . . | 29,000,000 |

Here, the ultimate compression is given as 64,000 pounds per square inch. This means that under such a unit pressure, a block made of medium structural steel would crush.

As will be noted, the ultimate tensile stress in this case has the same value; that is, a rod 1 square inch in cross-section would break if a weight of 64,000 pounds were hung on one end of it.

The elastic limit is given as 33,000 pounds per square inch. This means that if medium structural steel is stressed so as not to exceed this amount, the steel will not receive a permanent set, but will spring back to its normal position after the load is removed. However, if this material is stressed beyond 33,000 pounds per square inch, it will not entirely resume its original form, but will be distorted, the amount of distortion depending on how far the elastic limit is exceeded.

Following the elastic limit is given the ultimate shearing stress, which is the unit shearing stress that would cause the material to shear.

Following the ultimate shearing stress is given the modulus of rupture. This is a value, not the ultimate tensile or compression stress, but a quantity something like it, that is used in calculating the strength of beams. Its use will be discussed in *Theory of Beams*.

Following the modulus of rupture is given the modulus of elasticity, which is the ratio of unit stress to unit strain for any value of unit stress below the elastic limit.

These, then, are the values that experimenters have found and that are used in structural design. Different experimenters may find values that vary slightly from the ones given, as no two men can get exactly the same results. However, as structural steel is manufactured in quite a

uniform manner, the results obtained by different investigators should not differ so very much, and the values here given will be found to be about the average values obtainable.

EXAMPLE 1 —What pull will be required to break a medium structural-steel rod that is 2 inches in diameter?

SOLUTION —The area of the rod is equal to the area of a circle 2 in. in diameter, or $2^2 \times .7854 = 3.14$ sq in. The ultimate tensile, or breaking, strength of steel is 64,000 lb. per sq. in. Therefore, the ultimate strength of the rod in question is

$$3.14 \times 64,000 = 200,960 \text{ lb. Ans.}$$

EXAMPLE 2 —A rod of medium structural steel is 5 feet long and 1 square inch in cross-section At one end it carries a weight of 30,000 pounds. How much will it stretch?

SOLUTION.—Applying the formula of Art. **12,** $E = 29,000,000$ and $s = 30,000$. Therefore,

$$29,000,000 = \frac{30,000}{q}, \text{ or } q = \frac{30,000}{29,000,000},$$

which is the unit strain. Since the rod is 60 in. long, each inch of its length will elongate this much, or the total elongation will be

$$q \times 60 = \frac{60 \times 30,000}{29,000,000} = .062 \text{ in. Ans.}$$

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### EXAMPLES FOR PRACTICE

1. A medium-steel rod is 11 feet 8 inches long. What will be its total elongation when stressed to the elastic limit?        Ans. .1593 in.

2. A bar of medium structural steel is 4 in $\times \frac{1}{2}$ in. in cross-section. How many pounds will it take to shear it?        Ans. 100,000 lb.

3   A medium structural-steel column is 27 feet 6 inches high.   It compresses under a load an amount equal to .047 inch.   The area of the column section is 30 square inches.   What load does the column carry?        Ans. 123,909 lb.

4. A medium structural-steel tension member has an area of 13.5 square inches   It carries a load of 200,000 pounds and is 30 feet long   What total elongation is produced?        Ans. .1839 in.

5. What must be the area of the cross-section of a rod made of medium structural steel to just fail under a shear of 200,000 pounds?        Ans. 4 sq. in.

## FACTOR OF SAFETY

**17.** A value that is taken for the ultimate strength of any material is an average value of a number of experiments made on the material. As it is impossible to get two samples of the same material exactly alike, so is it also impossible to get two samples with the same ultimate strength. A certain sample may exhibit much more or much less strength than the average sample would. Then, again, when a material is stressed beyond the elastic limit the deformation is permanent. This is a feature to be avoided in a building if possible. It is therefore customary in design to avoid stressing a material up to its ultimate strength or even up to its elastic limit.

**18.** The **factor of safety,** or, as it is sometimes called, the **safety factor,** is the ratio of the ultimate strength of the material to the load that, under usual conditions, the material is called on to carry. Suppose that the load required to rupture a piece of steel is 5,000 pounds, and that the load it is called on to carry is 1,000 pounds, then the factor of safety may be obtained by dividing the 5,000 pounds by the 1,000 pounds; thus, $5,000 \div 1,000 = 5$, which is the factor of safety of this material.

The safety factor required for a structure depends on conditions and on the materials used; in other words, it is the *factor of ignorance.* When a piece of steel, wood, or cast iron is used in a building, the engineer does not know the exact strength of that particular piece of steel, wood, or cast iron. From his own experience and that of others he knows the approximate tensile strength of structural steel to be 64,000 pounds per square inch, and that it varies in different specimens more or less from this value. In regard to timber, the uncertainty is much greater, because of knots, shakes, and interior rot, which are not always evident on the surface but which affect the strength. Cast iron is even more unreliable, on account of the almost indeterminable blowholes, flaws, and imperfections in the castings.

**19. Deterioration.**—Another factor to be considered in selecting materials is deterioration, which is due to various causes. In metals, there is corrosion on account of moisture and gases in the atmosphere. This is especially noticeable in the steel trusses over railroad sheds, where the sulphur fumes from the stacks of the locomotives unite with the moisture in the air, forming free sulphuric acid, which attacks the steel vigorously and demands that the steelwork be painted frequently, in order to prevent its entire destruction.

Wood is subject to decay from either dry or wet rot, caused by local conditions; it may, like iron and steel, be subjected to *fatigue*, produced by constant stress due to the load it may have to sustain.

Cast iron does not deteriorate to any great extent, its corrosion not being so rapid as that of steel or wrought iron. There are, however, internal strains produced in cast iron by the irregular cooling of the metal in the mold. Castings under the slightest blow will sometimes, owing to these internal stresses, snap and break in a number of places.

**20. Factors of Safety in Common Use.**—The preceding reasons are, in truth, sufficiently cogent to require the factors now adopted by conservative constructors in all engineering work. These factors are 3 to 4 for structural steel and wrought iron, and 6 to 10 for cast iron.

Stone and brick are very unreliable and a high factor of safety should therefore be used. Usually, the factors employed are 10 for compressive stresses, 15 for tensile stresses, and from 10 to 20 for bending stresses. Some engineers, however, do not use such high factors. Stonework or brickwork is even more unreliable than stone or brick themselves, but the same factors are often employed. Usually, however, the strengths of stonework and brickwork, instead of being given as ultimate strength, are arranged in tables to give the *safe*, or *allowable*, *stress* per square inch or per square foot. This value embodies its own factor of

safety and is the stress that the material is intended to withstand.

Concrete is another material that sometimes employs a high factor of safety. In reinforced-concrete work a factor varying from 4 to 6 is usually employed, although some engineers use a higher one.

**21.** Ordinances in various cities throughout the country govern the allowable load that building materials should carry. Thus, the engineer is often limited by these laws as much as by any other conditions in selecting the unit stresses to be developed in the structure.

**22.** In regard to the factor of safety to be used for different kinds of stresses in timber, Table I gives values recommended by the Association of Railway Superintendents of Bridges and Buildings. All the factors of safety given in this table are for timbers used in trestles, bridges, and buildings.

### TABLE I

**FACTOR OF SAFETY FOR STRESSES IN TIMBER**

| Kind of Stress | Factor of Safety |
| --- | --- |
| Tension . . . . . . . . . . . . . . . | 10 |
| Compression with grain . . . . . . . . | 5 |
| Compression across grain . . . . . . . | 4 |
| Bending . . . . . . . . . . . . . . | 6 |
| Shearing . . . . . . . . . . . . . . . | 4 |

**23.** A structure withstanding shocks or containing rapidly vibrating machinery is more liable to fail than one in which the loads are quiet, even if the latter has the greater loads.

It is impossible to design accurately beams, columns, etc. to withstand these shocks, because it is usually impossible to calculate the effect of the shocks and what stresses they will produce. Also, when metal beams or columns are subjected to shocks, these shocks have a tendency to make the material crystallize and therefore withstand a less unit

stress.    Therefore, in designing a structure to withstand vibrating loads or shocks, the designer, to make the structure safe, uses a larger factor of safety.    Just how large this factor should be is entirely a matter of experience and also depends on the exact conditions of each case.    Usually, this factor averages from two to three times the factors already given, except those for timber, which need not as a rule be increased.

EXAMPLE —If the breaking strength of a cast-iron column is 200,000 pounds, what safe load will the column sustain if a factor of safety of 6 is used?

SOLUTION.—   200,000 ÷ 6 = 33,333 lb.   Ans.

---

### EXAMPLES FOR PRACTICE

1.    Provided a factor of safety of 3 is adopted, what will be the safe working strength of a medium structural-steel tension rod 2 inches in diameter?                                                    Ans. 67,021 lb.

2.    A certain beam will carry a total load of 1,000 pounds uniformly distributed with a factor of safety of 6.    What load will it take to break the beam?                                        Ans. 6,000 lb.

3.    A timber stick has an ultimate shearing value of 12,000 pounds. Using the factor given in Table I, what safe shear may be allowed?
                                                    Ans. 3,000 lb.

4.    A beam will break under a total load of 12,000 pounds    If a factor of safety of 6 is used, what safe load will it carry?
                                                    Ans. 2,000 lb.

# THEORY OF BEAMS

## QUANTITIES AFFECTING THE STRENGTH OF BEAMS

### PROPERTIES OF SECTIONS

#### INTRODUCTION

**1.** The strength and the stiffness of a beam depend on various factors. For instance, the load that a beam can bear depends on the material of the beam, on the manner in which the load is applied, and on the length and cross-section of the beam. As to the area of the cross-section, the strength does not depend on that area itself, because, as every-day experience shows, a plank used as a beam will sustain a greater load when placed edgewise than when placed on its broad side, although it has the same area in both cases. It will be shown later that the strength of a beam depends on the manner in which the area is distributed or disposed with respect to a certain line, called the *neutral axis*, of the cross-section. The effect of the cross-section is measured by a quantity that depends on such disposition or distribution of area, and is called the *moment of inertia* of the cross-section with respect to the neutral axis. It should be understood at the outset that the moment of inertia of a plane figure has really nothing to do with the property of matter known as inertia. The moment of inertia as here used is simply the name for a certain constant depending on the shape of the

cross-section of a beam. It is not a moment as the word is generally understood, and it might have been called anything so far as the structural engineer is concerned, but it has received its name from some relations in higher mathematics where it is derived. This quantity must be considered simply as a constant that has been found suitable for use in certain formulas for the design of beams, which will be explained later.

<div align="center">MOMENT OF INERTIA</div>

**2. Definitions.**—Let $B\ C$, Fig. 1, be any plane area, and $X'\ X$ a line, or axis, in its plane. Let the area be divided into
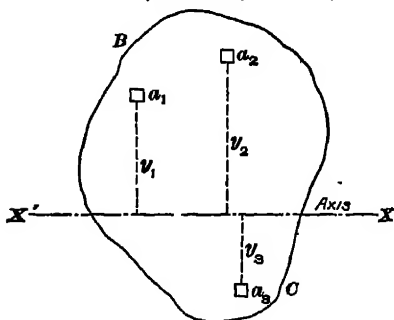


FIG 1

small areas $a_1, a_2, a_3$, etc., distant $y_1, y_2, y_3$, etc. from $X'\ X$. If each small area is multiplied by the square of its distance from the axis, a sum of the form $a_1 y_1^2 + a_2 y_2^2 + a_3 y_3^2 \ldots$ will be obtained. When the whole area is thus divided, giving to the small areas definite values, a certain value is found for the sum of all the products $a y^2$. If, now, the small areas are subdivided into smaller areas, a different value will be found for the sum of all the products $a y^2$. If the small areas are again subdivided, a new value will be found for the sum of all the products $a y^2$. As the areas are made smaller and smaller, the values of the sum of all the products $a y^2$ approach nearer and nearer a fixed value, depending on the form of the figure $B\ C$ and on the location of the axis $X'\ X$. This fixed value is called the **moment of inertia** of the area $B\ C$ with respect to the axis $X'\ X$.

**3. Computing the Moment of Inertia.**—In order to illustrate the method of computing the moment of inertia of a beam section, the relation of the I section shown in Fig. 2 to the axis $de$ through its center of gravity will be con-

sidered. Thus, divide the section into a number of little squares (in this case, each with an area of 1 square inch) and consider the distance from the center of gravity of each square to the axis as the distance of each square from the
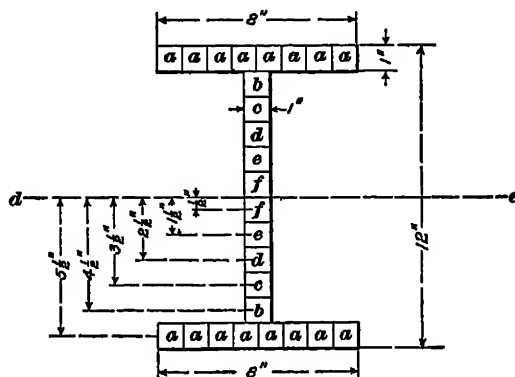


Fig. 2

axis. The products of the area of each square, multiplied by the square of its distance from the axis, are as follows:

Squares $a$, $1 \times (5\frac{1}{2})^2 = \frac{121}{4}$
Squares $b$, $1 \times (4\frac{1}{2})^2 = \frac{81}{4}$
Squares $c$, $1 \times (3\frac{1}{2})^2 = \frac{49}{4}$
Squares $d$, $1 \times (2\frac{1}{2})^2 = \frac{25}{4}$
Squares $e$, $1 \times (1\frac{1}{2})^2 = \frac{9}{4}$
Squares $f$, $1 \times (\frac{1}{2})^2 = \frac{1}{4}$

The sum of the products of each of the small areas multiplied by the square of its distance from the axis is as follows:

$$
\begin{array}{llll}
16 \text{ squares } a, & \frac{121}{4} \times 16 = & 4\,8\,4 \\
2 \text{ squares } b, & \frac{81}{4} \times 2 = & 4\,0\frac{1}{2} \\
2 \text{ squares } c, & \frac{49}{4} \times 2 = & 2\,4\frac{1}{2} \\
2 \text{ squares } d, & \frac{25}{4} \times 2 = & 1\,2\frac{1}{2} \\
2 \text{ squares } e, & \frac{9}{4} \times 2 = & 4\frac{1}{2} \\
2 \text{ squares } f, & \frac{1}{4} \times 2 = & \frac{1}{2} \\
& \text{Total,} & 5\,6\,6\frac{1}{2}
\end{array}
$$

This result, however, is only a rough approximation to the moment of inertia, owing to the fact that the assumed areas

are so large.   The actual value of the moment of inertia of the section is $568\frac{2}{3}$.

**4.   Moment of Inertia of Common Areas.**—Table I, at the end of this Section, contains, in the column marked *I*, the moments of inertia of the shapes of beam sections commonly used in practice.   The axis in each case passes through the center of gravity, and is designated *neutral axis* in the table for reasons that will be explained later.   The square may be regarded as a particular case of a rectangle whose base *b* and altitude *d* are equal.   The value of the area *A* is given in the second column.   The distance *c*, given in the third column, is the distance of the most remote part of the figure from the neutral axis.   The section modulus and radius of gyration, the character of which will be explained in a subsequent article, are given in the last two columns.

EXAMPLE —What is the area, the distance from the neutral axis to the extremities of the section, and the moment of inertia about the neutral axis of the section illustrated in Fig. 3?
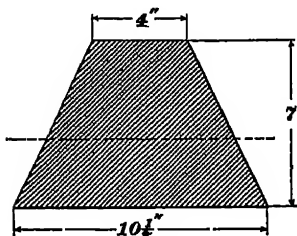


FIG. 3

SOLUTION.—Referring to Table I, it will be seen that $b_1 = 4$ in., $b = 10\frac{1}{2}$ in., and $d = 7$ in.   Therefore, the area is

$$\frac{b + b_1}{2} \times d = \frac{10\ 5 + 4}{2} \times 7$$

$$= 50.75 \text{ sq in.}$$

Likewise, the distance from the neutral axis to the longer parallel side is

$$c_1 = \frac{b + 2\,b_1}{b + b_1} \times \frac{d}{3} = \frac{10.5 + 2 \times 4}{10.5 + 4} \times \frac{7}{3} = 2.977 \text{ in.}$$

The distance *c*, which is the distance of the neutral axis from the shorter parallel side, is

$$c = \frac{b_1 + 2\,b}{b + b_1} \times \frac{d}{3} = \frac{4 + 2 \times 10\ 5}{10.5 + 4} \times \frac{7}{3} = 4\ 023 \text{ in.}$$

As a check, $c_1 + c$ should equal *d*, or 7 in.; thus, $2.977 + 4.023 = 7.000$ in , which indicates that the preceding solution is correct.

The moment of inertia is found by the formula

$$I = \frac{b^2 + 4\,b\,b_1 + b_1{}^2}{36(b + b_1)} \times d^4$$

Substituting values for the letters in this formula,

$$I = \frac{10.5^3 + 4 \times 10.5 \times 4 + 4^3}{36 \times (10.5 + 4)} \times 7^3 = 193.348. \quad \text{Ans.}$$

In general, if one of the values of $c$ and $c_1$ is found by means of the formula, the other may be found by subtracting the known one from $d$. Thus, after having calculated $c_1$ in this example, $c$ would be equal to $d - c_1 = 7 - 2.977 = 4.023$ in., which value corresponds to the one found by means of the formula.

### EXAMPLES FOR PRACTICE

1. What is the moment of inertia with respect to its neutral axis of a circular beam section 8 inches in diameter?                    Ans. 200.7

2. What is the moment of inertia with respect to its neutral axis of the beam section shown in Fig. 4?
                    Ans. 11
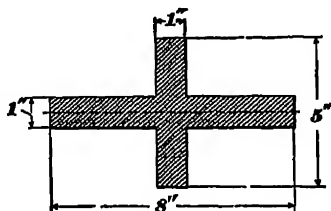
3. What is the moment of inertia with respect to its neutral axis of the beam section shown in Fig. 5?
                    Ans. 200.08



FIG 4
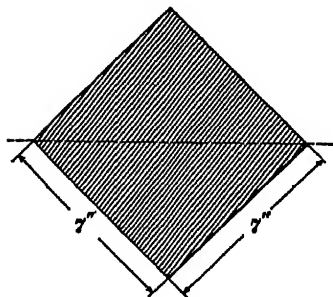


FIG. 5
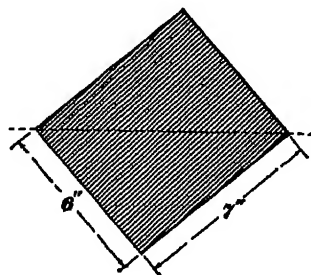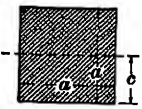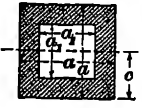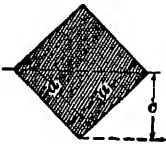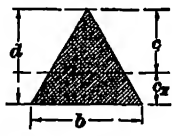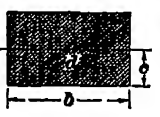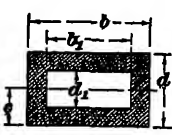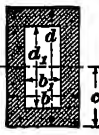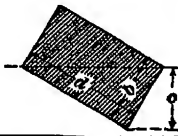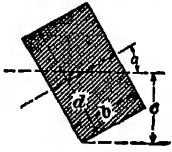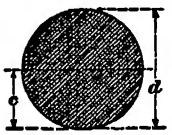


FIG. 6

(4) What is the moment of inertia with respect to its neutral axis of the beam section shown in Fig. 6?                    Ans. 145.27

### 5. Moment of Inertia of Structural-Steel Shapes.

Many of the beams used in building construction are made of steel. These steel beams are rolled with various cross-

sectional shapes, from which they derive their names. These shapes are *standard;* that is, they are rolled to conform to certain sizes that have been adopted by many of the large steel companies. In the first column of Table II, at the end of this Section, are shown the cross-sections of the various structural shapes. The first section shown is known as an *angle with equal legs;* the second, as an *angle with unequal legs;* the third, as a *channel;* the fourth, as a *bulb beam;* while the fifth, sixth, and seventh sections are known respectively as an I *beam*, a T *bar*, and a Z *bar*.
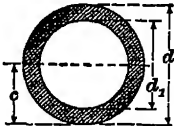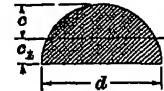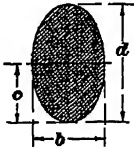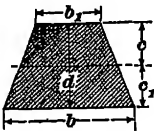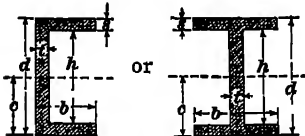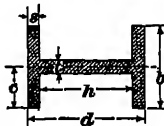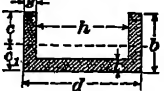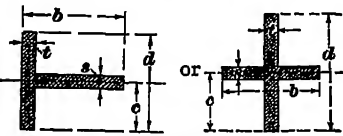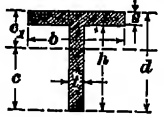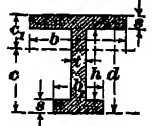
**6.** In this table is given the moment of inertia for each section about at least two axes, both of which pass through the center of gravity of the section; that is, the moments of inertia are given always with respect to neutral axes. The moments of inertia with respect to different neutral axes are, in general, different, and as a rule there is one neutral axis about which the moment of inertia is less than it is about any other. With the channel, bulb beam, I beam, and T bar, the smallest moment of inertia is about the axis $y'-y'$, or the vertical axis. However, with both styles of angles and with the Z bar, there is another moment of inertia about a neutral axis, not horizontal or vertical, that is smaller than the moment of inertia about any other neutral axis; that is, in certain work, more particularly in the design of columns, it is necessary to know about what neutral axis the moment of inertia will be smallest. It is also necessary to know what this moment of inertia will be. Therefore, in the section of the angle with equal legs, the section of the angle with unequal legs, and the Z-bar section, the moment of inertia is also given for the oblique axis giving the smallest moment of inertia. The position of this neutral axis is found by higher mathematics. With the angle with equal legs, this oblique neutral axis $y'-y'$ makes an angle of 45° with the horizontal. With the angle section with unequal legs and with the Z-bar section, this oblique neutral axis $y''-y''$ makes an angle $\alpha$ with a vertical line. Instead of giving this angle direct, it is more convenient to give a value for the trigono-

| Section | Area of Section $A$ | Distance From Neutral $A$ to Extremities of Section $c$ and $c_1$ |
|---|---|---|
|  | $a^2$ | $c = \dfrac{a}{2}$ |
|  | $a^2 - a_1^2$ | $c = \dfrac{a}{2}$ |
|  | $a^2$ | $c = \dfrac{a}{\sqrt{2}} = .707\,a$ |
|  | $\dfrac{b\,d}{2}$ | $c = \dfrac{2\,d}{3} \qquad c = d -$ <br> $c_1 = \dfrac{d}{3} \qquad c_1 = d -$ |
|  | $b\,d$ | $c = \dfrac{d}{2}$ |
|  | $b\,d - b_1\,d_1$ | $c = \dfrac{d}{2}$ |
|  | $b\,d$ | $c = \dfrac{b\,d}{\sqrt{b^2 + d^2}}$ |
|  | $b\,d$ | $c = \dfrac{d\cos a + b\sin}{2}$ |
|  | $\dfrac{\pi\,d^2}{4} = .785\,d^2$ | $c = \dfrac{d}{2}$ |

| Moment of Inertia With Respect to Neutral Axis $I$ | Section Modulus $S = \dfrac{I}{c}$ | Radius of Gyration $r = \sqrt{\dfrac{I}{A}}$ |
|:---:|:---:|:---:|
| $\dfrac{a^4}{12}$ | $\dfrac{a^3}{6}$ | $\dfrac{a}{\sqrt{12}} = .289\,a$ |
| $\dfrac{a^4 - a_1^4}{12}$ | $\dfrac{a^4 - a_1^4}{6\,a}$ | $\sqrt{\dfrac{a^2 + a_1^2}{12}}$ |
| $\dfrac{a^4}{12}$ | $\dfrac{a^3}{6\,\sqrt{2}} = .118\,a^3$ | $\dfrac{a}{\sqrt{12}} = .289\,a$ |
| $\dfrac{b\,d^3}{36}$ | $\dfrac{b\,d^2}{24}$ | $\dfrac{d}{\sqrt{18}} = .236\,d$ |
| $\dfrac{b\,d^3}{12}$ | $\dfrac{b\,d^2}{6}$ | $\dfrac{d}{\sqrt{12}} = .289\,d$ |
| $\dfrac{b\,d^3 - b_1\,d_1^3}{12}$ | $\dfrac{b\,d^3 - b_1\,d_1^3}{6\,d}$ | $\sqrt{\dfrac{b\,d^3 - b_1\,d_1^3}{12\,(b\,d - b_1\,d_1)}}$ |
| $\dfrac{b^3\,d^3}{6\,(b^2 + d^2)}$ | $\dfrac{b^2\,d^2}{6\,\sqrt{b^2 + d^2}}$ | $\dfrac{b\,d}{\sqrt{6\,(b^2 + d^2)}}$ |
| $\dfrac{b\,d}{12}(d^2 \cos^2 a + b^2 \sin^2 a)$ | $\dfrac{b\,d}{6}\left(\dfrac{d^2 \cos^2 a + b^2 \sin^2 a}{d \cos a + b \sin a}\right)$ | $\sqrt{\dfrac{d^2 \cos^2 a + b^2 \sin^2 a}{12}}$ |
| $\dfrac{\pi\,d^4}{64} = .049\,d^4$ | $\dfrac{\pi\,d^3}{32} = .098\,d^3$ | $\dfrac{d}{4}$ |

| Section | Area of Section $A$ | Distance From Neutral Axis to Extremities of Section $c$ and $c_1$ |
|---|---|---|
| | $\dfrac{\pi(d^2 - d_1^2)}{4}$ $= .785(d^2 - d_1^2)$ | $c = \dfrac{d}{2}$ |
| | $\dfrac{\pi d^2}{8} = .393\, d^2$ | $c_1 = \dfrac{2\,d}{3\,\pi} = .212\, d$ $c = \dfrac{(3\,\pi - 4)d}{6\,\pi} = .288\, d$ |
| | $\dfrac{\pi\, b\, d}{4} = .785\, b\, d$ | $c = \dfrac{d}{2}$ |
| | $\dfrac{b + b_1}{2} \times d$ | $c_1 = \dfrac{b + 2\,b_1}{b + b_1} \times \dfrac{d}{3}$ $c = \dfrac{b_1 + 2\,b}{b + b_1} \times \dfrac{d}{3}$ |
| | $b\, d - h(b - t)$ | $c = \dfrac{d}{2}$ |
| | $b\, d - h(b - t)$ | $c - \dfrac{b}{2}$ |
| | $b\, d - h(b - t)$ | $c_1 = \dfrac{2\,b^2\,s + h\,t^2}{2\,A}$ $c = b - c_1$ |
| | $t\, d + s(b - t)$ | $c = \dfrac{d}{2}$ |
| | $b\, s + h\, t$ | $c_1 = \dfrac{d^2\,t + s^2(b - t)}{2\,A}$ $c = d - c_1$ |
| | $b\, s + h\, t + b_1\, s$ | $c_1 = \dfrac{t\, d^2 + s^2(b - t) + s(b_1 - t)(2\,d}{2\,A}$ $c = d - c_1$ |

| ment of Inertia With spect to Neutral Axis $I$ | Section Modulus $S = \dfrac{I}{c}$ | Radius of Gyration $r = \sqrt{\dfrac{I}{A}}$ |
|---|---|---|
| $\dfrac{\pi(d^4 - d_1^4)}{64}$ $= .049(d^4 - d_1^4)$ | $\dfrac{\pi}{32}\left(\dfrac{d^4 - d_1^4}{d}\right) = .098\left(\dfrac{d^4 - d_1^4}{d}\right)$ | $\dfrac{\sqrt{d^2 + d_1^2}}{4}$ |
| $\dfrac{{}^2 - 64}{152\,\pi}d^4 = .007\,d^4$ | $\dfrac{9\,\pi^2 - 64}{192(3\,\pi - 4)}d^3 = .024\,d^3$ | $\dfrac{\sqrt{9\,\pi^2 - 64}}{12\,\pi}d = .132\,d$ |
| $\dfrac{\tau\,b\,d^3}{64} = .049\,b\,d^3$ | $\dfrac{\pi\,b\,d^2}{32} = .098\,b\,d^2$ | $\dfrac{d}{4}$ |
| $\dfrac{+ 4\,?\,o_1 + b_1^2}{36(b + b_1)} \times d^3$ | $\dfrac{b^2 + 4\,b\,b_1 + b_1^2}{12(b_1 + 2\,b)} \times d^2$ | $\dfrac{d}{6(b + b_1)}\sqrt{2(b^2 + 4\,b\,b_1 + b_1^2)}$ |
| $\dfrac{b\,d^3 - h^3(b - t)}{12}$ | $\dfrac{b\,d^3 - h^3(b - t)}{6\,d}$ | $\sqrt{\dfrac{b\,d^3 - h^3(b - t)}{12[b\,d - h(b - t)]}}$ |
| $\dfrac{2\,s\,b^3 + h\,t^3}{12}$ | $\dfrac{2\,s\,b^3 + h\,t^3}{6\,b}$ | $\sqrt{\dfrac{2\,s\,b^3 + h\,t^3}{12[b\,d - h(b - t)]}}$ |
| $\dfrac{s\,b^3 + h\,t^3}{3} - A\,c_1^2$ | $\dfrac{I}{b - c_1}$ | $\sqrt{\dfrac{I}{A}}$ |
| $\dfrac{t\,d^3 + s^3(b - t)}{12}$ | $\dfrac{t\,d^3 + s^3(b - t)}{6\,d}$ | $\sqrt{\dfrac{t\,d^3 + s^3(b - t)}{12[t\,d + s(b - t)]}}$ |
| $\dfrac{b\,c_1^3 - (b - t)(c_1 - s)^3}{3}$ | $\dfrac{I}{d - c_1}$ | $\sqrt{\dfrac{t\,c^3 + b\,c_1^3 - (b - t)(c_1 - s)^3}{3(b\,s + h\,t)}}$ |
| $\dfrac{+ b_1\,c^3 - (b - t)(c_1 - s)^3}{3}$ $- \dfrac{(b_1 - t)(c - s)^3}{3}$ | $\dfrac{I}{d - c_1}$ | $\left[\dfrac{b\,c_1^3 + b_1\,c^3 - (b - t)(c_1 - s)^3}{3(b\,s + h\,t + b_1\,s)} - \dfrac{(b_1 - t)(c - s)^3}{3(b\,s + h\,t + b_1\,s)}\right]^{\frac{1}{2}}$ |

metric tangent of twice the angle, and this is done in the last column of the table.

**7.** The formulas given in Table II are long and difficult to use. The values of the properties of various structural-steel sections have therefore been calculated for all the standard sizes of angles, channels, etc. that are ordinarily manufactured. The table containing these values will be found at the end of this Section. Their use will be explained more fully later on.

**8. Reduction Formula.**—In Tables I and II the moment of inertia is taken about the neutral axis. Sometimes, however, it is desirable to find the moment of inertia of the section about some axis other than the neutral axis. This may be accomplished as follows:

Let $A$ = area of any section;

$I$ = moment of inertia with respect to an axis through the center of gravity of the section;

$I_x$ = moment of inertia with respect to any other axis parallel to the former;

$p$ = distance between axes.

Then, $I_x = I + A p^2$

EXAMPLE 1.—Determine the moment of inertia of a triangle with respect to its base (see fourth section, Table I).

SOLUTION.—The distance between the base and the neutral axis $c_1$ is $\frac{1}{3} d$, the area is $\frac{1}{2} b d$, and $I = \frac{1}{36} b d^2$. Then, by the preceding formula,

$$I_x = \frac{1}{36} b d^2 + \frac{1}{2} b d \times (\frac{1}{3} d)^2 = \frac{b d^2}{36} + \frac{b d^2}{18} = \frac{b d^2}{12}. \quad \text{Ans.}$$

EXAMPLE 2.—Find the moment of inertia of a rectangle about its side $b$ (see fifth section, Table I).

SOLUTION.—From Table I, the moment of inertia $I$ about a neutral axis parallel to $b$ is $\frac{1}{12} b d^2$. Also, $A = b d$, and $p = c = \frac{d}{2}$. Therefore,

$$I_x = \frac{b d^2}{12} + b d \left(\frac{d}{2}\right)^2 = \frac{b d^2}{12} + \frac{b d^2}{4} = \frac{b d^2}{3}. \quad \text{Ans.}$$

**EXAMPLES FOR PRACTICE**

Find the moment of inertia of a square about an axis through ᴏrner and parallel to the diagonal that does not pass through ᴏrner (see third section, Table I). Ans. $I_x = \frac{7}{12} a^4$

Find the moment of inertia of a circle about a tangent.

Ans. $I_x = \frac{5}{64} \pi d^4 = .245 d^4$

---

**Least Moment of Inertia.**—*The moment of inertia section with respect to a neutral axis is less than with ·t to any other line parallel to that axis,* for, according ɪe principle, or formula, of the preceding article, the



FIG. 7

moment of the section with respect to the parallel line is equal to the moment of inertia with respect to the neutral axis plus a positive quantity.
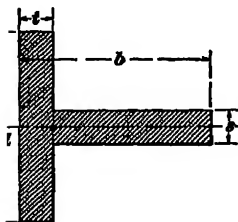
**10. Moment of Inertia of Compound Sections or Areas.**—Many sections may be regarded as built up of ler parts; they are called **compound sections.** For ɪple, a hollow square consists of a large square and a ler square; the **T** shown in Table I consists of two ler rectangles, one horizontal and one vertical.

ɪe *moment of inertia of a compound section with respect to ɪxis may be found by adding, algebraically, the moments of ia, with respect to the same axis, of the component parts ɪ figure.*

ᴀᴍᴘʟᴇ 1.—Determine the value of $I$ given for the hollow square, ɪ second section in Table I.

ʟᴜᴛɪᴏɴ.—The section may be regarded as the difference between ɪrge outside square and the small inside square. In Table I, the of the squares are, respectively, $a$ and $a_1$. According to the first ɪn, given in Table I, the moments of inertia of the squares are and $\frac{1}{12} a_1^4$; hence, the moment of inertia of the hollow square is

$$I = \frac{1}{12} a^4 - \frac{1}{12} a_1^4 = \frac{1}{12}(a^4 - a_1^4). \text{ Ans.}$$

EXAMPLE 2.—Determine the value of $I$ given for the T in Table I.

SOLUTION —The section may be regarded as consisting of two rectangles, as shown in Fig. 7. The axis passes through the center of gravity of each rectangle, and is parallel to the base of each. Hence, according to Table I, the moment of inertia of the vertical rectangle is $\frac{1}{12} t d^3$, and the moment of inertia of the other rectangle is $\frac{1}{12}(b - t)s^3$. The moment of inertia of the entire figure is the sum of these; that is,

$$I = \tfrac{1}{12} t d^3 + \tfrac{1}{12}(b - t)s^3 = \frac{t d^3 + (b - t)s^3}{12}. \quad \text{Ans.}$$

EXAMPLE 3.—Find the moment of inertia of the Z bars shown in Fig. 8, about the neutral axis $X' X$, the dimensions being as shown.

SOLUTION.—The figure may be divided into the three rectangles $efgh$, $e'f'g'h'$, and $jgig'$. The moment of inertia of $efgh$ about an axis through its center of gravity and parallel to $X'X$ is $\frac{1}{12} a' t^3$; that of $e'f'g'h'$ about an axis through its center of gravity and parallel to $X'X$ is also $\frac{1}{12} a' t^3$. The distance between this axis and the axis $X'X$ is $\frac{1}{2}(b - t)$. The moment of inertia of the rectangle $efgh$ and also of the rectangle $e'f'g'h'$ about the axis $X'X$ is



FIG. 8

then, $\frac{1}{12} a' t^3 + a' t \left[\frac{1}{2}(b - t)\right]^2$. The moment of inertia of the rectangle $jgig'$ about the axis $X'X$ is $\frac{1}{12} t b^3$. The moment of inertia of the entire figure is, therefore,

$$2\left\{\tfrac{1}{12} a' t^3 + a' t \left[\tfrac{1}{2}(b - t)\right]^2\right\} + \tfrac{1}{12} t b^3$$

Expanding and reducing this expression,

$$I = \frac{a b^3 - a'(b - 2t)^3}{12}. \quad \text{Ans}$$

By comparing this Z section with the C section in Table I, it will be evident that the disposition of the material on both sides of the neutral axis is alike; hence, their moments of inertia should also be equal. This is found to be the case when the letters on the section in Table I are replaced by those in Fig. 8.
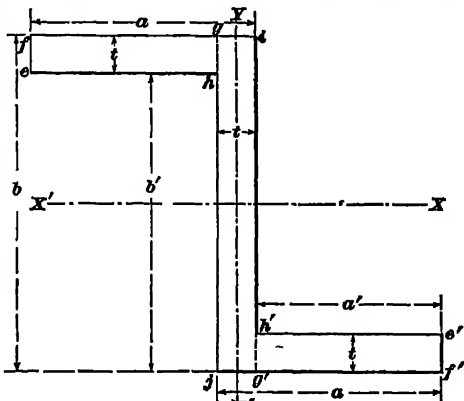
an expression for the moment of inertia $I_y$ of the Z bar shown
8, about the neutral axis $Y'Y$.

$$\text{Ans. } I_y = \frac{t}{12} \left[ (a + a')^3 + b' t^2 \right]$$

---

**Units Used for Moment of Inertia.**—As will be
·om the formulas in Table I, the value of the moment
rtia always involves the product of four quantities
or all of which may be identical) representing lengths.
.he moment of inertia of a rectangle involves the prod-
·, or $b \times d \times d \times d$. When the value of a moment of
is given, it is necessary to specify the unit of length
iince such expressions as $b\,d^3$, $d^4$, etc. evidently have
nt values, according to whether $b$ and $d$ are expressed
ies, feet, meters, etc. Generally, the dimensions of
is for which moments of inertia are computed are
in inches. As the moment of inertia represents the
t of four separate dimensions, this product will, of
ity, give the unit dimension in the fourth power; that
will be *biquadratic*. If the dimensions are given in
some writers state the moment of inertia as so many
*ratic inches*, for which they use the term *inches*⁴. Thus,
he dimensions of a section are to be expressed in inches
ie moment of inertia is 54, these writers call this
it 54 biquadratic inches, and write it *54 in⁴*. Another
£ expressing the same thing is to state the moment
*referred to the inch*. However, unless stated to the
·y, the value given for the moment of inertia is referred
inch and not to the foot or meter.

---

**Definition.**—The **radius of gyration** of a section
:spect to an axis is a quantity whose square multiplied
area of the section is equal to the moment of inertia
section with respect to the same axis. If $r$ and $I$ denote,

respectively, the radius of gyration and the moment of inertia of a section, and $A$ the area in square inches, then,

$$A r^2 = I, \qquad (1)$$

whence
$$r = \sqrt{\frac{I}{A}} \qquad (2)$$

The last column of Table I gives radii of gyration corresponding to the moments of inertia given in the fourth column.

The name radius of gyration, like the name moment of inertia, is derived from relations in higher mathematics. It is simply the name for a certain quantity, that may be defined as in formula $2$. The radius of gyration is used more for the design of columns than anything else, but is occasionally used in the design of beams.

**13. Computation of Radius of Gyration.**—The radius of gyration of a section or figure may be found directly from its moment of inertia by means of formula $2$ of the preceding article. For example, the radii of gyration for the rectangle and the hollow square in Table I are found as follows:

For the rectangle,

$$r = \sqrt{\tfrac{1}{12} \, b \, d^3 \div b \, d} = \frac{d}{\sqrt{12}} = .289 \, d$$

For the hollow square,

$$r = \sqrt{\tfrac{1}{12}(a^4 - a_1^4) \div (a^2 - a_1^2)} = \sqrt{\frac{a^2 + a_1^2}{12}}$$

**14. Reduction Formula.**—Dividing both sides of the formula of Art. $8$ by $A$,

$$\frac{I_x}{A} = \frac{I}{A} + p^2$$

Now, $\dfrac{I}{A}$ is the square of the radius of gyration of the section with respect to a neutral axis, and $\dfrac{I_x}{A}$ is the square of the radius of gyration with respect to an axis parallel to and distant $p$ from that axis. Denoting these radii by $r$ and $r_x$, respectively, the preceding equation becomes

$$r_x^2 = r^2 + p^2$$

That is, *the square of the radius of gyration of a section with respect to any axis equals the square of the radius of gyration of the section with respect to a parallel neutral axis plus the square of the distance between the two axes.*

EXAMPLE.—What is the radius of gyration of a square with respect to an axis coinciding with a side?

SOLUTION.—Call the desired radius of gyration $r_x$. From Table I, $r^2 = \frac{a^2}{12}$; also, $p = \frac{a}{2}$. Then, by the preceding formula,

$$r_x{}^2 = \frac{a^2}{12} + \frac{a^2}{4} = \frac{a^2}{3}, \text{ and } r_x = \sqrt{\frac{a^2}{3}} = \frac{a}{3}\sqrt{3}. \quad \text{Ans.}$$

## BENDING STRESSES

### BENDING AND RESISTING MOMENTS

**15. Bending Moment.**—If, in a cantilever loaded as shown in Fig. 9, any point $x$ on the center line $a\,b$ be taken as a center of moments, and a section made by a vertical plane $c\,d$ through this center be considered, it will be evident
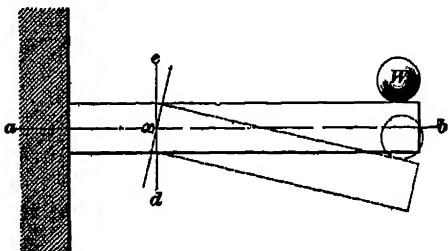


FIG. 9

that the moment of the force due to the downward thrust of the load tends to turn the end of the beam to the right of $c\,d$, around the center $x$. The measure of this tendency is the product of the weight $W$ multiplied by its distance from $c\,d$, and, since this tendency is the moment of a force that tends to bend the beam, it is called the **bending moment**. The methods of finding the bending moments at various sections of beams with different loading is explained in *Forces Acting on Beams*.

**16. Resisting Moment.**—A further inspection of Fig. 9 will show that if the end of the beam turns around the

center $x$ until it takes the position shown by the dotted lines, the parts of the two surfaces formed by the cutting plane $c\,d$, which are above the center $x$, must be pulled away from each other, while those below are pushed closer together. Thus, if a vertical section through any point on the center line $a\,b$, between the load and the point of support, is considered, it will be seen that the tendency of the load is to separate the particles, or fibers, in this section above the center line and to push those below the center line closer together; in other words, through the bending action of the load, the upper part of the beam is subjected to a tensile stress, while the lower is subjected to a compressive stress.

**17.** Fig. 10, which illustrates a cantilever beam that has been broken by a load, as shown, will perhaps make this matter more clear. As will be noticed, the fibers at the top of the beam have been pulled apart while those in the lower part of the section have been
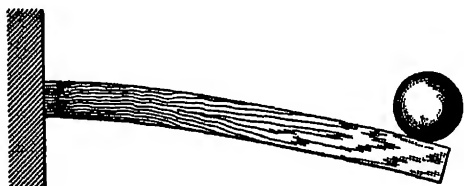


Fig. 10

pressed together. In other words, the fibers at the top of the beam have been subjected to tension while those at the bottom have been subjected to compression.

This action is reversed when the beam is a simple one, instead of a cantilever, in which case the fibers at the bottom of the beam are pulled apart while those at the top are crushed.

**18.** Returning to Fig. 9, it also shows that the greater the distance of the particles in the assumed section above or below the center $x$, the greater will be their displacement. In other words, since the stress in a loaded body is directly proportional to the strain, or relative displacement of the particles, it follows that the stress in a particle of any section is proportional to its distance from the center line $a\,b$, and that the greatest stress is in the particles composing the upper and lower surfaces of the beam.

In accordance with the conditions of equilibrium, the alge-braic sum of the moments of all the forces tending to pro-duce rotation around a given center must be zero. It has been shown that the weight of the load is a force that tends to produce right-hand rotation around the center $x$; therefore, if the beam does not break under the action of the load, there must be forces acting whose moments, with respect to the center $x$, balance the moment of the load. These forces are the resistances with which the particles of the beam oppose any effort to change their relative position. The tensile stresses in the particles above the center $x$ and the com-pressive stresses in those below it constitute a set of forces that resists the tendency of the load to turn the end of the beam, and, when the effect of the load is just balanced by the effect of this set of forces, it is evident that the sum of the moments of these resisting stresses is equal to the moment of the load. The sum of the moments of the stresses of all the particles, or fibers, composing any section of a beam is called the **resisting moment, or moment of resistance,** of that section.

### NEUTRAL AXIS

**19.** The neutral axis of a section of a beam has been defined in *Statics* as a line passing through the center of gravity of a section. This definition is correct in part, but
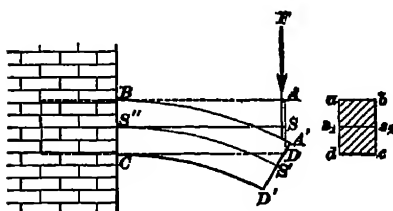


FIG. 11

the term neutral axis re-quires a further explana-tion.

In Fig. 11, let $A B C D$ represent a cantilever. A force $F$ acts on it at its extremity $A$. This force will tend to bend the beam into something like the shape shown by $A'B C D'$. It is evident from what has preceded that the effect of the force $F$ in bending the beam is to lengthen the upper fibers and to shorten the lower ones. Hence, the upper part $A'B$ is now

longer than it was before the force was applied; that is, $A'B$ is longer than $AB$. It is also evident that $D'C$ is shorter than $DC$. Further consideration will show that there must be a fiber, as $SS''$, that is neither lengthened nor shortened when the beam is bent; that is, $SS'' = S'S''$. A line drawn through this fiber $SS''$ when the beam is straight is called the *neutral line*, and the horizontal plane in which this line lies is known as the *neutral surface* of the beam. The neutral line corresponds to the center line $ab$, Fig. 9, on which the center of moments $x$ was taken.

**20.** The relations between the effect of a load and the resulting stresses in a beam have been thoroughly proved, both by mathematical investigations and by numerous experiments. The results of these experiments on beams may be briefly expressed by the following:

**Experimental Law.**—*When a beam is bent, the horizontal elongation (or compression) of any fiber is directly proportional to its distance from the neutral surface, and, since the strains are directly proportional to the horizontal stresses in each fiber, the stresses are also directly proportional to their distances from the neutral surface, provided the elastic limit is not exceeded.*

The line $s_1 s_2$, which passes through any beam section, as $abcd$, Fig. 11, at the neutral line, perpendicular to the direction in which the beam bends, is called the **neutral axis.** It is shown in works on mechanics that *the neutral axis always passes through the center of gravity of the cross-section of a beam made of uniform material.*

**21.** Thus, in a beam, it is evident that the neutral axis of any section is a horizontal line at which the fibers composing the beam are neither stretched nor compressed. The axis is horizontal because it is at right angles to the direction of the load, which, in structural problems, usually acts downwards. It so happens that in a beam made of one material, this line will pass through the center of gravity of the section. It is for this reason that the neutral axis has been spoken of as a line through the center of gravity of a section. It happens, however, that in many beams made

of two materials, as, for example, concrete and steel, the neutral axis—that is, the horizontal line along which the particles of the beam are neither stretched nor compressed—does not pass through the center of gravity of the section. For this reason, the preceding definition is limited in its scope and should be no longer used; the neutral axis should be defined as the horizontal line in any section of a beam along which the fibers are neither lengthened nor shortened when a beam is bent.

### SECTION MODULUS

**22.** The modulus of a section of a beam is equal to the moment of inertia of the section about its neutral axis divided by the distance from the outermost fiber in that section to the neutral axis. This fiber may be either above or below the neutral axis, and it is immaterial whether the beam is a simple beam, a cantilever, or any other kind of beam. If the moment of inertia of a section about its neutral axis is represented by $I$, the distance from the neutral axis to the outermost fiber by $c$, and the section modulus by $S$, then

$$S = \frac{I}{c}$$

EXAMPLE.—Prove the formula for the modulus of a section as shown in Fig. 12 to be correct as given in Table I.
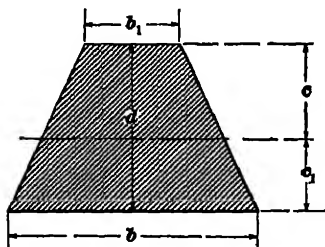


FIG 12

SOLUTION.—According to Table I, the moment of inertia of the section is $\dfrac{b^2 + 4\,b\,b_1 + b_1{}^2}{36(b + b_1)} \times d^2$. It is a question whether $c$ or $c_1$ is the longer. However, on examining the values given for these two quantities in the table, it is seen that they are the same, with the exception of the first part of the numerator. For $c_1$, this part of the numerator is $b + 2\,b_1$, while for $c$ it is $b_1 + 2\,b$. In Fig. 12, $b$ is greater than $b_1$, therefore, $b_1 + 2\,b$ is greater than $b + 2\,b_1$, and it follows that $c$ is greater than $c_1$. Accordingly, the formula is

$$S = \frac{I}{c} = \frac{b^3 + 4\,b\,b_1 + b_1^3}{36\,(b + b_1)}\, d^3 \div \frac{b^3 + 2\,b}{b + b_1} \times \frac{d}{3}$$

$$= \frac{(b^3 + 4\,b\,b_1 + b_1^3)\, d^3 (b + b_1)\,3}{36\,(b + b_1)\,(b_1 + 2\,b)\,d} = \frac{b^3 + 4\,b\,b_1 + b_1^3}{12\,(b_1 + 2\,b)} \times d^3,$$

which is the same as the one given in the table.    Ans.

---

#### EXAMPLES FOR PRACTICE

1.  Find the section modulus of an I beam, as shown in Table II, about axis $y\!-\!y$.  Ans. $\dfrac{b\,d^3}{6} - \dfrac{h^4 - t^4}{4\,d}$

2.  Find the section modulus of a hollow square, using the notation and moment of inertia given in Table I.  Ans. $\dfrac{a^4 - a_1^4}{6\,a}$

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### PROPERTIES OF ROLLED-STEEL SHAPES

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#### EXPLANATION OF TABLES

**23.**  Table II, already mentioned, shows various cross-sections of steel beams.   Each one of these beams is made in various sizes and weights.   The majority of steel manufacturers roll beams of the same size.   On account of the confusion that ensued from using beams of different sizes from different manufacturers, the latter agreed to adopt standard sizes, which are known as *American standard.* Therefore, knowing the dimensions of the section of any beam that is rolled according to the American standard, the properties of the section may be found by using Table II. Table II, however, while quite accurate, is inconvenient to use, because the formulas it contains are long and complicated. Then, also, to use Table II for American standard sections, it is necessary to know the dimensions of the sections, which would require another table giving these dimensions.   The steel manufacturers, therefore, have each published a handbook.   But even standards vary somewhat at different mills. Moreover, some shapes are not uniformly standardized.   The engineer must therefore use *the handbook of the mill that supplies the steel.*   Tables III to XIII of shapes of the Cambria Steel

Company give results uniform for the examples in the text. The beams and channels conform to the American standard of 1896; the angles conform to the standard of 1895; the weight of angles, **Z** bars and **T** bars are the standards of 1902.

**24.**    Table III gives the properties of standard I beams. The bevel on the inside of the flange is always made 1 to 6;
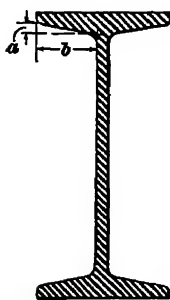


that is, as in Fig. 13, $\frac{a}{b} = \frac{1}{6}$.    This same bevel is also used for channels.    The moment of inertia is given about two axes; namely, axis *1–1* and axis *2–2*.    These axes pass through the center of gravity of the section. Thus, the beam may be used standing on one flange or lying on edge, and the moment of inertia about the neutral axis (which is, of course, always the horizontal axis through the center of gravity) will in either case be known.

Fɪɢ 18

Column 1, Table III, gives the depth of the beam.    As will be seen, standard beams are rolled all the way from 3 to 24 inches deep.    Some manufacturers, however, make special beams as deep as 30 inches.    The weights of the beams, in pounds per foot, is given in column 2.    It will be noted that several beams have the same depth, but different weights.    This is caused by differences in the width of flanges and thickness of webs.    The area of the beam section is given in column 3.    Of course, the area of the section governs the weight per foot.    Column 4 gives the thickness of the web marked *t* on the figure at the head of the table, while column 5 gives the width of the flange marked *b*. Column 6 gives the moment of inertia about axis *1–1*, which is the neutral axis when the beam is standing on one flange. Column 7 gives the section modulus, while column 8 gives the radius of gyration, both about the same axis.    As was previously shown, the section modulus and the radius of gyration can be obtained from the moment of inertia, but with this table these calculations are not required.

To prove that the table is correct up to column 8, take, for example, the case of an 8-inch, 18-pound beam. From the Section on *Loads in Structures*, the weight of steel per cubic foot is 489.6 pounds. The weight of a bar 1 square inch in cross-section and 12 inches long is therefore $\frac{12}{1728} \times 489.6$ $= \frac{489.6}{144} = 3.4$ pounds. This is a very useful value to remember. If the area of a steel beam section, in square inches, is multiplied by it, the product will be the weight of the beam per foot. Thus, the weight of the beam under consideration is $5.33 \times 3.4 = 18.122$ pounds. As the weights are given in the table to within only $\frac{1}{4}$ pound, this result is correct. Taking the moment of inertia of the beam as correct, the section modulus is found by dividing it by one-half the depth; thus, $56.9 \div \frac{8}{2} = 14.225$, which agrees, to one place of decimals, with the table. This radius of gyration may be found as follows: $r = \sqrt{\frac{I}{A}} = \sqrt{\frac{56.9}{5.33}} = \sqrt{10.6754}$ $= 3.27$ inches, which is the same as the value given in the table.

Continuing with the remaining columns in Table III, columns 9 and 10 give the moment of inertia and radius of gyration, respectively, of the section about axis 2–2. These values are seldom used except in brick-arch floor construction and column design.

Column 11 gives the increase of thickness of web required for each pound per foot increase in weight of beam. Thus, a 7-inch beam with a web .25 inch thick weighs, according to the table, 15 pounds. If it is desired to make a 7-inch beam that will weigh 20 pounds, the thickness of web would be $.25 + 5 \times .042 = .46$ inch. By referring to column 4, this value is found to be correct.

Columns 12 and 13 are functions of the deflection, or sag, that beams have when loaded. The use of these columns will be explained later.

**25.** Table IV contains the properties of sections of **standard channels.** The properties given are practically

the same as the ones given for I beams in Table III. In Table IV, however, the section modulus is also given for axis *2–2*, as channels are sometimes used flat for door lintels and the like.

The value $x$ given in column 12 is one that is not needed for I beams. This value represents the distance from an axis through the center of gravity to the back of the channel. This axis is, of course, axis *2–2*. By referring to the illustration at the head of the table, it will be observed that this axis is nearer the back of the channel than it is the tips of the flanges. From the formula of Art. **22**, $S = \dfrac{I}{c}$. Substituting for the values used in this formula those referring to axis *2–2*, $S' = \dfrac{I'}{b - x}$.

Around axis *1–1*, the section modulus is found as explained in the description of Table III. In order to check up $S'$ from Table IV, find, as an example, the value of $S'$ for a 6-inch, 8-pound channel. Thus, $S' = \dfrac{.70}{1.92 - .52} = .50$, which is the value given in the table.

The use of columns 14 and 15 will be explained later.

**26.** Table V contains the properties of steel **angles with equal legs.** Column 1 gives the size of the leg, measured, according to custom, on the outside of the section; column 2, the thickness of metal; column 3, the weight per foot; column 4, the area of the section in square inches; and column 5, the distance from the center of gravity to the back of each leg. Since both legs are equal, axis *1–1* can be drawn either in the direction shown in the figure or at right angles to this direction; that is, a beam made of an angle may have either leg vertical without affecting the strength, provided, of course, both legs are equal. It will also be noted in column 5 that $x$ is not the distance from the neutral axis to the outermost fiber; that is, axis *1–1* is nearer to the back of the angle than it is to the end of one of the legs. Therefore, $S = \dfrac{I}{a - x}$. Checking the value of $S$ for, say, a 2-inch,

2 5-pound angle, $S = \dfrac{.27}{2 - .57} = .1888$, or practically .19, as given in the table.

Axis *2–2* makes an angle of 45° with axis *1–1*. Mathematicians have found that the moment of inertia is smaller



*(a)*

*(b)*

*(c)*

*(d)*
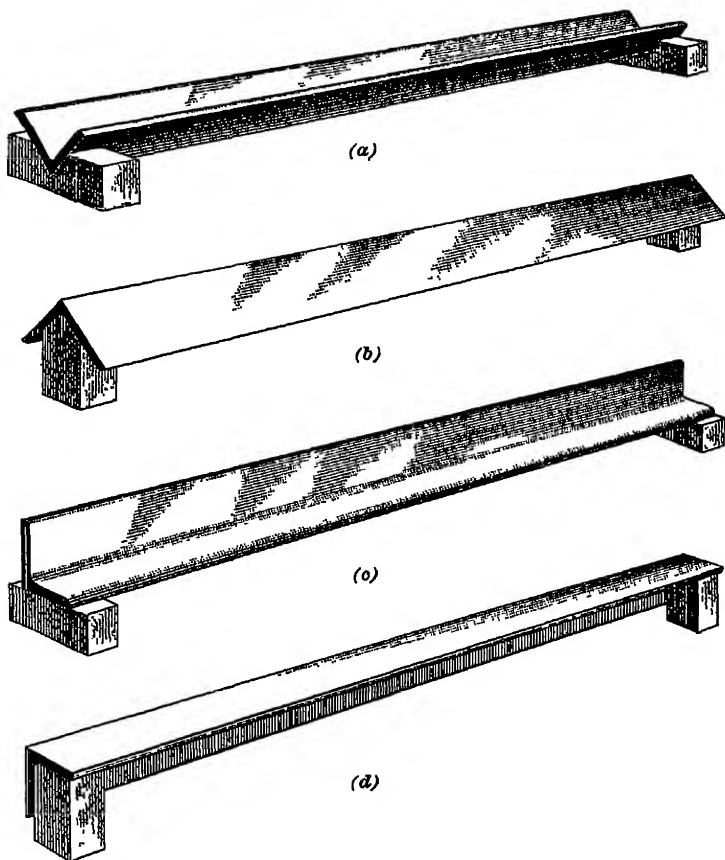
Fig 14

about such a neutral axis than about any other. Thus, in Fig. 14, a beam made of an angle placed as shown at (*a*) or (*b*) is weaker than one made of an angle placed as shown at (*c*) or (*d*). As may be seen from the table, $S' = \dfrac{I'}{x'}$.

The remaining columns in Table V should present no great difficulty.

**27.** Table VI gives the properties of standard steel **angles with unequal legs.** Since the legs are not alike, the moment of inertia will be different, according to whether, when used as a beam, the long or the short leg is vertical. Therefore, the moment of inertia, etc. are given about both axis *1–1* and axis *2–2*. The moment of inertia about axis *3–3*, as can be proved, is the least moment of inertia that the section can have. Axis *3–3* will pass through the center of gravity of the section, and its direction will vary from that of axis *1–1* by the angle *a*, the tangent of which is given in column 13. Column 14 gives the radius of gyration about axis *3–3*. If required, the moment of inertia about this axis may be found by means of formula **2,** Art. **12.** Axis *3–3* is of importance in column design. However, in the design of beams, it is seldom used, because, for practical reasons, the designer usually makes one of the legs horizontal.

**28.** Table VII gives the properties of **Z-bar sections.** This table should be easy to understand without any explanation, as it is arranged along the same lines as those just given. Axis *3–3*, about which the moment of inertia is least, is given. Columns 14 and 15 will be explained later.

**29.** Table VIII gives the properties of **T bars.** It will be noted that the thickness of the T-bar section is not uniform throughout, like the standard angle or **Z** bar, but varies in different places.

**30.** Table IX gives the properties and principal dimensions of *standard railroad rails*, commonly called **T rails** (not to be confused with the **T** bars). In specifying any standard I beam, channel, etc., it is necessary to give both the depth and the weight of the shape, because beams of the same depth may vary in weight. Thus, a certain I beam is known as a 10-inch, 40-pound beam. With **T** rails, however, each weight of rail is of a different size all over; that is, only one weight of rail of a certain depth is rolled. It is custom-

ary, therefore, when specifying rails, simply to specify them by weight. It will also be noted that the weight is given for a yard of rail instead of for a foot. This is also a matter of custom; that is, while the weight of I beams, channels, etc. are always given for 1 foot of length, the weight of T rails is always given for 1 yard of length.

**31.** Table X gives the moment of inertia of rectangular sections of varying depths and widths. This table will be found useful in determining the moment of inertia of steel plates that are used on edge. Thus, to find the moment of inertia about the neutral axis of a plate 18 inches deep and ⅝ inch wide, trace down the first column until the number 18 is reached; then, from this point move horizontally to the right until the column headed ⅝ is reached; here, the moment of inertia about the neutral axis $y$–$y$ is found to be 182.25.

**32.** Tables XI, XII, and XIII give the radii of gyration of two angles bolted, or riveted, back to back. Table XI deals with angles having equal legs; Table XII, with angles having unequal legs and the two longer legs placed back to back; and Table XIII, with angles having unequal legs and the shorter legs placed back to back. The first column in each table gives the back dimensions of the angles, while the second column gives the thickness of the metal. The third column gives the area of the combined section of the two angles. This, of course, is twice the area of one angle, as given in Tables V and VI. Thus, in Table XI, the area of two angles each 2 in. × 2 in. and $\frac{7}{16}$ inch thick is 3 12 square inches; this is twice 1.56 square inches, or the area given for one angle in Table V. The fourth column, headed $r_0$, gives the radius of gyration of the two angles about a horizontal axis through the center of gravity of the section. Of course, the moment of inertia of the two angles about this axis is twice the moment of inertia of one angle. To find the radius of gyration, divide the moment of inertia, by the area and extract the square root from the quotient. Now, the area of two angles is twice the area of one angle; there-fore, the quotient of the moment of inertia divided by the

area will be the same for two angles as it is for one.    There-
fore, the values of $r_o$ in Tables XI, XII, and XIII, for axis *1–1*
are similar to those given in Tables V and VI for similar
angles.

As will be noted, $r_1$, $r_2$, etc. are radii of gyration about
neutral axes $x$–$x$.    Thus, if two angles were used as shown
in Fig. 15   the values of $r_1$, $r_2$, etc., given in Table XI,
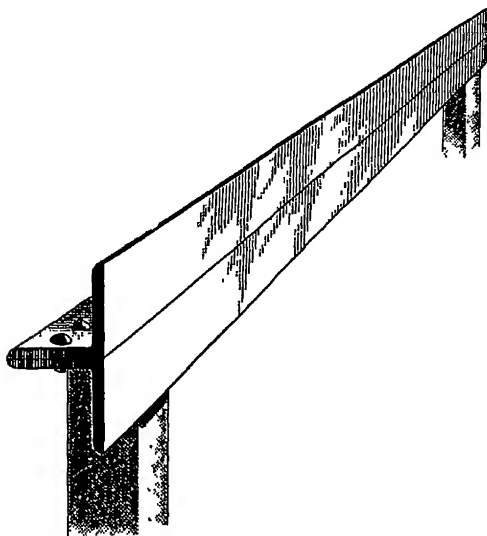


Fig. 15

would represent the radius of gyration of the combined sec-
tion about the neutral axis of the beam section, which is, of
course, the axis of symmetry of the section.    It will be
noted that for the radius $r_1$ the angles are riveted or bolted
together with their backs touching.    For $r_2$, the angles are
held $\frac{1}{4}$ inch apart by small separators made of plate washers,
etc.; for $r_3$, the angles are held still farther apart; and so on.
The values given in Tables XI, XII, and XIII are intended
more for the design of columns than for the design of beams.
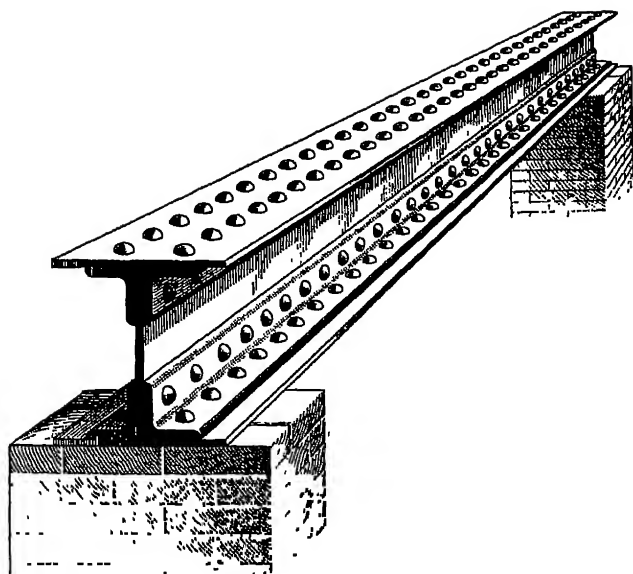
$(a)$
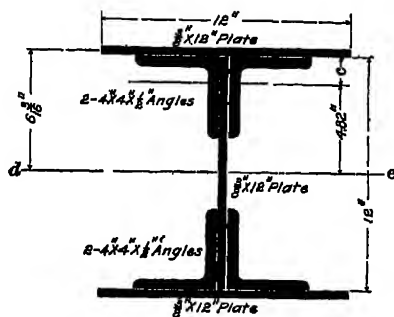


$(b)$

FIG 16

## COMPOUND SECTIONS

**33.** Frequently, steel beams are built up of several steel sections riveted together. To find the strength of a beam of this kind it is first necessary to determine the moment of inertia of the entire section about its neutral axis. This is done by the methods described in Arts. **8** and **10**, but it may probably be best illustrated by an actual problem. Thus, assume that it is desired to find the moment of inertia about the neutral axis of the cross-section of the compound beam shown in Fig. 16 (*a*). The cross-section of the beam is shown in (*b*). It is evident that the neutral axis is the line *de* that divides the figure into equal parts. From Table V it is found that the moment of inertia of a $4'' \times 4'' \times \frac{1}{2}''$ angle about the axis *1–1* through its own center of gravity is 5.56, and that the area of each angle is 3.75 square inches. The distance $c$, Fig. 16 (*b*), is 1.18 inches. The distance from the neutral axis of the angle to the neutral axis *de* of the entire figure is therefore $\frac{12}{2} - 1.18 = 4.82$ inches. The moment of inertia of one angle about *de*, according to the formula of Art. **8**, is $I_x = I + A p^2 = 5.56 + 3.75 \times 4.82^2 = 92.68$. The moment of inertia of the four angles about *de* is therefore $92.68 \times 4 = 370.72$. According to Table I, the moment of inertia of either the top or the bottom plate about its own neutral axis is $\dfrac{b\,d^3}{12} = \dfrac{12 \times (\frac{3}{8})^3}{12} = .05$.

The distance from the neutral axis of the plate itself to the neutral axis of the entire section is $6 + \dfrac{\frac{3}{8}}{2} = 6\frac{3}{16}$. The moment of inertia of the top or the bottom plate about the axis *de* is, according to the formula of Art. **8**, $I_x = I + A p^2 = .05 + 12 \times \frac{3}{8} \times (6\frac{3}{16})^2 = 172.33$. The moment of inertia of both the top and the bottom plate is therefore $2 \times 172.33 = 344.66$. The neutral axis of the central plate, or web plate, by itself is identical with the neutral axis *de* of the entire figure. The moment of inertia of this web plate can be found from Table I, or more easily from Table X, which shows that it is 54. The moment of inertia of the entire

beam section about the neutral axis is therefore 370.72 + 344.66 + 54.00 = 769.38.

EXAMPLE —What is the moment of inertia, with respect to the axis $de$, of the beam section shown in Fig. 17?

SOLUTION —The moment of inertia of one of the cover-plates, with respect to an axis through its center of gravity, parallel to $de$, is $\dfrac{12 \times (\frac{1}{2})^3}{12} = .125$, the area of the plate is $12 \times \frac{1}{2} = 6$ sq. in , and the distance of its center of gravity from $de$ is $5\frac{1}{4}$ in.; therefore, its moment of inertia, with respect to $de$, is $.125 + 6 \times (5\frac{1}{4})^2 = 165.5$. From Table IV, the area of a 10-in., 15-lb. channel is 4.46 sq in., and its moment of inertia,



FIG 17

with respect to an axis through its center of gravity, corresponding in this case with the axis $de$, is 66.9; therefore, the moment of inertia of the whole section, with respect to $de$, is $2 \times 165\ 5 + 2 \times 66.9 = 464.8$. Ans.
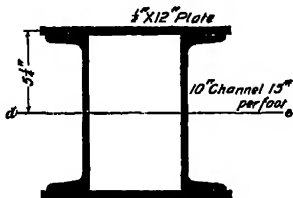
**34.** In each of the examples just given, the neutral axis of the entire figure was known at once, because the figure was symmetrical. Sometimes, however, the sections are not symmetrical and the neutral axis must be found. It is known that the neutral axis of a beam made of the same material throughout is a horizontal line running through the center of gravity of the section. The method of locating the neutral axis and the center of gravity of the section was explained in *Statics*. As an example, consider the beam shown in Fig. 18 (*a*). It is composed of an I beam and a channel riveted together. Such a beam is often used to carry the track for a traveling crane.

The first problem is to locate the neutral axis $c$–$d$, Fig. 18 (*b*). Assume any axis, as $a$–$b$, about which to take ordinary moments of the areas. The moment of each area about $ab$ is equal in each case to the product of the area and the distance from its center of gravity to $ab$. The areas and location of the centers of gravity of the beam and column sections can be obtained from Tables III and IV. Adding these moments, the work is as follows:

Moment of I beam $= 20.59 \times 9$ . . . . $= 185.31$
Moment of channel $= 11.76 \times (18 + .78) = \underline{220\ 85}$

Total . . . . . . . . . . . . . . . . . 406.16

The total area of the section is $20.59 + 11.76 = 32.35$ square inches. Therefore, the distance from the line $a\ b$ to the neutral axis $c\ d$ is $\dfrac{406.16}{32.35} = 12.56$ inches.

It now remains, by the method given in Art. 8, to find the moments of inertia of the channel and the I beam about the



(a)

(b)
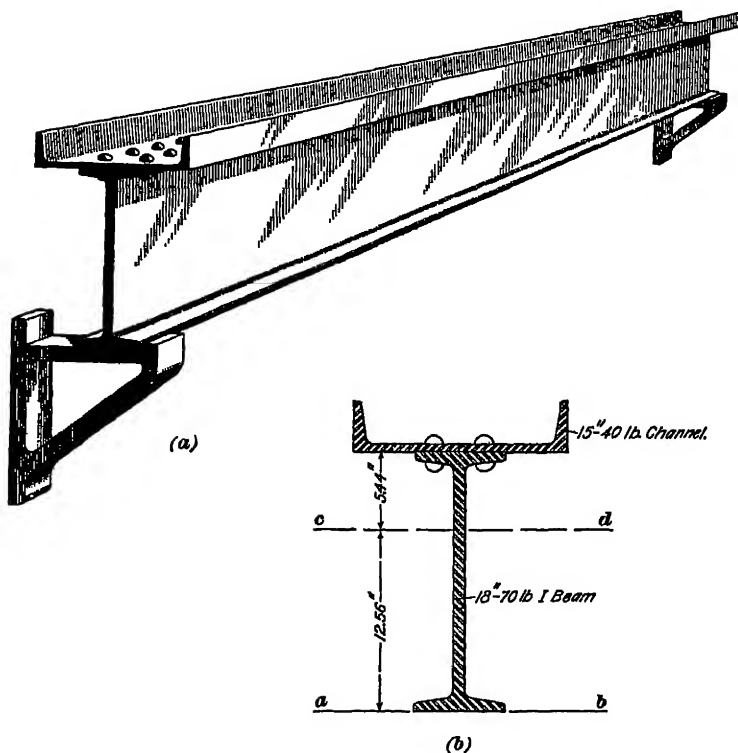
Fig. 18

axis $c$–$d$ and to add them together. This can be done with the help of Tables III and IV as follows: The moment of inertia of the channel about $c\ d$ is $9.39 + 11.76 \times (5.44 + .78)^2$

10ᴸ-35#I Beam

15ᴸ-42#I Beam

10ᴸ-35#I Beam

FIG. 19



18ᴸ-55#I Beam

6"x6"x¾"Angles
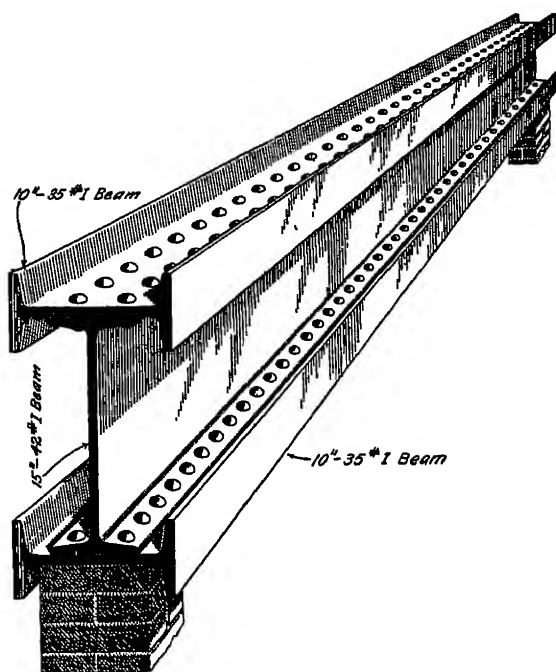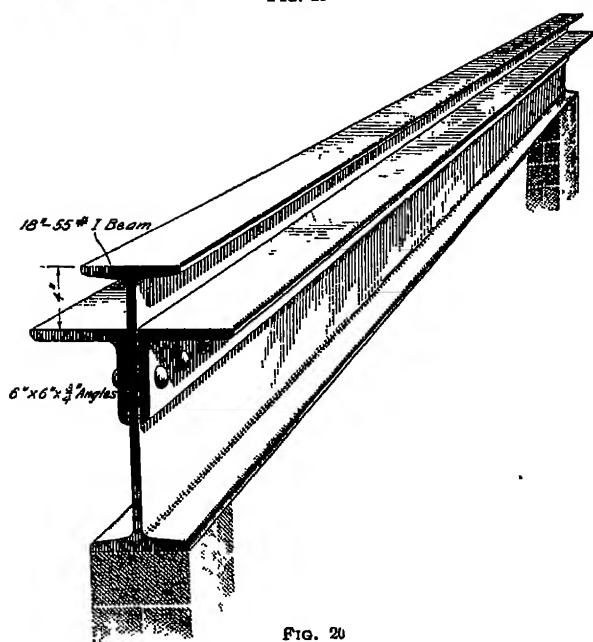
FIG. 20

= 464.36. Likewise, the moment of inertia of the I beam about the axis $cd$ is $921.2 + 20.59 \times (12.56 - 9)^{2} = 1,182.15$; therefore, the total moment of inertia is $464.36 + 1,182.15 = 1,646.51$ inches[4].

### EXAMPLES FOR PRACTICE

1. Find the moment of inertia about the neutral axis of the beam shown in Fig. 19.                    Ans. 1,710.93 in.[4]

2. Find the moment of inertia about the neutral axis of the beam shown in Fig. 20, remembering that it is first necessary to locate the neutral axis.                    Ans. 936.88 in.[4]

# FORMULAS FOR DESIGN

## BENDING-MOMENT FORMULAS

### INTRODUCTION

**35.** In *Forces Acting on Beams*, it was shown that the external forces—that is, the loads and the reactions—caused a bending moment about every section of the beam. This bending moment is known as the *external bending moment*. At every section of the beam, this external bending moment is resisted by the strength of the beam itself. This resistance offered by the fibers of the beam at any section is known as the *resisting moment*. Of course, if the beam does not break, the resisting moment is equal to the external bending moment. The method of obtaining the value of the external bending moment at any section and the method of obtaining its maximum value, commonly called the *maximum bending moment*, are explained in *Forces Acting on Beams*. In the present Section, the methods of calculating the resisting, or internal, moment of a beam will be considered. With a knowledge, then, of the value of the external and resisting moments, which are equal to each other, a beam may be designed to carry any desired load.

**36.**   Fig. 21 shows a beam carrying a load $W$.   The cen-
tral part of the beam is supposed to have been removed so
as to show the cross-section.   The neutral axis is represented
by the line $m\,n$, and the fibers along this line are not stressed.
Above this line, the fibers are subjected to compression, and
below it they are subjected to tension.   As usual, the per-
pendicular distance from the neutral axis $m\,n$ to the outer-
most fiber is denoted by $c$.   The distance $c$ will be measured
either above the neutral axis, as shown, or below it, depend-
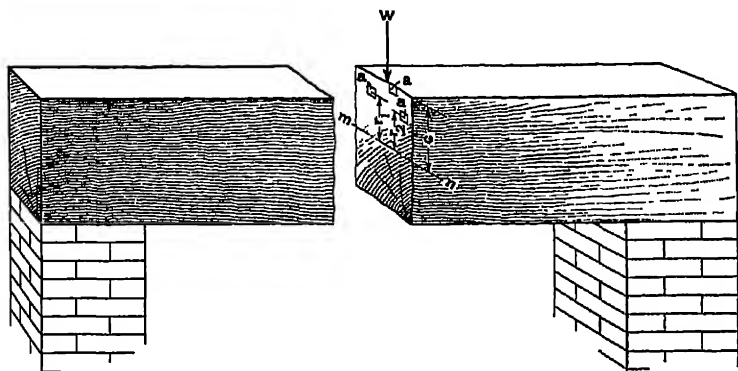


Fig 21

ing on whether the fiber farthest from the neutral axis is at the
top or at the bottom of the section.   In the case at hand, the
section is rectangular and the material is the same through-
out; therefore, the distance from the neutral axis to the top
and that to the bottom of the beam are identical.   However,
the formula about to be given is derived in the same way, no
matter what shape the section may have; but it should
always be borne in mind that $c$ is the distance from the
neutral axis to the outermost fiber in the section.

**37.**   Let the horizontal unit stress (compression if above,
and tension if below) at the distance $c$ from the neutral axis
be denoted by $s$.   If $a$ is the sectional area, in square inches,
of each fiber of the section, the stress on the outermost

fibers will be $a\,s$. Since the stress in each fiber varies as its distance from the neutral axis, as stated in Art. **18**, it follows that the stress in a fiber $a$ at the unit distance (1 inch) from $m\,n$ is $\dfrac{a\,s}{c}$; also, the stress on a fiber at the distance $r_1$ is $\dfrac{a\,s}{c} \times r_1 = \dfrac{a\,s\,r_1}{c}$. The moment of this stress about the axis is $\dfrac{a\,s\,r_1}{c} \times r_1 = \dfrac{a\,s\,r_1^2}{c} = \dfrac{s}{c}a\,r_1^2$. The moment of the stress in the fiber at a distance $r_2$ from the neutral axis $m\,n$ is evidently $\dfrac{s}{c}a\,r_2^2$, and for a distance $r_3$ it is $\dfrac{s}{c}a\,r_3^2$. The sum of the moments of all these little fibers about the neutral axis $m\,n$ is equal to the resisting moment; that is, it is equal to the moment with which the strength of the beam at the section under consideration resists the external bending moment. Let this resisting moment be called $M_1$; then,

$$M_1 = \frac{s}{c}a\,r_1^2 + \frac{s}{c}a\,r_2^2 + \frac{s}{c}a\,r_3^2 + \frac{s}{c}a\,r_4^2,\ \text{etc.}$$

$$= \frac{s}{c}(a\,r_1^2 + a\,r_2^2 + a\,r_3^2 + a\,r_4^2,\ \text{etc.})$$

The quantity shown in parenthesis is the same as described in Art. **2**; that is, it is the moment of inertia of the section about the neutral axis $m\,n$. The equation may then be written

$$M_1 = \frac{s}{c}I$$

The resisting moment must be equal to the bending moment caused by the external forces. If this external moment is $M$, then

$$M_1 = M = \frac{s}{c}I \qquad (1)$$

This is the general equation for beams. It expresses the bending moment about any section, caused by the loads and reactions, in terms of the maximum unit stress at the section and the dimensions of the beam at that section $\left(\text{that is, } \dfrac{I}{c}\right)$.

The stress $s$ may be either tension or compression, but it is always the maximum stress in the section.

It is evident that if a beam is going to break, the break will occur at that section about which the external bending moment is maximum. If this section holds, every other part of the beam will be strong enough. Therefore, it is the section of a beam about which the external bending moment is maximum that is always investigated.

The quantity $\dfrac{I}{c}$ is equal to the section modulus; therefore, formula 1 is often written

$$M = s\,S, \qquad (2)$$

in which $S$ is the section modulus.

**38.** In the tables at the end of this Section, the moments of inertia, etc. are given with the inch as a base, and $s$ is usually measured in pounds per square inch. Therefore, in the preceding formulas $M$ must be given in inch-pounds and not in foot-pounds. This is a point that should always be kept in mind, as it is very easy to make the mistake of giving $S$ in inches and $s$ in pounds per square inch, and then giving the moment of the external forces in foot-pounds. Often, the external bending moment can be calculated direct in inch-pounds, but as a rule the easiest way is to calculate it in foot-pounds and then change it to inch-pounds by multiplying it by 12 just before equating it to

$$\frac{s\,I}{c}, \text{ or } S\,s$$

EXAMPLE 1.—What is the maximum stress in a simple beam 12 feet long and uniformly loaded with 1,000 pounds per foot of length? The beam is rectangular in section, 10 inches broad and 14 inches deep

SOLUTION.—The total load on the beam is $1,000 \times 12 = 12,000$ lb. The maximum bending moment, in foot-pounds, according to *Forces Acting on Beams*, is $\dfrac{W\,l}{8}$. In this case, $W = 12,000$ lb. and $l = 12$ ft. Therefore, the maximum moment, which shall be called $M$, equals $\dfrac{12,000 \times 12}{8} = 18,000$ ft.-lb. To change this to inch-pounds, multiply by 12. Thus, $12 \times 18,000 = 216,000$ in -lb The section modulus of the beam, according to Table I, is $\dfrac{b\,d^2}{6} = \dfrac{10 \times 14 \times 14}{6} = 326.67$. Substituting these values in formula 2, Art 37, $216,000 = s \times 326.67$, and $s = 216,000 \div 326\ 67 = 661.22$ lb per sq. in. Ans

EXAMPLE 2 —An 8-inch, 18-pound I beam has one end firmly fixed in a wall, while its other end projects out of the wall without support for a distance of 7 feet 4 inches. There is a concentrated load on the free end of the beam of 384 pounds. What is the maximum unit stress developed in the beam?

SOLUTION —This beam is a cantilever. The maximum bending moment developed, according to *Forces Acting on Beams*, is $Wl$ If $l$ is taken in inches, then it is equal to $7 \times 12 + 4 = 88$ in. The maximum bending moment is therefore $384 \times 88 = 33,792$ in.-lb. According to Table III, the section modulus of an 8-in , 18-lb. beam is 14.2 Substituting these values in formula 2, Art. **37**, $33,792 = 14.2 \times s$; therefore, $s = 33,792 \div 14.2 = 2,379.72$ lb. per sq. in.

### EXAMPLES FOR PRACTICE

1. A beam firmly fixed at both ends has a span of 11 feet 6 inches. It carries a load of 4,500 pounds concentrated at the middle, and is 10 inches wide and 15 inches deep. What is the maximum unit stress produced?    Ans. 207 lb. per sq in.

2. In the preceding problem, substitute for the beam of rectangular section, a 15-inch, 50-pound channel and find the maximum unit stress produced.    Ans. 1,445 53 lb. per sq. in.

3. A load of 460 pounds is supported from the center of a piece of pipe. The pipe is supported at the ends and is 54 inches long. The outside diameter of the pipe is 4.5 inches, and the inside diameter, 4.026 inches. Find the maximum stress developed in the pipe.
    Ans. 1,935.18 lb. per sq. in.

### MODULUS OF RUPTURE

**39.** Assume that the load on a beam is increased until it breaks. From the loading that causes failure, $M$ can be found, while $S$ can be found from the shape of the beam section. Substituting these values in the formula $M = Ss$, a value for $s$ will be obtained. This value of $s$, which corresponds to the moment that causes the beam to break, is called the **modulus of rupture**, or **ultimate unit bending stress**, of the material. Since the formula $M = Ss$ is strictly correct only for values of $s$ below the elastic limit, the modulus of rupture will not agree exactly with either the ultimate unit tensile or the ultimate unit compressive strength of the material. The modulus of rupture is a

valuable constant, however, because by substituting it in the formula $M = Ss$ the breaking moment of a beam can be ascertained.

The moduli of rupture of various materials are given in Table XV, at the end of this Section. Other moduli of rupture are given in Sections where the material under consideration is discussed.

**40.** In designing a beam, of course the beam is not intended to develop the ultimate unit bending stress, but only a fraction of it, depending on the factor of safety used. For timber, a factor of 6 is usually employed; for wrought iron, one of 4 is sufficient; while for cast iron, from 6 to 10 is employed. For structural steel, the factor of safety generally used is about 4. With structural steel, however, instead of dividing the modulus of rupture by the factor of safety it is customary to use certain approved unit working bending stresses. For light roof construction, this value is often taken as high as 18,000 to 20,000 pounds per square inch. In ordinary building construction, 16,000 pounds is usually employed, while in bridge work, 12,500 pounds per square inch is used. In this Section, unless otherwise stated, it will be understood that the safe unit transverse stress is to be taken at 16,000 pounds per square inch.

EXAMPLE 1.—Design a rectangular white-oak beam to carry a safe load of 2,000 pounds located at the center, the span being 11 feet 5 inches.

SOLUTION.—The maximum external bending moment is $\dfrac{Wl}{4}$ $= \dfrac{2,000 \times 137}{4} = 68,500$ in.-lb. The modulus of rupture for white oak is 7,000, and a factor of safety of 6 will be used. Therefore, $s = 7,000 - 6 = 1,167$ lb. per sq in. Substituting these values in formula **2**, Art **37**, $68,500 = S \times 1,167$, and $S = 68,500 + 1,167 = 58.71$. From Table I, $S = \dfrac{b\,d^2}{6}$, therefore, either the breadth or the depth of the beam may be assumed and the other dimension found. It will also be noted that in the value of $S$ the breadth is involved only as a first power, while the depth is squared. It is therefore evident that to design an economical beam the better plan is to make the beam narrow and as deep as possible. Of course, there are practical

considerations that govern this matter, such as obtaining commercial sizes of material and the like

In the problem at hand, let it be assumed that the beam will be 10 in deep. Then, $d = 10$ in, $S = 58.71 = \dfrac{b\,d^2}{6} = \dfrac{b \times 10^2}{6}$, and $b = \dfrac{6 \times 58.71}{100} = 3.523$ in. The next larger size of commercial timber is 4 in. $\times$ 10 in., and is therefore the size to be used. Ans.

EXAMPLE 2.—A hemlock beam 10 inches broad and 12 inches deep is on a span of 18 feet. What uniformly distributed safe load per foot will it carry?

SOLUTION —In this case, $S = \dfrac{b\,d^2}{6} = \dfrac{10 \times 12 \times 12}{6} = 240$, and, using the values given in Table XV, with a factor of safety of 6, $s = \dfrac{3500}{6}$. Therefore, $M = S s = 240 \times \dfrac{3500}{6} = 140{,}000$ in -lb. According to *Forces Acting on Beams*, $M = \dfrac{W l}{8}$. Thus, $l = 18 \times 12 = 216$ in. Therefore, $140{,}000 = M = \dfrac{W \times 216}{8}$ and $W = \dfrac{8 \times 140{,}000}{216} = \dfrac{140000}{27}$ lb. This is the total uniformly distributed load over the entire 18 ft. of span. The load per foot is therefore

$$\frac{W}{18} = \frac{140{,}000}{27 \times 18} = 288 \text{ lb per ft. Ans.}$$

EXAMPLE 3.—A beam has a span of 20 feet. It carries a load of 8,000 pounds concentrated at the center. What size of I beam will be required to carry this load safely?

SOLUTION.—The bending moment, according to *Forces Acting on Beams*, is $\dfrac{W l}{4} = \dfrac{8{,}000 \times 20}{4} = 40{,}000$ ft.-lb , or $480{,}000$ in -lb. $= S s$; therefore, $s = 16{,}000$, which is the usual working transverse stress for structural steel. Therefore, $480{,}000 = S\ 16{,}000$ and $S = 480{,}000 \div 16{,}000 = 30$. Referring to Table III, it will be seen that a 10-in., 40-lb beam has a section modulus of 31.7 and will therefore fulfil the requirements On further examination, however, it will be observed that a 12 in., $31\frac{1}{2}$-lb. beam has a section modulus of 36, which is more than sufficient. On account of its being deeper, this increased section modulus is obtained with less weight per foot; in fact, by using this beam, $8\frac{1}{2}$ lb per foot is saved. Therefore, a 12-in., $31\frac{1}{2}$-lb. I beam is the proper beam to use. Ans.

**41.** As an example of the subject of beams, the following problem is given: Fig. 22 shows the transverse sectional elevation of a store. It will require two I beams to form the girder *B*. What will be the size of these steel beams?

Before commencing the calculations, draw the outline diagram as shown in Fig. 23. The two supports for the girder
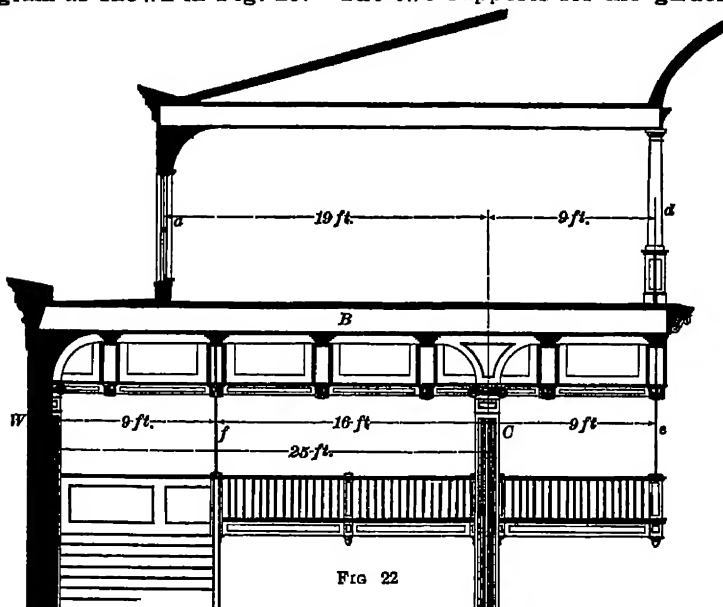


Fɪɢ 22

are the wall $W$ and the column $C$, Fig. 22. The loads on the girder are the two uniform loads $g$ and $h$, Fig. 23. The
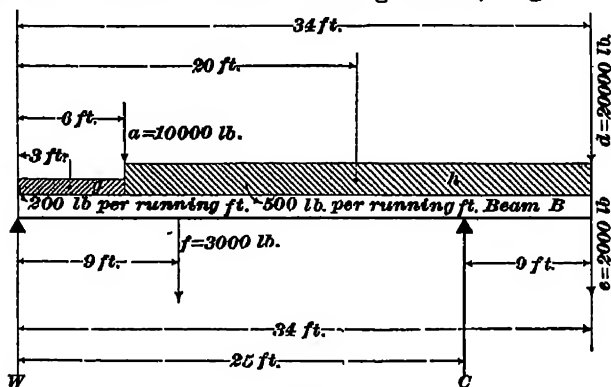


Fɪɢ. 23

load $h$ is due to the weight of the floor, the girder, and the ceiling, together with the live load on the floor, due to the

people, furniture, etc. This load has been assumed to amount to 500 pounds per running foot of the girder. The load $g$, being due only to the ceiling and a portion of the roof, and there being no live load here, and but small snow load, has been considered as amounting to 200 pounds per running foot.

The girder is also loaded with four concentrated loads: $a$ of 10,000 pounds, due to the weight of the light wall and a portion of the roof; $d$ of 20,000 pounds, due to the load coming down the small column from a portion of the roof; and two hanging loads $f$ and $e$, of 3,000 and 2,000 pounds, respectively, the weight of the stair landing or hall.

Now calculate the reactions. The moments about $W$, Fig. 22, due to the various loads, are as follows:

FOOT-POUNDS

| | | |
|---|---|---:|
| Load $g$ ($200 \times 6 = 1{,}200$ lb.) . $1{,}200 \times 3 =$ | | 3 6 0 0 |
| Load $h$ ($500 \times 28 = 14{,}000$ lb.) . $14{,}000 \times 20 =$ | | 2 8 0 0 0 0 |
| Load $a$ . . . . . . . . . . . . $10{,}000 \times 6 =$ | | 6 0 0 0 0 |
| Load $f$ . . . . . . . . . . . . $3{,}000 \times 9 =$ | | 2 7 0 0 0 |
| Load $d$ . . . . . . . . . . . . $20{,}000 \times 34 =$ | | 6 8 0 0 0 0 |
| Load $e$ . . . . . . . . . . . . $2{,}000 \times 34 =$ | | 6 8 0 0 0 |
| Total . . . . . . . . . . . . . . . . . . . | | 1 1 1 8 6 0 0 |

This, divided by the distance between the supports, or the span, 25 feet, gives 44,744, the load, in pounds, coming on the column $C$; or, in other words, the reaction at $C$. The loads, in pounds, are as follows:

$$\begin{array}{rl} \text{Load } g = & 1\,2\,0\,0 \\ \text{Load } h = & 1\,4\,0\,0\,0 \\ \text{Load } a = & 1\,0\,0\,0\,0 \\ \text{Load } f = & 3\,0\,0\,0 \\ \text{Load } d = & 2\,0\,0\,0\,0 \\ \text{Load } e = & 2\,0\,0\,0 \\ \hline \text{Total load} = & 5\,0\,2\,0\,0 \end{array}$$

Then, the reaction at $W$ is $50{,}200 - 44{,}744 = 5{,}456$ pounds.

Next, find the point between the two supports, $W$ and $C$, where the shear changes sign. Starting from $W$, the first

load encountered and to be deducted from the reaction $W$ is the uniform load $g$, equal to $200 \times 6 = 1,200$ pounds. Then, 5,456 (reaction at $W$) − 1,200 (load $g$) = 4,256 pounds. The next load on the beam is the concentrated load $a$ of 10,000 pounds, which is much greater than the remaining portion of the reaction $W$. The greatest bending moment occurring between the supports is therefore at the point $a$, and is equal to 5,456 (reaction at $W$) × 6 = 32,736 foot-pounds, less the moment of the load $g$ about the same point, or 1,200 × 3 = 3,600 foot-pounds. Thus, 32,736 − 3,600 = 29,136 foot-pounds.

Again referring to the diagram, Fig. 23, it will be seen that there is a large bending moment directly over the column $C$, due to the two concentrated loads $d$ and $e$ on the end of the beam and the portion of the uniform load $h$ overhanging the support $C$. It may be seen by calculation that the shear also changes sign at this section; therefore, this point must also be considered. This portion of the beam may be considered as a cantilever; the bending moment at $C$ is equal to the sum of the moments of all the loads on the overhanging portion of the beam, which are:

|  | | FOOT-POUNDS |
|---|---|---|
| Load $d$ . . . . . . . . . . | 20,000 × 9 = | 1 8 0 0 0 0 |
| Load $e$ . . . . . . . . . . | 2,000 × 9 = | 1 8 0 0 0 |
| Load $h$ (overhanging portion = 500 × 9 = 4,500 pounds) | 4,500 × 4.5 = | 2 0 2 5 0 |
| Total . . . . . . . . . . . . . | | . . 2 1 8 2 5 0 |

This sum, in inch-pounds, is 218,250 × 12 = 2,619,000 inch-pounds.

Since this bending moment is greater than that under the load $a$, it is used in determining the size of the beam, as of course the beam must withstand the maximum bending moment. This bending moment divided by 16,000, the safe unit transverse stress for structural steel, gives 163.69, the required section modulus in the two beams. Then, the section modulus required in one of the beams. since two are to be used side by side, is 163.69 ÷ 2 = 81.85.

Referring to Table III, it will be seen that the section modulus of an 18-inch, 55-pound beam is 88.4. While this is in excess of the required amount, it is, in this case, the most economical beam to use, and two of this kind are required.

#### EXAMPLES FOR PRACTICE

1. Design a Georgia-pine beam 6 inches wide to carry safely a uniformly distributed load of 150 pounds per foot over a span of 17 feet.  Ans. 6 in. × 7.47 in., say 6 in. × 8 in.

2. A 10-inch, 40-pound I beam projects 13 feet from a wall. What safe concentrated load will it carry at its end?  Ans 3,251 3 lb.

3. Design an I beam to carry a centrally concentrated load of 100,000 pounds on a span of 4 feet 3½ inches.
Ans. 18-in., 55-lb. I beam

## SHEAR AND DEFLECTION

### SHEAR

**42.** A 10-inch, 25-pound I beam with a unit bending stress of 16,000 pounds will carry on a span of 30 feet a total uniformly distributed load of 8,683 pounds. This value can be proved by calculation. If the span is reduced to 20 feet,
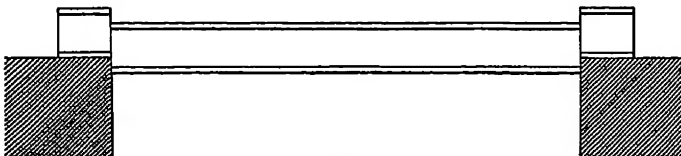


FIG. 24

the safe uniformly distributed load, by calculation, increases to 13,025 pounds. For a 10-foot span, the safe load is 26,049 pounds, and for a 1-foot span, it is 260,490 pounds. From these figures it will be noted that the total safe uniform load increases in the same ratio as the span decreases.

**43.** So far, the bending of a beam has been considered as the cause producing fracture. A beam may, however,

fail by shear. This method of failure is illustrated in Fig. 24. The method of ascertaining the maximum shear on beams has been explained in *Forces Acting on Beams*. From that Section, it will be noted that the value of the shear depends on the total load that the beam carries. From the preceding article, it is evident that a short beam will carry a greater safe load than a long beam. It can therefore be seen that a short beam is much more liable to shear than is a long beam. Taking the actual case given in the preceding article, the maximum shear is equal to one-half the load. The area of the beam section, from Table III, is 7.37 square inches; the maximum total shear in the case considered, with a span of 30 feet, is $8,683 \div 2 = 4,341.5$; and the unit shear is $4,340 \div 7.37 = 589$ pounds (approximately). The safe unit shear of medium steel may be taken at 12,500 pounds per square inch. Thus, in the case under consideration, if the beam is on a 30-foot span, it will fail by bending before it will fail by shear. But, as just stated, the safe load may be increased as the span decreases. This will also increase the unit shear. Thus, for a span of 20 feet, the unit shear is found to be about 884 pounds per square inch. If the span is 10 feet, the unit shear is 1,767 pounds per square inch, and if the span is 1 foot, the unit shear is 17,672. The allowable unit shear is only 12,500. It is therefore clear that the shear on the beam with a 1-foot span is excessive. Of course, a span of only 1 foot is very rare, but is occasionally encountered.

The above demonstration that beams fail by shear requires explanation. As can be proved by mathematics, shear is not distributed uniformly over a section as just assumed. The external shear divided by the section area is the *average shear*, and not the maximum unit shear at the section, which, for rectangular sections, is 50 per cent. greater. In steel beams the total shear is often assumed to be uniformly distributed over the web alone, but steel webs, if not stiffened, may buckle before they shear. There is also at every point in a beam, a horizontal unit shear equal to the vertical unit shear. This is of importance in wooden beams which shear easily with the grain.

### WEIGHT OF BEAMS

**44.** So far it has been assumed that a beam has no weight. Of course, every beam, whether it is made of wood, steel, or stone, has a certain weight, and the question is whether it should be considered or not.

It will be interesting to consider the beam mentioned in Art. **42.** When the span is 30 feet the safe load is 8,683 pounds. Now, the beam weighs 25 pounds per foot, and its total weight is therefore $30 \times 25 = 750$ pounds. This reduces the load that the beam can safely carry outside of its own weight to $8,683 - 750 = 7,933$ pounds, a reduction of over $8\frac{6}{10}$ per cent. Consider, now, the same beam on a span of 1 foot. Here, the total weight of the beam is 25 pounds and, therefore, the total load the beam will carry outside of its own weight is $260,490 - 25 = 260,365$ pounds. This is a reduction of less than $\frac{1}{100}$ of 1 per cent.

It is quite evident that neglecting the weight of the beam itself in beam design does not make so much difference on a short span with heavy loads as it does on a long span with comparatively light loads. Just when the weight of the beam itself should be considered and just when it should not, is a matter of experience, and no set rule can be laid down. Usually, however, if the weight of the beam is less than 5 per cent. of the load it is intended to carry, its weight may be neglected.

**45.** Since, in some cases, it has been decided that the weight of the beam itself must be taken into account, the methods of attaining these results will be considered. It can easily be seen that all the problems relating to beams can be divided into two classes. In one class, the beam is already made and is perhaps in place, and it is desired to know what load the beam will carry. In the other class, a given load is known and it remains to design the beam so as to carry this load.

**46.** As an example of a problem of the first class, let it be supposed that the safe external load is to be calculated

for a hemlock beam that is 10 inches wide and 12 inches deep, the span being 12 feet and the load being concentrated at the center. If it is decided to consider the weight of the beam itself, the problem is still quite simple and may be solved as follows: The safe unit bending stress may be taken from Table XV, using a factor of safety of 6, as 16,000. The section modulus, from Table I, is $\dfrac{b\,d^2}{6} = \dfrac{10 \times 12^2}{6}$ = 240. The resisting moment is $Ss = 240 \times \dfrac{3500}{6} = 140{,}000$ inch-pounds, which is equal to the external bending moment. This external moment is caused by a uniformly distributed load, due to the weight of the beam, and an unknown central concentrated load. The moments caused by both of these loads is maximum about the center of the span of the beam, and the total external moment is therefore equal to the sum of these two moments. The total volume of the beam is $\dfrac{12 \times 10 \times 12}{12 \times 12} = 10$ cubic feet. According to *Loads in Structures*, the weight of a cubic foot of hemlock is 26 pounds. Therefore, the total weight of the beam is $26 \times 10 = 260$ pounds, and the bending moment due to the weight of the beam itself, is $\dfrac{260 \times 12 \times 12}{8} = 4{,}680$ inch-pounds. Subtracting this from the resisting moment, the moment left to resist the unknown concentrated external load is $140{,}000 - 4{,}680 = 135{,}320$ inch-pounds. If this unknown load is called $W$, then from *Forces Acting on Beams*,

$$\dfrac{Wl}{4} = \dfrac{W \times 12 \times 12}{4} = 135{,}320, \text{ or } W = 3{,}759-$$

**47.** In the second case just mentioned, the load is already known and the size of beam is required. As the weight of the beam cannot be obtained until its size is known, and as the size of the beam cannot be found until the total bending moment is known, this problem can be solved only by trial. The following example will serve to illustrate the method to be pursued. It is desired to calculate the size of I beam required to carry, besides its own load, a uniformly

distributed load of 960 pounds per foot over a span of 20 feet. The total load on the beam, exclusive of its own weight, is $960 \times 20 = 19,200$ pounds. From *Forces Acting on Beams*, the maximum bending moment is $\dfrac{Wl}{8} = \dfrac{19,200 \times 20 \times 12}{8}$ $= 576,000$ inch-pounds. From formula **2**, Art. **37**, $M = sS$ $= \dfrac{Wl}{8} = 576,000$. Giving $s$ a value of 16,000 and neglecting the weight of the beam, $16,000 \times S = 576,000$, or $S = 576,000$ $\div 16,000 = 36$. On consulting Table III, it will be seen that the value of $S$ here found corresponds to that of a 12-inch, 31.5-pound I beam. This beam would satisfy the requirements if the weight of the beam itself were left out of consideration, but as it is necessary to provide for this additional load, the next larger size may be chosen and a trial calculation made to see whether it will support the combined load. This beam is a 12-inch one, weighing 35 pounds per foot; hence, the weight of the beam is $35 \times 20 = 700$ pounds. From the preceding formula $\dfrac{Wl}{8}$, the maximum bending moment due to the weight of the beam alone is $\dfrac{700 \times 20 \times 12}{8} = 21,000$ inch-pounds. The sum of this moment and that of the external load is $576,000 + 21,000$ $= 597,000$ inch-pounds $= M$. From formula **2**, Art. **37**, $M = sS$, or $597,000 = 16,000 \times S$; therefore, $S = 597,000$ $\div 16,000 = 37.31$. As the value given for $S$ in Table III is greater than this, the beam selected is of ample strength.

**48.** As was stated, it is usually considered safe to neglect the weight of the beam itself in calculations of beam design. As the amount of load a beam must carry, particularly the live load, is very uncertain at best, the addition of a slight weight due to the weight of the beam itself is seldom considered to be a factor of great importance. When beams carry floors, it is customary, as was explained in *Loads in Structures*, to find the weight of the floor per square foot and then multiply this value by the distance

between beams and by the span to get the total load on the beam. Many engineers assume that the weight of the beams themselves add a certain weight per square foot to the weight of the floor. This added weight is assumed to be 8 pounds for wooden beams and 6 pounds for steel beams. Thus, if all the materials composing a floor, exclusive of beams, were calculated to weigh 12 pounds per square foot, a weight of 20 pounds per square foot would be taken to constitute the total load of a floor supported by wooden beams, while 18 pounds per square foot would be taken for the total load of one supported by steel beams. This method, while not absolutely accurate, is one way of estimating the weight of the beams in a floor. However, as the weight of beams varies a great deal with the span, the load to be carried, etc., this method is only approximate.

### EXAMPLES FOR PRACTICE

1. A 24-inch, 100-pound I beam is on a 32-foot span. What uniform load per foot will it carry, provided allowance is made for its own weight?                                  Ans. 1,966 lb. per ft.

2. Design an I beam on a 21-foot span to carry, besides its own weight, a concentrated load of 408 pounds at the center of span.
                                        Ans. 4-in., 7½-lb. I beam

### DEFLECTION OF BEAMS

**49.** The stresses of tension and compression created in a loaded beam cause elongation and shortening of the fibers above and below the neutral axis at every section, the result of which is a curvature of the beam. The amount of this curvature depends on the amount and distribution of the load, the material of which the beam is composed, its span and manner of support, and the dimensions and form of the cross-section.

**50. Deflection** is the name applied to the distortion or bending produced in a beam when subjected to bending stresses. The measure of the deflection at any point on a beam is the perpendicular displacement of the point from its

original position.   If, on the removal of the bending stresses or loads on the beam, it returns to the straight or original form, the material in the beam has not been stressed beyond the elastic limit.   On the other hand, if the internal stresses exceed the elastic limit of the material, a permanent set will be given the beam.

**51.   Stiffness** is a measure of the ability of a body to resist bending; this property is very different from the strength of the material or its power to resist rupture.

The stiffness of a beam does not depend so much on the elasticity of the material of which it is composed as on its length of span.   This property of stiffness is as important in building construction as mere strength, and the two should be considered together; thus, the floor joists of a building may be strong enough to resist breaking, but so long as to lack stiffness, in which case the floor will be springy and will vibrate from persons walking on it.   If there is a plastered ceiling on the under side of the joists of such a floor, the deflection of the joists may cause the plaster to crack and fall into the room below.   The allowable deflection of a plastered ceiling is usually placed at $\frac{1}{360}$ of the span, or $\frac{1}{30}$ inch for each foot of span.   Where stiffness is lacking in the rafters of a roof, they will be liable to sag, thereby causing unsightly hollows in the surface in which moisture and snow may lodge, which is very detrimental to the roof covering.

**52.**   From the foregoing, it is evident that not only must the strength of the beams composing a structure be calculated to withstand rupture, but the beams must be stiff, or rigid, enough to resist bending beyond a certain limit.   It is therefore important to be able to calculate the deflection of any beam under its load, and if found excessive, the size of the beam may be increased and the deflection reduced to working limits.   While the calculation of the deflection of a beam is important, it will usually be found that, unless the span is very long, the safe unit fiber stress will be exceeded before the deflection is excessive.

The amount of deflection that exists in beams loaded and supported in different ways may be calculated by the formulas given in Table XIV. In using these formulas, all the loads should be expressed in pounds and the lengths in inches. The modulus of elasticity is denoted by $E$, and the moment of inertia of the section by $I$.

EXAMPLE 1.—A 10-inch, 35-pound steel I beam supported at the ends must sustain a uniformly distributed load of 10,000 pounds. The span of the beam is 20 feet, and its moment of inertia is 146 4. There is to be a plastered ceiling on its under side, the allowable deflection of which is $\frac{1}{30}$ inch for each foot of span. Will the deflection of the beam be excessive?

SOLUTION.—The formula for the deflection of a beam of this character, from the table, is $\dfrac{5\ W\,l^3}{384\ E\,I}$, and from *Steel and Other Metals*, the modulus of elasticity of structural steel is 29,000,000. Substituting the values of the example in the formula, the deflection equals

$$\frac{5 \times 10,000 \times 240^3}{384 \times 29,000,000 \times 146.4} = .42, \text{ or about } \tfrac{7}{16} \text{ in.}$$

Since the allowable deflection is $\frac{1}{360}$ of the span, the total allowable deflection is $\frac{1}{360} \times 240 = \frac{2}{3}$ in. This is greater than the calculated deflection, and the beam therefore satisfies the required conditions. Ans.

EXAMPLE 2.—A $12'' \times 16''$ yellow-pine girder must support a symmetrically placed triangular piece of brickwork that weighs about 12,000 pounds. What will be the deflection of the timber if the span is 20 feet and the modulus of elasticity is 1,500,000?

SOLUTION —The formula for the deflection, in this case, from the table, is $\dfrac{W\,l^3}{60\ E\,I}$. The value of the modulus of elasticity is 1,500,000. The moment of inertia of the section, from the formula $I = \dfrac{b\,d^3}{12}$, is $I = \dfrac{12 \times 16^3}{12} = 4,096$. Then, by substituting the given values, the deflection is

$$\frac{12,000 \times 240^3}{60 \times 1,500,000 \times 4,096} = .45, \text{ about } \tfrac{7}{16} \text{ in.} \quad \text{Ans.}$$

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### EXAMPLES FOR PRACTICE

1. The moment of inertia of a 12-inch, 35-pound steel I beam is 228.3, and its span is 25 feet   If the ends of the beam are simply supported, what will be its deflection under a concentrated load of 10,000 pounds suspended from its center?    Ans. .85 in.

2. A $12'' \times 16''$ cantilever beam of yellow pine extends from a building wall 10 feet, and is loaded on the end with a concentrated load of 12,500 pounds. What will be the greatest deflection of the beam?

Ans. 1.17+ in.

3. The span of a 15-inch, 45-pound steel I beam is 30 feet, and the load on the beam is uniformly distributed and amounts to 3,000 pounds per linear foot. If the ends of the beam are firmly fixed, what will be its deflection?     Ans. .83 in.

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**53.   Coefficients of Deflection.**—The values for $N$ and $N'$, the coefficients of deflection for uniform and center loads, respectively, given in Tables III, IV, and VII, at the end of this Section, were obtained from the formulas $N = \dfrac{5\ Wl^3}{384\ EI}$ and $N' = \dfrac{Wl^3}{48\ EI}$, in which $W$ equals 1,000 pounds; $l$, 12 inches; $E$, 29,000,000; and $I$, the moment of inertia about the axis *1–1*. Therefore, these coefficients represent the deflection, in inches, of a beam 1 foot long having a load of 1,000 pounds. Multiplying the proper coefficient by the cube of the span, in feet, and by the number of 1,000-pound units in the given load, will give the deflection of a beam for any load and span.

EXAMPLE 1.—What is the deflection of a 20-inch, 65-pound I beam that carries a center load of 28,000 pounds and has a span of 20 feet?

SOLUTION.—The amount of deflection is obtained by multiplying the coefficient of deflection for beams with center loads (column 13, Table III) by the cube of the span, in feet, and the number of 1,000-lb. units in the load. Hence, the deflection equals

$$.00000106 \times 20^3 \times \tfrac{28000}{1000} = .237 \text{ in. Ans.}$$

EXAMPLE 2.—Two channels, placed back to back with webs vertical, form the support of a column. The load is concentrated at the center of the channels and is equal to 10,000 pounds. If the span of the channels is 8 feet and they are 7 inches in depth and weigh 14 75 pounds per foot, what will be the deflection?

SOLUTION.—The deflection is equal to the coefficient multiplied by the cube of the span and the number of 1,000-lb. units in the load. From column $N'$, Table IV, the coefficient of deflection for a 7-in , 14.75-lb. channel is 0000457. As there are two channels, each may be considered as supporting one-half of the load, or 5,000 lb. Then the deflection will be equal to

$$.0000457 \times 8^3 \times \tfrac{5000}{1000} = .117 \text{ in. Ans.}$$

Two 12-inch, 20½-pound channels, placed back to back, are to be used to support a uniform load of 1,000 pounds per linear foot over a span of 20 feet.   What will be the deflection?          Ans. .487 in.

SUDDENLY APPLIED LOADS

54.  In the formulas and investigations so far discussed, it has been assumed that the loads on the beams were laid gently in place.   This, however, is not always the case, for the load may be suddenly or almost instantaneously applied, or it may even be dropped on the beam.   Of course, in designing such beams, a large factor of safety may be employed, as suggested in *Stresses and Strains*, but if the load is dropped or very suddenly applied, this method is at best a matter of guesswork and experience.

Careful designers sometimes make allowance for the accidental load caused by a heavy body falling on the floor, or by a mass of snow dropping from one roof to another.   The latter may usually be ignored, because it is taken care of in the factor of safety, within the limit of which every member in the structure is designed.

Members subjected to suddenly applied loads are seldom encountered in building construction, and still less frequently are members required to resist the effect of impact, or the blow imparted by a falling load.   The beams supporting the mechanism at the heads of elevator shafts are at times subjected not only to suddenly applied loads, but also to falling loads; therefore, they should always be proportioned to withstand at least a suddenly applied load.

55.  The investigation of sudden loads divides itself naturally into two classes.   The first class includes loads that are not raised above a beam and whose weight is suddenly applied to the beam.   A railroad train suddenly running on a bridge may be called a suddenly applied load. The second class of loads includes those that fall vertically

on a beam, as, for example, when heavy boxes or crates are dropped on the beams of a floor.

**56.** As the problems of the first class are the simplest to solve, they will be taken up first. Where a load is placed suddenly on a beam, the stress produced is twice as great as if the same load had been at rest; that is, a beam to sustain a suddenly applied load should have twice the transverse strength required to sustain the same load at rest.

EXAMPLE.—Design a steel I beam to carry safely a suddenly applied load of 42,000 pounds concentrated at the center. The span is 12 feet 6 inches.

SOLUTION.—The maximum bending moment, if the load were not suddenly applied, would be $\dfrac{Wl}{4} = \dfrac{42,000 \times 12\ 5}{4} = 131,250$ ft.-lb., or $131,250 \times 12 = 1,575,000$ in.-lb. However, since the load is suddenly applied, the resisting moment should be twice this amount, or $1,575,000 \times 2 = 3,150,000$ in.-lb ; that is, $3,150,000 = Ss$, and $s = 16,000$; therefore, $S = 3,150,000 \div 16,000 = 196.88$. Referring to Table III, it will be found that a 24-in , 100-lb. beam is required. Ans.

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### EXAMPLES FOR PRACTICE

1. What size of I beam on a 21-foot span will be required to withstand safely a suddenly applied load of 800 pounds concentrated at the center? Ans 6 in., 12.25-lb. I beam

2. A 12-inch, 40-pound I beam is on a 10-foot span What safe uniformly distributed, suddenly applied load will it withstand?
Ans. 21,867 lb.

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**57.** Very often, a problem occurs concerning suddenly applied loads in which the beam has two loads, one a quiet load, which is the dead load, and the other a suddenly applied load, which is the live load. Such a problem should present no difficulty and is solved as follows:

EXAMPLE.—Design an I beam to carry a uniformly distributed load of 140 pounds per foot on a span of 12 feet, and also a centrally concentrated, suddenly applied load of 3,700 pounds.

SOLUTION.—The bending moment due to the uniformly distributed load is $\dfrac{Wl}{8} = \dfrac{(140 \times 12) \times 12}{8} = 2,520$ ft.-lb., or $2,520 \times 12 = 30,240$ in.-lb. The concentrated load, if gently applied, would cause a bend-

ing moment of $\dfrac{3{,}700 \times 12}{4} = 11{,}100$ ft.-lb., or $11{,}100 \times 12 = 133{,}200$ in.-lb. Since, however, the load is suddenly applied, it will produce stresses equivalent to twice this bending moment, or $133{,}200 \times 2 = 266{,}400$ in.-lb. The total moment that the beam must be designed to withstand is therefore $266{,}400 + 30{,}240 = 296{,}640 = S s$, since $s = 16{,}000$. Therefore, $S = 296{,}640 \div 16{,}000 = 18\ 54$. Referring to Table III, it will be found that a 9-in., 21.5-lb. I beam is required.

Ans.

### EXAMPLES . FOR PRACTICE

1. What size of steel I beam will be required on an 18-foot span to carry a quiet load of 500 pounds per foot and a suddenly applied load of 350 pounds per foot?          Ans. 12-in , 35-lb. I beam

2. What centrally concentrated, suddenly applied safe load will a 20-inch, 65-pound I beam carry on a 20-foot span? Besides its own weight, the beam carries a dead load of 200 pounds per foot

Ans. 14,275 lb.

**58.** The other class of loads referred to are those which drop on a beam from a distance above it. A falling load produces a greater stress on a beam than does a load simply suddenly applied, owing to the impact produced. It is customary in considering the effect of a falling concentrated load (which is by far the most common kind of falling load and the only one considered in this Section), to determine the statical or quiet load concentrated at the center that would produce the same stress, and then to design the beam for this statical load. The formula used to accomplish this is

$$W_1 = W\left(1 + \sqrt{\frac{2\,a\,h}{d} + 1}\right), \qquad (1)$$

in which  $W_1$ = static load, in pounds, concentrated at the center, that would produce the same stress in the beam as the falling load;

$W$ = falling load, in pounds, which strikes the beam in the center of the span;

$h$ = distance, in inches, that the load falls;

$d$ = deflection of beam, in inches, produced by load $W$ statically applied;

$a$ = constant.

The value of $d$ may be determined in the manner previously explained, while the value of $a$ must be determined by the formula

$$a = \frac{1}{1 + .489 \frac{W_1}{W}}, \qquad (2)$$

in which $W_1$ = combined weight, in pounds, of beam and dead load that it supports;

$W$ = falling load.

From the construction of formulas 1 and 2, it will be noted that the size of a beam required to sustain a certain falling load cannot be found direct. The size of beam must be assumed; then the formulas are used to ascertain whether the beam will meet the requirements.

The solution of problems relating to loads that drop on beams has one great advantage in that all problems must be stated in the same way. Thus, by knowing how to solve one problem, any other one may be solved.

EXAMPLE.—A 12-inch, 40-pound I beam carries, besides its own weight, a uniform load of 260 pounds per foot. The span is 10 feet. If a load of 400 pounds drops on the beam from a distance of 18 inches, will it develop a unit stress beyond the safe unit stress of 12,500 pounds?

SOLUTION.—The total static load per foot on the beam is 260 + weight of beam per foot = 260 + 40 = 300 lb per ft. The total static load on the beam, therefore, is 300 × span = 300 × 10 = 3,000 lb. The deflection due to the falling load of 400 lb , according to column 13, Table III, is .00000505 × 10⁵ × .4 = .00202 in. The constant $a$ thus equals

$$\frac{1}{1 + .489 \frac{W_1}{W}} = \frac{1}{1 + .489 \times \frac{3,000}{400}} = .2142$$

Therefore,

$$W_1 = W\left(1 + \sqrt{\frac{2\,a\,h}{d} + 1}\right) = 400\left(1 + \sqrt{\frac{2 \times .2142 \times 18}{.00202} + 1}\right)$$
$$= 25,117 \text{ lb.}$$

The maximum bending moment due to this load is $\dfrac{25,117 \times 10}{4}$ = 62,792.5 ft.-lb., or 62,792 5 × 12 = 753,510 in.-lb. The maximum bending moment due to the static or dead load is $\dfrac{3,000 \times 10}{8} = 3,750$

ft.-lb , or $3,750 \times 12 = 45,000$ in.-lb. The total bending moment of both the static and sudden load is therefore $753,510 + 45,000 = 798,510$ in -lb. $= Ss$. From Table III, $S = 41$. Therefore, $798,510 = 41\ s$, and $s = 798,510 \div 41 = 19,476$ lb per sq in.

This is greater than 12,500, which was assumed as the allowable unit stress. Even if 16,000 pounds were taken as the allowable unit stress, the actual stress would still be too large and a larger size of beam would have to be assumed. Ans.

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### EXAMPLES FOR PRACTICE

1. If an 18-inch, 65-pound I beam were used in the example just given, what would be the maximum unit stress developed?

Ans. 14,508 lb per sq. in.

2. A 24-inch, 100-pound I beam is on a span of 10 feet. It carries no static load except its own weight A load of 1,000 pounds drops on its center from a height of 4 inches. What is the stress produced?

Ans. 15,605 lb. per sq. in.

## TABLE III

## PROPERTIES OF STANDARD I BEAMS



| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 18 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Depth of Beam, Inches | Weight per Foot, Pounds | Area of Section, Square Inches | Thickness of Web, Inches | Width of Flange, Inches | Moment of Inertia, Axis 1–1, Inches$^4$ | Section Modulus, Axis 1–1, Inches$^3$ | Radius of Gyration, Axis 1–1, Inches | Moment of Inertia, Axis 2–2, Inches$^4$ | Radius of Gyration, Axis 2–2, Inches | Increase of Thickness of Web for Each Pound Increase in Weight, Inches | Coefficient of Deflection, Uniform Load | Coefficient of Deflection, Center Load |
| $d$ | | $A$ | $t$ | $b$ | $I$ | $S$ | $r$ | $I'$ | $r'$ | $f$ | $N$ | $N'$ |
| 3 | 5 50 | 1 63 | .17 | 2 33 | 2 5 | 1 7 | 1 23 | .46 | .53 | | .00031253 | .0005006 |
| 3 | 6 50 | 1 91 | .26 | 2 42 | 2 7 | 1 8 | 1 19 | .53 | .52 | .098 | .00028827 | .00046124 |
| 3 | 7 50 | 2 21 | .36 | 2 52 | 2 9 | 1 9 | 1 15 | .60 | .52 | | .00026644 | .00042630 |
| 4 | 7 50 | 2.21 | .19 | 2 66 | 6 0 | 3 0 | 1 64 | .77 | .59 | | .00013009 | .00028815 |
| 4 | 8 50 | 2 50 | .26 | 2 73 | 6 4 | 3 2 | 1 59 | .85 | .58 | .074 | .00012209 | .00019535 |
| 4 | 9 50 | 2.79 | .34 | 2 81 | 6 7 | 3 4 | 1 54 | .93 | .58 | | .00011500 | .00018400 |
| 4 | 10 50 | 3 09 | .41 | 2 88 | 7 1 | 3 6 | 1 52 | 1 01 | .57 | | .00010868 | .00017389 |
| 5 | 9 75 | 2 87 | .21 | 3 00 | 12 1 | 4 8 | 2 05 | 1 23 | .65 | | .00006417 | .00010267 |
| 5 | 12 25 | 3 60 | .36 | 3 15 | 13 6 | 5 4 | 1 94 | 1 45 | .63 | .059 | .00005698 | .00009117 |
| 5 | 14 75 | 4 34 | .50 | 3 29 | 15 1 | 6 1 | 1 87 | 1 70 | .63 | | .00005122 | .00008195 |
| 6 | 12 25 | 3 61 | .23 | 3 33 | 21 8 | 7 3 | 2 46 | 1 85 | .72 | | .00003561 | .00005698 |
| 6 | 14 75 | 4.34 | .35 | 3.45 | 24.0 | 8.0 | 2.35 | 2 09 | .69 | .049 | .00003235 | .00005177 |
| 6 | 17.25 | 5.07 | .47 | 3.57 | 26.2 | 8.7 | 2.27 | 2 36 | .68 | | .00002963 | .00004741 |

| No. | Size | K | J | I | H | G | F | E | D | C | B | A |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 7 | 15 00 | 4 42 | 25 | 3 66 | 36.2 | 10 4 | 2 86 | 2 67 | 78 | 042 | .00002142 | .00003427 |
| 7 | 17 50 | 5 15 | 35 | 3 76 | 39 2 | 11 2 | 2 76 | 2 94 | 76 | | .00001980 | .00003168 |
| 7 | 20 00 | 5 88 | 46 | 3 87 | 42 2 | 12 1 | 2 68 | 3 24 | .74 | | .00001839 | .00002943 |
| 8 | 18 00 | 5 33 | 27 | 4 00 | 56 9 | 14.2 | 3 27 | 3 78 | 84 | 037 | .00001364 | .00002183 |
| 8 | 20 25 | 5 96 | 35 | 4 08 | 60 2 | 15 0 | 3 18 | 4 04 | 82 | | .00001289 | .00002062 |
| 8 | 22 75 | 6 69 | 44 | 4 17 | 64 1 | 16 0 | 3 10 | 4 36 | 81 | | .00001210 | .00001936 |
| 8 | 25 25 | 7 43 | 53 | 4 26 | 68 0 | 17 0 | 3.03 | 4 71 | 80 | | .00001140 | .00001825 |
| 9 | 21 50 | 6 31 | 29 | 4 33 | 84 9 | 18 9 | 3 67 | 5 16 | 90 | 033 | .00000914 | .00001462 |
| 9 | 25 00 | 7 35 | 41 | 4 45 | 91 9 | 20 4 | 3 54 | 5 65 | 88 | | .00000844 | .00001350 |
| 9 | 30 00 | 8 82 | 57 | 4 61 | 101 9 | 22 6 | 3 40 | 6 42 | 85 | | .00000762 | .00001219 |
| 9 | 35 00 | 10 29 | 73 | 4 77 | 111 8 | 24 8 | 3 30 | 7 31 | 84 | | .00000694 | .00001110 |
| 10 | 25 00 | 7 37 | .31 | 4 66 | 122 1 | 24 4 | 4 07 | 6 89 | 97 | .029 | .00000635 | .00001017 |
| 10 | 30 00 | 8 82 | .45 | 4 80 | 134 2 | 26 8 | 3 90 | 7 65 | 93 | | .00000578 | .00000925 |
| 10 | 35 00 | 10 29 | 60 | 4 95 | 146 4 | 29 3 | 3 77 | 8 52 | 91 | | .00000530 | .00000848 |
| 10 | 40 00 | 11 76 | 75 | 5 10 | 158 7 | 31 7 | 3 67 | 9 50 | 90 | | .00000489 | .00000782 |
| 12 | 31 50 | 9 26 | 35 | 5 00 | 215 8 | 36 0 | 4 83 | 9 50 | 1 01 | .025 | .00000360 | .00000575 |
| 12 | 35 00 | 10 29 | 44 | 5 09 | 228 3 | 38 0 | 4 71 | 10 07 | .99 | | .00000340 | .00000544 |
| 12 | 40 00 | 11 76 | 56 | 5 21 | 245 9 | 41 0 | 4 57 | 10 95 | 96 | | .00000316 | .00000505 |
| 15 | 42 00 | 12 48 | 41 | 5 50 | 441 8 | 58 9 | 5 95 | 14 62 | 1 08 | 020 | .00000176 | .00000281 |
| 15 | 45 00 | 13 24 | 46 | 5 55 | 455 8 | 60 8 | 5 87 | 15 09 | 1 07 | | .00000170 | .00000272 |
| 15 | 50 00 | 14 71 | 56 | 5 65 | 483 4 | 64 5 | 5 73 | 16 04 | 1 04 | | .00000161 | .00000257 |
| 15 | 55 00 | 16 18 | 66 | 5 75 | 510 0 | 68 1 | 5 62 | 17 06 | 1 03 | | .00000152 | .00000243 |
| 15 | 60 00 | 17 65 | 76 | 5 84 | 538 0 | 71 8 | 5 52 | 18 17 | 1 01 | | .00000144 | .00000231 |
| 18 | 55 00 | 15 93 | .46 | 6 00 | 795 6 | 88 4 | 7 07 | 21 19 | 1 15 | .016 | .00000098 | .00000156 |
| 18 | 60 00 | 17 65 | 56 | 6 10 | 841 8 | 93 5 | 6 91 | 22 38 | 1 13 | | .00000092 | .00000148 |
| 18 | 65 00 | 19 12 | 64 | 6 18 | 881.5 | 97 9 | 6 79 | 23 47 | 1 11 | | .00000088 | .00000141 |
| 18 | 70 00 | 20 59 | 72 | 6 26 | 921 2 | 102 4 | 6 69 | 24 62 | 1 09 | | .00000084 | .00000135 |
| 20 | 65 00 | 19 08 | 50 | 6 25 | 1,169 5 | 117 0 | 7 83 | 27 86 | 1 21 | 015 | .00000066 | .00000106 |
| 20 | 70 00 | 20 59 | 58 | 6 33 | 1,219 8 | 122 0 | 7 70 | 29 04 | 1 19 | | .00000064 | .00000102 |
| 20 | 75 00 | 22 06 | 65 | 6 40 | 1,268 8 | 126 9 | 7 58 | 30 25 | 1 17 | | .00000061 | .00000098 |
| 24 | 80 00 | 23 32 | 50 | 7 00 | 2,087 2 | 173 9 | 9 46 | 42 86 | 1 36 | 0123 | .00000037 | .00000060 |
| 24 | 85 00 | 25 00 | 57 | 7 07 | 2,167 8 | 180 7 | 9 31 | 44 35 | 1 33 | | .00000036 | .00000057 |
| 24 | 90 00 | 26 47 | 63 | 7 13 | 2,238 4 | 186 5 | 9 20 | 45 70 | 1.31 | | .00000035 | .00000056 |
| 24 | 95 00 | 27 94 | .69 | 7 19 | 2,309 0 | 192 4 | 9 09 | 47 10 | 1 30 | | .00000034 | .00000054 |
| 24 | 100 00 | 29 41 | .75 | 7 25 | 2,379 6 | 198 3 | 8 99 | 48 55 | 1.28 | | .00000033 | .00000052 |

# TABLE IV
## PROPERTIES OF STANDARD CHANNELS



| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 |
|---|---|---|---|---|---|---|---|---|----|----|----|----|----|----|
| Depth of Channel, Inches | Weight per Foot, Pounds | Area of Section, Square Inches | Thickness of Web, Inches | Width of Flange, Inches | Moment of Inertia, Axis 1-1, Inches⁴ | Section Modulus, Axis 1-1, Inches³ | Radius of Gyration, Axis 1-1, Inches | Moment of Inertia, Axis 2-2, Inches⁴ | Section Modulus, Axis 2-2, Inches³ | Radius of Gyration, Axis 2-2, Inches | Distance of Center of Gravity From Outside of Web, Inches | Increase of Thickness of Web for Each Pound Increase in Weight, Inches | Coefficient of Deflection, Uniform Load | Coefficient of Deflection, Center Load |
| $a$ | | $A$ | $t$ | $b$ | $I$ | $S$ | $r$ | $I'$ | $S'$ | $r'$ | $x$ | $f$ | $N$ | $N'$ |
| 3 | 4.00 | 1.19 | .17 | 1.41 | 1.6 | 1.1 | 1.17 | .20 | .21 | .41 | .44 | .098 | .0004743 | .0007589 |
| 3 | 5.00 | 1.47 | .26 | 1.50 | 1.8 | 1.2 | 1.12 | .25 | .24 | .41 | .44 | | .0004199 | .0006718 |
| 3 | 6.00 | 1.76 | .36 | 1.60 | 2.1 | 1.4 | 1.08 | .31 | .27 | .42 | .46 | | .0003751 | .0006001 |
| 4 | 5.25 | 1.55 | .18 | 1.58 | 3.8 | 1.9 | 1.56 | .32 | .29 | .45 | .46 | .074 | .0002046 | .0003273 |
| 4 | 6.25 | 1.84 | .25 | 1.65 | 4.2 | 2.1 | 1.51 | .38 | .32 | .45 | .46 | | .0001858 | .0002973 |
| 4 | 7.25 | 2.13 | .33 | 1.73 | 4.6 | 2.3 | 1.46 | .44 | .35 | .46 | .46 | | .0001698 | .0002717 |
| 5 | 6.50 | 1.93 | .19 | 1.75 | 7.4 | 3.0 | 1.95 | .48 | .38 | .50 | .49 | .059 | .0001046 | .0001674 |
| 5 | 9.00 | 2.65 | .33 | 1.89 | 8.9 | 3.5 | 1.83 | .64 | .45 | .49 | .48 | | .0000875 | .0001399 |
| 5 | 11.50 | 3.38 | .48 | 2.04 | 10.4 | 4.2 | 1.75 | .82 | .54 | .49 | .51 | | .0000746 | .0001193 |
| 6 | 8.00 | 2.38 | .20 | 1.92 | 13.0 | 4.3 | 2.34 | .70 | .50 | .54 | .52 | .049 | .0000597 | .0000855 |
| 6 | 10.50 | 3.09 | .32 | 2.04 | 15.1 | 5.0 | 2.21 | .88 | .57 | .53 | .50 | | .0000513 | .0000821 |
| 6 | 13.00 | 3.82 | .44 | 2.16 | 17.3 | 5.8 | 2.13 | 1.07 | .65 | .53 | .52 | | .0000448 | .0000717 |
| 6 | 15.50 | 4.56 | .56 | 2.28 | 19.5 | 6.5 | 2.07 | 1.28 | .74 | .53 | .55 | | .0000397 | .0000636 |

| | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 7 | 9 75 | 2 85 | 21 | 2 09 | 21 1 | 6 0 | 2 72 | 98 | 63 | 59 | 55 | 042 | 0000368 | 0000588 |
| 7 | 12 25 | 3 60 | 32 | 2 20 | 24 2 | 6 9 | 2 59 | 1 19 | 71 | 57 | 53 | | 0000321 | 0000514 |
| 7 | 14 75 | 4 34 | 42 | 2 30 | 27 2 | 7 8 | 2 50 | 1 40 | 79 | 57 | 53 | | 0000286 | 0000457 |
| 7 | 17 25 | 5 07 | 53 | 2 41 | 30 2 | 8 6 | 2 44 | 1 62 | 87 | 56 | 55 | | 0000257 | 0000411 |
| 7 | 19 75 | 5 81 | 63 | 2 51 | 33 2 | 9 5 | 2 39 | 1 85 | 96 | 56 | 58 | | 0000234 | 0000374 |
| 8 | 11 25 | 3 35 | 22 | 2 26 | 32 3 | 8 1 | 3 10 | 1 33 | 79 | 63 | 58 | 037 | 0000240 | 0000384 |
| 8 | 13 75 | 4 04 | 31 | 2 35 | 36 0 | 9 0 | 2 98 | 1 55 | 87 | 62 | 56 | | 0000216 | 0000345 |
| 8 | 16 25 | 4 78 | 40 | 2 44 | 39 8 | 10 0 | 2 89 | 1 78 | 95 | 61 | 56 | | 0000194 | 0000311 |
| 8 | 18 75 | 5 51 | 49 | 2 53 | 43 8 | 11 0 | 2 82 | 2 01 | 1 02 | 60 | 57 | | 0000177 | 0000283 |
| 8 | 21 25 | 6 25 | 58 | 2 62 | 47 8 | 11 9 | 2 76 | 2 25 | 1 11 | 60 | 59 | | 0000162 | 0000260 |
| 9 | 13 25 | 3 89 | 23 | 2 43 | 47 3 | 10 5 | 3 49 | 1 77 | 97 | 67 | 61 | 033 | 0000164 | 0000262 |
| 9 | 15 00 | 4 41 | 29 | 2 49 | 50 8 | 11 3 | 3 40 | 1 95 | 1 03 | 66 | 59 | | 0000153 | 0000244 |
| 9 | 20 00 | 5 88 | 45 | 2 65 | 60 8 | 13 5 | 3 21 | 2 45 | 1 19 | 65 | 58 | | 0000128 | 0000204 |
| 9 | 25 00 | 7 35 | 61 | 2 81 | 70 7 | 15 7 | 3 10 | 2 98 | 1 36 | 64 | 62 | | 0000110 | 0000176 |
| 10 | 15 00 | 4 46 | 24 | 2 60 | 66 9 | 13 4 | 3 87 | 2 30 | 1 17 | 72 | 64 | 029 | 0000116 | 0000186 |
| 10 | 20 00 | 5 88 | 38 | 2 74 | 78 7 | 15 7 | 3 66 | 2 85 | 1 34 | 70 | 61 | | 0000099 | 0000158 |
| 10 | 25 00 | 7 35 | 53 | 2 89 | 91 0 | 18 2 | 3 52 | 3 40 | 1 50 | 68 | 62 | | 0000085 | 0000136 |
| 10 | 30 00 | 8 82 | 68 | 3 04 | 103 2 | 20 6 | 3 42 | 3 99 | 1 67 | 67 | 65 | | 0000075 | 0000120 |
| 10 | 35 00 | 10 29 | 82 | 3 18 | 115 5 | 23 1 | 3 35 | 4 66 | 1 87 | 67 | 69 | | 0000067 | 0000107 |
| 12 | 20 50 | 6 03 | 28 | 2 94 | 128 1 | 21 4 | 4 61 | 3 91 | 1 75 | 81 | 70 | 025 | 0000061 | 0000097 |
| 12 | 25 00 | 7 35 | 39 | 3 05 | 144 0 | 24 0 | 4 43 | 4 53 | 1 91 | 78 | 68 | | 0000054 | 0000086 |
| 12 | 30 00 | 8 82 | 51 | 3 17 | 161 6 | 26 9 | 4 28 | 5 21 | 2 09 | 77 | 68 | | 0000048 | 0000077 |
| 12 | 35 00 | 10 29 | 64 | 3 30 | 179 3 | 29 9 | 4 17 | 5 90 | 2 27 | 76 | 69 | | 0000043 | 0000069 |
| 12 | 40 00 | 11 76 | 76 | 3 42 | 196.9 | 32 8 | 4 09 | 6 63 | 2 46 | 75 | 72 | | 0000039 | 0000063 |
| 15 | 33 00 | 9 90 | 40 | 3 40 | 312 6 | 41 7 | 5 62 | 8 23 | 3 16 | 91 | 79 | 020 | 0000025 | 0000040 |
| 15 | 35 00 | 10 29 | 43 | 3 43 | 319 9 | 42 7 | 5 57 | 8 48 | 3 22 | 91 | 79 | | 0000024 | 0000039 |
| 15 | 40 00 | 11 76 | 52 | 3 52 | 347 5 | 46 3 | 5 44 | 9 39 | 3 43 | 89 | 78 | | 0000022 | 0000036 |
| 15 | 45 00 | 13 24 | 62 | 3 62 | 375.1 | 50 0 | 5 32 | 10 29 | 3 63 | 88 | 79 | | 0000021 | 0000033 |
| 15 | 50 00 | 14 71 | 72 | 3 72 | 402.7 | 53 7 | 5 23 | 11 22 | 3 85 | 87 | 80 | | 0000019 | 0000031 |
| 15 | 55 00 | 16 18 | 82 | 3 82 | 430 2 | 57 4 | 5 16 | 12 19 | 4 07 | 87 | 82 | | 0000018 | 0000029 |

TABLE V

# PROPERTIES OF STANDARD ANGLES HAVING EQUAL LEGS



| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Dimensions Inches | Thickness Inches | Weight per Foot Pounds | Area of Section Square Inches | Distance of Center of Gravity From Back of Leg Inches | Moment of Inertia, Axis 1-1 Inches⁴ | Section Modulus, Axis 1-1 Inches³ | Radius of Gyration, Axis 1-1 Inches | Distance of Center of Gravity From External Apex Inches | Least Moment of Inertia, Axis 2-2 Inches⁴ | Section Modulus, Axis 2-2 Inches³ | Least Radius of Gyration, Axis 2-2 Inches |
| $a \times a$ | $t$ | | $A$ | $x$ | $I$ | $S$ | $r$ | $x'$ | $I'$ | $S'$ | $r'$ |
| ¾×¾ | 1/8 | 6 | 18 | 23 | 009 | 017 | .22 | .33 | 004 | 011 | .14 |
| ¾×¾ | 3/16 | 9 | .25 | .25 | 012 | 024 | 22 | .36 | .005 | 014 | 14 |
| 1×1 | 1/8 | 8 | .24 | .30 | 022 | 031 | 30 | .42 | 009 | 021 | .19 |
| 1×1 | 3/16 | 1 2 | 34 | 32 | 030 | 044 | 30 | 45 | 013 | 028 | 19 |
| 1×1 | 1/4 | 1 5 | 44 | 34 | 037 | 056 | 29 | .48 | 016 | 034 | 19 |
| 1¼×1¼ | 1/8 | 1 1 | 30 | 36 | 044 | 049 | 38 | .51 | 018 | 035 | 24 |
| 1¼×1¼ | 3/16 | 1 5 | 44 | .38 | 061 | 071 | 38 | .54 | 025 | 047 | 24 |
| 1¼×1¼ | 1/4 | 2 0 | 57 | 40 | 077 | 091 | .37 | 57 | .033 | 057 | 24 |
| 1¼×1¼ | 5/16 | 2 4 | 69 | 42 | .090 | 109 | 36 | .60 | 040 | 066 | 24 |
| 1½×1½ | 1/8 | 1 3 | 36 | .42 | 08 | 072 | 47 | 60 | 031 | 053 | .30 |
| 1½×1½ | 3/16 | 1 8 | 53 | 44 | 11 | 104 | 46 | 63 | 045 | 072 | 29 |
| 1½×1½ | 1/4 | 2.4 | 69 | 47 | 14 | 134 | 45 | .66 | 058 | 088 | 29 |
| 1½×1½ | 5/16 | 2 9 | 84 | 49 | 16 | 162 | 44 | 69 | 070 | 101 | 29 |
| 1½×1½ | 3/8 | 3 4 | 99 | 51 | 19 | 188 | 44 | 72 | 082 | 114 | 29 |
| 1½×1½ | 7/16 | 3 9 | 1 13 | 53 | 21 | 214 | 43 | 75 | 094 | 126 | 29 |
| 1¾×1¾ | 3/16 | 2 2 | 63 | 51 | .18 | 14 | 54 | .72 | 073 | 10 | .34 |
| 1¾×1¾ | 1/4 | 2 8 | 82 | 53 | 23 | 19 | .53 | 75 | 094 | 13 | .34 |
| 1¾×1¾ | 5/16 | 3 4 | 1 00 | 55 | 27 | 23 | 52 | 78 | 113 | 15 | .34 |
| 1¾×1¾ | 3/8 | 4 0 | 1 18 | 57 | 31 | 26 | 51 | 81 | 133 | 16 | .34 |
| 1¾×1¾ | 7/16 | 4 6 | 1 34 | 59 | .35 | 30 | 51 | .84 | 152 | 18 | 34 |
| 1¾×1¾ | 1/2 | 5 1 | 1 50 | .61 | .38 | 33 | 50 | 87 | 171 | .20 | 34 |
| 2×2 | 3/16 | 2 5 | 72 | 57 | .27 | 19 | 62 | 80 | 11 | 14 | 39 |
| 2×2 | 1/4 | 3 2 | 94 | 59 | 35 | 25 | 61 | 84 | 14 | 17 | 39 |
| 2×2 | 5/16 | 4 0 | 1 16 | .61 | .42 | 30 | 60 | 87 | 17 | 20 | 39 |
| 2×2 | 3/8 | 4 7 | 1 36 | .64 | .48 | 35 | 59 | 90 | 20 | 22 | 39 |
| 2×2 | 7/16 | 5 3 | 1 56 | .66 | 54 | .40 | 59 | .93 | .23 | 25 | 38 |
| 2×2 | 1/2 | 6 0 | 1 75 | .68 | 59 | .45 | .58 | .96 | 26 | 27 | 38 |
| 2½×2½ | 3/16 | 3 1 | 91 | 69 | 55 | 30 | 78 | .98 | 22 | 22 | 49 |
| 2½×2½ | 1/4 | 4 1 | 1 19 | 72 | 70 | .39 | .77 | 1 01 | 29 | 28 | .49 |
| 2½×2½ | 5/16 | 5 0 | 1 47 | .74 | 85 | .48 | 76 | 1 05 | 35 | 33 | .49 |
| 2½×2½ | 3/8 | 5 9 | 1 74 | .76 | 98 | .57 | 75 | 1 08 | 41 | 38 | .48 |
| 2½×2½ | 7/16 | 6 8 | 2 00 | 78 | 1 11 | 65 | 75 | 1 11 | 46 | 42 | 48 |
| 2½×2½ | 1/2 | 7.7 | 2 25 | .81 | 1 23 | 72 | 74 | 1 14 | 52 | 46 | 48 |
| 2½×2½ | 9/16 | 8 5 | 2 50 | 83 | 1 34 | 80 | 73 | 1 17 | .58 | 49 | 48 |
| 3×3 | 1/4 | 4 9 | 1 44 | 84 | 1 24 | .58 | 93 | 1 19 | 50 | 42 | 59 |
| 3×3 | 5/16 | 6 1 | 1 78 | 87 | 1 51 | 71 | 92 | 1 22 | 61 | 50 | 59 |
| 3×3 | 3/8 | 7 2 | 2 11 | .89 | 1 76 | 83 | 91 | 1 26 | 72 | 57 | 58 |
| 3×3 | 7/16 | 8.3 | 2 44 | 91 | 1 99 | 95 | 91 | 1 29 | 82 | 64 | 58 |
| 3×3 | 1/2 | 9 4 | 2 75 | .93 | 2 22 | 1 07 | 90 | 1 32 | 92 | 70 | 58 |
| 3×3 | 9/16 | 10 4 | 3 06 | .95 | 2 43 | 1 19 | .89 | 1 35 | 1 02 | 76 | 58 |
| 3×3 | 5/8 | 11.5 | 3 36 | .98 | 2 62 | 1 30 | 88 | 1 38 | 1 12 | 81 | 58 |
| 3×3 | 11/16 | 12 5 | 3.66 | 1 00 | 2 81 | 1 40 | 88 | 1.41 | 1.22 | 86 | .58 |

TABLE V—(*Continued*)



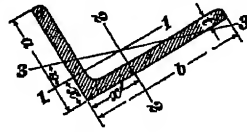| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Dimensions Inches | Thickness Inches | Weight per Foot Pounds | Area of Section Square Inches | Distance of Center of Gravity From Back of Leg Inches | Moment of Inertia, Axis 1–1 Inches[4] | Section Modulus, Axis 1–1 Inches[3] | Radius of Gyration, Axis 1–1 Inches | Distance of Center of Gravity From External Apex Inches | Least Moment of Inertia, Axis 2–2 Inches[4] | Section Modulus, Axis 2–2 Inches[3] | Least Radius of Gyration, Axis 2–2 Inches |
| $a \times a$ | $t$ | | $A$ | $x$ | $I$ | $S$ | $r$ | $x'$ | $I'$ | $S'$ | $r'$ |
| 3½×3½ | $\frac{5}{16}$ | 7 2 | 2 09 | 99 | 2 45 | 98 | 1 08 | 1 40 | 99 | .71 | .69 |
| 3½×3½ | $\frac{3}{8}$ | 8 5 | 2 49 | 1 01 | 2 87 | 1.15 | 1 07 | 1.43 | 1 16 | 81 | 68 |
| 3½×3½ | $\frac{7}{16}$ | 9 8 | 2 88 | 1.04 | 3 26 | 1 32 | 1.07 | 1 46 | 1.33 | .91 | 68 |
| 3½×3½ | $\frac{1}{2}$ | 11 1 | 3 25 | 1 06 | 3 64 | 1.49 | 1 06 | 1 50 | 1 50 | 1 00 | 68 |
| 3½×3½ | $\frac{9}{16}$ | 12 4 | 3 63 | 1 08 | 3 99 | 1 65 | 1.05 | 1 53 | 1 66 | 1 09 | 68 |
| 3½×3½ | $\frac{5}{8}$ | 13.6 | 3 99 | 1 10 | 4 33 | 1 81 | 1.04 | 1 56 | 1 82 | 1.17 | 68 |
| 3½×3½ | $\frac{11}{16}$ | 14 8 | 4 34 | 1 12 | 4 65 | 1 96 | 1.04 | 1 59 | 1.97 | 1 24 | 67 |
| 3½×3½ | $\frac{3}{4}$ | 16 0 | 4 69 | 1 15 | 4 96 | 2 11 | 1.03 | 1 62 | 2 13 | 1 31 | 67 |
| 3½×3½ | $\frac{13}{16}$ | 17 1 | 5 03 | 1.17 | 5 25 | 2 25 | 1.02 | 1 65 | 2 28 | 1 38 | 67 |
| 3½×3½ | $\frac{7}{8}$ | 18 3 | 5 36 | 1 19 | 5 53 | 2 39 | 1 02 | 1 68 | 2.43 | 1 45 | .67 |
| 4×4 | $\frac{5}{16}$ | 8 2 | 2 41 | 1 12 | 3 71 | 1.29 | 1 24 | 1 58 | 1 50 | .95 | 79 |
| 4×4 | $\frac{3}{8}$ | 9 8 | 2 86 | 1 14 | 4 36 | 1 52 | 1.23 | 1 61 | 1 77 | 1.10 | 79 |
| 4×4 | $\frac{7}{16}$ | 11 3 | 3 31 | 1 16 | 4 97 | 1 75 | 1.23 | 1 64 | 2.02 | 1.23 | .78 |
| 4×4 | $\frac{1}{2}$ | 12 8 | 3 75 | 1 18 | 5 56 | 1 97 | 1.22 | 1 67 | 2 28 | 1 36 | .78 |
| 4×4 | $\frac{9}{16}$ | 14 3 | 4 19 | 1 21 | 6 12 | 2.19 | 1 21 | 1 71 | 2 52 | 1 48 | .78 |
| 4×4 | $\frac{5}{8}$ | 15 7 | 4 62 | 1.23 | 6 66 | 2.40 | 1 20 | 1 74 | 2.76 | 1 59 | .77 |
| 4×4 | $\frac{11}{16}$ | 17 1 | 5 03 | 1 25 | 7 17 | 2 61 | 1 19 | 1 77 | 3 00 | 1.70 | .77 |
| 4×4 | $\frac{3}{4}$ | 18 5 | 5 44 | 1 27 | 7 66 | 2 81 | 1 19 | 1.80 | 3.23 | 1 80 | .77 |
| 4×4 | $\frac{13}{16}$ | 19 9 | 5 84 | 1 29 | 8 14 | 3 01 | 1 18 | 1 83 | 3.46 | 1 89 | .77 |
| 4×4 | $\frac{7}{8}$ | 21 2 | 6 24 | 1.31 | 8.59 | 3.20 | 1 17 | 1.86 | 3 69 | 1 99 | .77 |
| 6×6 | $\frac{3}{8}$ | 14 9 | 4.36 | 1.64 | 15 39 | 3 53 | 1 88 | 2 32 | 6 19 | 2 67 | 1.19 |
| 6×6 | $\frac{7}{16}$ | 17 2 | 5 06 | 1.66 | 17 68 | 4 07 | 1.87 | 2 34 | 7 13 | 3 04 | 1.19 |
| 6×6 | $\frac{1}{2}$ | 19 6 | 5 75 | 1 68 | 19 91 | 4 61 | 1.86 | 2 38 | 8 04 | 3 37 | 1.18 |
| 6×6 | $\frac{9}{16}$ | 21 9 | 6 44 | 1 71 | 22 07 | 5.14 | 1 85 | 2 41 | 8.94 | 3 70 | 1 18 |
| 6×6 | $\frac{5}{8}$ | 24 2 | 7.11 | 1 73 | 24 16 | 5 66 | 1 84 | 2 45 | 9 81 | 4.01 | 1.17 |
| 6×6 | $\frac{11}{16}$ | 26 5 | 7 78 | 1 75 | 26 19 | 6 17 | 1 83 | 2 48 | 10 67 | 4 31 | 1 17 |
| 6×6 | $\frac{3}{4}$ | 28 7 | 8 44 | 1 78 | 28 15 | 6 66 | 1 83 | 2.51 | 11 52 | 4 59 | 1 17 |
| 6×6 | $\frac{13}{16}$ | 31 0 | 9 09 | 1 80 | 30 06 | 7 15 | 1 82 | 2 54 | 12 35 | 4 86 | 1 17 |
| 6×6 | $\frac{7}{8}$ | 33 1 | 9 74 | 1 82 | 31.92 | 7 63 | 1.81 | 2 57 | 13 17 | 5 12 | 1 16 |
| 6×6 | $\frac{15}{16}$ | 35 3 | 10 38 | 1 84 | 33 72 | 8 11 | 1.80 | 2 60 | 13 98 | 5 37 | 1.16 |
| 6×6 | 1 | 37 4 | 11 00 | 1 86 | 35 46 | 8 57 | 1 80 | 2 64 | 14 78 | 5 61 | 1.16 |
| 8×8 | $\frac{1}{2}$ | 26 4 | 7 75 | 2 19 | 48.65 | 8 37 | 2 51 | 3 09 | 19 56 | 6.33 | 1.59 |
| 8×8 | $\frac{9}{16}$ | 29 6 | 8 69 | 2 21 | 54 09 | 9 34 | 2 50 | 3.12 | 21.79 | 6 98 | 1 58 |
| 8×8 | $\frac{5}{8}$ | 32 7 | 9 61 | 2 23 | 59 43 | 10 30 | 2 49 | 3 16 | 23 97 | 7 60 | 1 58 |
| 8×8 | $\frac{11}{16}$ | 35 8 | 10 53 | 2 25 | 64 64 | 11 25 | 2 48 | 3 19 | 26 13 | 8 20 | 1.58 |
| 8×8 | $\frac{3}{4}$ | 38 9 | 11 44 | 2 28 | 69 74 | 12 18 | 2 47 | 3.22 | 28 24 | 8 77 | 1 57 |
| 8×8 | $\frac{13}{16}$ | 42 0 | 12 34 | 2 30 | 74 72 | 13 11 | 2 46 | 3 25 | 30 33 | 9 33 | 1 57 |
| 8×8 | $\frac{7}{8}$ | 45 0 | 13 24 | 2 32 | 79 58 | 14 02 | 2 45 | 3 28 | 32 38 | 9 86 | 1 56 |
| 8×8 | $\frac{15}{16}$ | 48 1 | 14 13 | 2 34 | 84 34 | 14 91 | 2 44 | 3 32 | 34 40 | 10 38 | 1 56 |
| 8×8 | 1 | 51 0 | 15 00 | 2 37 | 88 98 | 15.80 | 2.44 | 3 35 | 36 40 | 10 88 | 1.56 |
| 8×8 | $1\frac{1}{16}$ | 54 c | 15 88 | 2 39 | 93 53 | 16 67 | 2 43 | 3 38 | 38 38 | 11 36 | 1 56 |
| 8×8 | $1\frac{1}{8}$ | 56 9 | 16 74 | 2 41 | 97 97 | 17.53 | 2.42 | 3.41 | 40 33 | 11 83 | 1.55 |

# TABLE VI

## PROPERTIES OF STANDARD ANGLES HAVING UNEQUAL LEGS



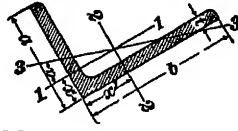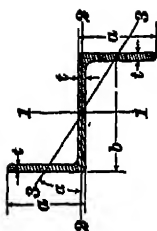| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Dimensions Inches | Thickness Inches | Weight per Foot Pounds | Area of Section Square Inches | Distance of Center of Gravity From Back of Longer Leg Inches | Moment of Inertia, Axis 1-1 Inches⁴ | Section Modulus, Axis 1-1 Inches³ | Radius of Gyration, Axis 1-1 Inches | Distance of Center of Gravity From Back of Shorter Leg Inches | Moment of Inertia, Axis 2-2 Inches⁴ | Section Modulus, Axis 2-2 Inches³ | Radius of Gyration, Axis 2-2 Inches | Tangent of Angle | Least Radius of Gyration, Axis 3-3 Inches |
| $b \times a$ | $t$ | | $A$ | $x$ | $I$ | $S$ | $r$ | $x'$ | $I'$ | $S'$ | $r'$ | $a$ | $r''$ |
| 2½×2 | ³⁄₁₆ | 2.8 | 81 | .51 | .29 | 20 | 60 | .76 | .51 | .29 | .79 | 632 | 43 |
| 2½×2 | ¼ | 3 7 | 1 07 | .54 | .37 | 25 | 59 | .79 | .65 | 38 | .78 | 626 | .42 |
| 2½×2 | ⁵⁄₁₆ | 4.5 | 1.31 | 56 | .45 | .31 | 58 | .81 | .79 | .47 | .78 | 620 | 42 |
| 2½×2 | ⅜ | 5 3 | 1 55 | 58 | .51 | .36 | 58 | 83 | .91 | 55 | .77 | 614 | .42 |
| 2½×2 | ⁷⁄₁₆ | 6 1 | 1.78 | .60 | .58 | 41 | 57 | .85 | 1 03 | .62 | 76 | 607 | 42 |
| 2½×2 | ½ | 6 8 | 2 00 | 63 | .64 | .46 | 56 | .88 | 1.14 | .70 | .75 | 600 | .42 |
| 2½×2 | ⁹⁄₁₆ | 7 6 | 2.22 | .65 | .69 | 51 | 56 | 90 | 1 24 | .77 | .75 | 592 | 42 |
| 3×2½ | ¼ | 4 5 | 1 32 | .66 | .74 | .40 | 75 | 91 | 1.17 | .56 | 95 | 684 | 53 |
| 3×2½ | ⁵⁄₁₆ | 5 6 | 1 63 | .68 | .90 | 49 | .74 | .93 | 1 42 | .69 | 94 | 680 | 53 |
| 3×2½ | ⅜ | 6.6 | 1 93 | 71 | 1 04 | .58 | .74 | .96 | 1 66 | .81 | 93 | 676 | 52 |
| 3×2½ | ⁷⁄₁₆ | 7.6 | 2.22 | .73 | 1.18 | 66 | .73 | .98 | 1.88 | 93 | 92 | 672 | 52 |
| 3×2½ | ½ | 8 5 | 2 50 | .75 | 1 30 | 74 | .72 | 1 00 | 2 08 | 1.04 | 91 | 666 | 52 |
| 3×2½ | ⁹⁄₁₆ | 9 5 | 2.78 | 77 | 1 42 | 82 | .72 | 1 02 | 2 28 | 1 15 | .91 | 661 | 52 |
| 3×2½ | ⅝ | 10.4 | 3.05 | .79 | 1.53 | 90 | .71 | 1.04 | 2 46 | 1 26 | 90 | 655 | 52 |
| 3½×2½ | ¼ | 4 9 | 1 44 | 61 | 78 | 41 | .74 | 1 11 | 1 80 | 75 | 1 12 | .506 | .54 |
| 3½×2½ | ⁵⁄₁₆ | 6.1 | 1.78 | .64 | .94 | .50 | .73 | 1 14 | 2 19 | 93 | 1.11 | 501 | 54 |
| 3½×2½ | ⅜ | 7 2 | 2.11 | 66 | 1 09 | .59 | .72 | 1 16 | 2 56 | 1 09 | 1 10 | 496 | 54 |
| 3½×2½ | ⁷⁄₁₆ | 8.3 | 2 44 | 68 | 1 23 | .68 | 71 | 1 18 | 2 91 | 1 26 | 1 09 | 491 | 54 |
| 3½×2½ | ½ | 9.4 | 2 75 | .70 | 1.36 | .76 | 70 | 1 20 | 3 24 | 1 41 | 1 09 | 486 | .53 |
| 3½×2½ | ⁹⁄₁₆ | 10 4 | 3 06 | .73 | 1.49 | 84 | 70 | 1 23 | 3 55 | 1 56 | 1 08 | 480 | 5: |
| 3½×2½ | ⅝ | 11 5 | 3 36 | .75 | 1 61 | 92 | 69 | 1 25 | 3 85 | 1 71 | 1.07 | 472 | 5: |
| 3½×2½ | ¹¹⁄₁₆ | 12 5 | 3.66 | 77 | 1 72 | .99 | 69 | 1 27 | 4 13 | 1.85 | 1 06 | 468 | 5: |
| 3½×2½ | ¾ | 13 4 | 3.94 | 79 | 1 83 | 1 07 | 68 | 1 29 | 4 40 | 1 99 | 1 06 | 461 | 54 |
| 3½×3 | ⁵⁄₁₆ | 6 6 | 1.94 | .81 | 1.58 | 72 | 90 | 1 06 | 2 33 | 95 | 1 10 | 724 | .6: |
| 3½×3 | ⅜ | 7 9 | 2 30 | 83 | 1 85 | .85 | .90 | 1 08 | 2 72 | 1 13 | 1 09 | 721 | .62 |
| 3½×3 | ⁷⁄₁₆ | 9 1 | 2 66 | .85 | 2 09 | 98 | 89 | 1 10 | 3 10 | 1 29 | 1 08 | 718 | 6: |
| 3½×3 | ½ | 10.2 | 3.00 | 88 | 2 33 | 1.10 | 88 | 1 13 | 3 45 | 1 45 | 1 07 | 714 | 6: |
| 3½×3 | ⁹⁄₁₆ | 11 4 | 3 34 | 90 | 2 55 | 1 21 | 87 | 1 15 | 3 79 | 1 61 | 1 07 | 711 | 6· |
| 3½×3 | ⅝ | 12 5 | 3 68 | 92 | 2 76 | 1 33 | 87 | 1 17 | 4 11 | 1 76 | 1 06 | 707 | 6· |
| 3½×3 | ¹¹⁄₁₆ | 13.6 | 4 00 | 94 | 2 96 | 1 44 | 86 | 1 19 | 4 41 | 1 91 | 1 05 | 703 | 6· |
| 3½×3 | ¾ | 14.7 | 4 32 | .96 | 3 15 | 1 54 | 85 | 1 21 | 4 70 | 2 05 | 1 04 | 698 | 6· |
| 3½×3 | ¹³⁄₁₆ | 15 8 | 4 63 | 98 | 3 33 | 1 65 | 85 | 1 23 | 4 98 | 2 20 | 1 04 | 694 | 6, |
| 3½×3 | ⅞ | 16 8 | 4 93 | 1.00 | 3.50 | 1 75 | .84 | 1 25 | 5 24 | 2 33 | 1 03 | 689 | 6, |
| 4×3 | ⁵⁄₁₆ | 7 2 | 2 09 | 76 | 1 65 | 73 | .89 | 1.26 | 3 38 | 1 23 | 1 27 | 554 | 6. |
| 4×3 | ⅜ | 8.5 | 2 49 | .78 | 1 92 | 87 | 88 | 1 28 | 3 96 | 1 46 | 1 26 | 551 | 6. |
| 4×3 | ⁷⁄₁₆ | 9 8 | 2 88 | 80 | 2.18 | 99 | 87 | 1.30 | 4.52 | 1 68 | 1 25 | 547 | 6 |
| 4×3 | ½ | 11 1 | 3 25 | 83 | 2 42 | 1 12 | 86 | 1 33 | 5 05 | 1 89 | 1 25 | .543 | 6 |
| 4×3 | ⁹⁄₁₆ | 12 4 | 3 63 | .85 | 2 66 | 1 23 | 86 | 1 35 | 5 55 | 2 09 | 1 24 | 538 | 6 |
| 4×3 | ⅝ | 13 6 | 3 99 | .87 | 2.87 | 1 35 | 85 | 1.37 | 6 03 | 2 30 | 1 23 | 534 | 6 |
| 4×3 | ¹¹⁄₁₆ | 14.8 | 4.34 | .89 | 3 08 | 1 46 | 84 | 1 39 | 6 49 | 2 49 | 1 22 | 529 | .6 |
| 4×3 | ¾ | 16.0 | 4 69 | .92 | 3 28 | 1 57 | 84 | 1.42 | 6 93 | 2 68 | 1 22 | 524 | 6 |
| 4×3 | ¹³⁄₁₆ | 17.1 | 5 03 | 94 | 3.47 | 1 68 | 83 | 1 44 | 7 35 | 2.87 | 1 21 | 518 | 6 |
| 4×3 | ⅞ | 18.3 | 5.36 | .96 | 3.66 | 1 79 | 83 | 1 46 | 7 75 | 3 05 | 1 20 | 512 | 6 |

TABLE VI—(*Continued*)



| Inches | Thickness Inches | Weight per Foot Pounds | Area of Section Square Inches | Distance of Center of Gravity From Back of Longer Leg Inches | Moment of Inertia, Axis 1-1 Inches⁴ | Section Modulus, Axis 1-1 Inches³ | Radius of Gyration, Axis 1-1 Inches | Distance of Center of Gravity From Back of Shorter Leg Inches | Moment of Inertia, Axis 2-2 Inches⁴ | Section Modulus, Axis 2-2 Inches³ | Radius of Gyration, Axis 2-2 Inches | Tangent of Angle | Least Radius of Gyration, Axis 3-3 Inches |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 |
| $a$ | $t$ | | $A$ | $x$ | $I$ | $S$ | $r$ | $x'$ | $I'$ | $S'$ | $r'$ | $\alpha$ | $r''$ |
| 3 | 5/16 | 8 2 | 2.41 | 68 | 1 75 | .75 | 85 | 1.68 | 6 26 | 1 89 | 1 61 | 368 | .66 |
| 3 | 3/8 | 9 8 | 2.86 | 70 | 2 04 | .89 | 84 | 1 70 | 7 37 | 2 24 | 1.61 | 364 | 65 |
| 3 | 7/16 | 11 3 | 3.31 | 73 | 2 32 | 1 02 | 84 | 1.73 | 8 43 | 2 58 | 1 60 | 361 | 65 |
| 3 | 1/2 | 12 8 | 3 75 | 75 | 2 58 | 1 15 | .83 | 1 75 | 9 45 | 2 91 | 1 59 | 357 | 65 |
| 3 | 9/16 | 14 3 | 4 19 | 77 | 2 83 | 1.27 | 82 | 1 77 | 10 43 | 3 23 | 1 58 | .353 | 65 |
| 3 | 5/8 | 15 7 | 4 61 | 80 | 3.06 | 1.39 | 82 | 1 80 | 11 37 | 3 55 | 1 57 | .349 | 65 |
| 3 | 11/16 | 17 1 | 5 03 | 82 | 3.29 | 1 51 | .81 | 1 82 | 12.28 | 3 86 | 1 56 | .345 | 64 |
| 3 | 3/4 | 18 5 | 5 44 | .84 | 3.51 | 1 62 | .80 | 1 84 | 13.15 | 4.16 | 1 55 | 340 | 64 |
| 3 | 13/16 | 19 9 | 5 84 | 86 | 3 71 | 1 74 | 80 | 1.86 | 13.98 | 4 46 | 1.55 | 336 | 64 |
| 3 | 7/8 | 21.2 | 6 24 | 88 | 3 91 | 1.85 | 79 | 1 88 | 14.78 | 4.75 | 1 54 | 331 | 64 |
| 3½ | 5/16 | 8 7 | 2 56 | .84 | 2 72 | 1 02 | 1.03 | 1 59 | 6 60 | 1.94 | 1 61 | 489 | 77 |
| 3½ | 3/8 | 10 4 | 3 05 | 86 | 3.18 | 1 21 | 1 02 | 1 61 | 7.78 | 2 29 | 1 60 | 485 | 76 |
| 3½ | 7/16 | 12 0 | 3 53 | 88 | 3 63 | 1 39 | 1 01 | 1.63 | 8 90 | 2 64 | 1.59 | 482 | .76 |
| 3½ | 1/2 | 13 6 | 4 00 | 91 | 4 05 | 1 56 | 1.01 | 1 66 | 9 99 | 2 99 | 1 58 | 479 | 75 |
| 3½ | 9/16 | 15 2 | 4 47 | 93 | 4 45 | 1.73 | 1.00 | 1 68 | 11 03 | 3 32 | 1 57 | 476 | .75 |
| 3½ | 5/8 | 16.8 | 4 93 | 95 | 4 83 | 1 90 | 99 | 1 70 | 12 03 | 3 65 | 1 56 | .472 | .75 |
| 3½ | 11/16 | 18 3 | 5 38 | 97 | 5.20 | 2 06 | 98 | 1.72 | 12 99 | 3 97 | 1 56 | .468 | 75 |
| 3½ | 3/4 | 19.8 | 5 82 | 1 00 | 5 55 | 2 22 | 98 | 1 75 | 13 92 | 4 28 | 1 55 | 464 | 75 |
| 3½ | 13/16 | 21.3 | 6 25 | 1.02 | 5 89 | 2.37 | 97 | 1 77 | 14 81 | 4 58 | 1 54 | .460 | 75 |
| 3½ | 7/8 | 22 7 | 6 68 | 1.04 | 6 21 | 2 52 | 96 | 1.79 | 15 67 | 4 88 | 1 53 | .455 | 75 |
| 3½ | 15/16 | 24 2 | 7 09 | 1 06 | 6 52 | 2 67 | 96 | 1 81 | 16 49 | 5.17 | 1 53 | 451 | .75 |
| 3½ | 3/8 | 11 7 | 3 43 | 79 | 3.34 | 1.23 | .99 | 2 04 | 12 86 | 3 24 | 1 94 | 350 | .77 |
| 3½ | 7/16 | 13 5 | 3 97 | 81 | 3.81 | 1 41 | 98 | 2 06 | 14.76 | 3 75 | 1 93 | 347 | 76 |
| 3½ | 1/2 | 15.3 | 4 50 | 83 | 4.25 | 1.59 | 97 | 2.08 | 16 59 | 4 24 | 1.92 | 344 | 76 |
| 3½ | 9/16 | 17 1 | 5 03 | 86 | 4 67 | 1 77 | 96 | 2.11 | 18.37 | 4.72 | 1.91 | .341 | 75 |
| 3½ | 5/8 | 18 9 | 5.55 | 88 | 5 08 | 1.94 | 96 | 2.13 | 20 08 | 5 19 | 1 90 | 338 | 75 |
| 3½ | 11/16 | 20.6 | 6 06 | 90 | 5 47 | 2 11 | 95 | 2 15 | 21.74 | 5.65 | 1 89 | 334 | 75 |
| 3½ | 3/4 | 22 4 | 6 57 | 93 | 5 84 | 2 27 | 94 | 2 18 | 23 34 | 6 10 | 1 89 | 331 | .75 |
| 3½ | 13/16 | 24 0 | 7 06 | .95 | 6 20 | 2 43 | 94 | 2.20 | 24 89 | 6.55 | 1 88 | 327 | 75 |
| 3½ | 7/8 | 25.7 | 7 55 | .97 | 6 53 | 2 59 | 93 | 2.22 | 26 39 | 6 98 | 1 87 | .323 | 75 |
| 3½ | 15/16 | 27 3 | 8 03 | 99 | 6 88 | 2 74 | 93 | 2 24 | 27 84 | 7 41 | 1 86 | 320 | .75 |
| 3½ | 1 | 28 9 | 8 50 | 1 01 | 7.21 | 2 90 | 92 | 2 26 | 29.15 | 7 80 | 1 85 | 317 | .75 |
| 4 | 3/8 | 12.3 | 3 61 | 94 | 4 90 | 1 60 | 1 17 | 1 94 | 13 47 | 3 32 | 1 93 | .446 | .88 |
| 4 | 7/16 | 14 3 | 4 19 | 96 | 5 60 | 1.85 | 1.16 | 1.96 | 15 46 | 3.83 | 1 92 | 443 | .87 |
| 4 | 1/2 | 16 2 | 4 75 | 99 | 6 27 | 2 08 | 1 15 | 1 99 | 17.40 | 4 33 | 1 91 | 440 | .87 |
| 4 | 9/16 | 18 1 | 5 31 | 1 01 | 6 91 | 2 31 | 1 14 | 2 01 | 19 26 | 4.83 | 1 91 | 438 | .87 |
| 4 | 5/8 | 20 0 | 5 86 | 1 03 | 7.52 | 2 54 | 1 13 | 2.03 | 21 07 | 5 31 | 1 90 | 434 | 86 |
| 4 | 11/16 | 21 8 | 6 41 | 1 06 | 8 11 | 2 76 | 1 13 | 2 06 | 22 82 | 5.78 | 1 89 | 431 | .86 |
| 4 | 3/4 | 23 6 | 6 94 | 1 08 | 8 68 | 2 97 | 1 12 | 2 08 | 24 51 | 6 25 | 1 88 | 428 | .86 |
| 4 | 13/16 | 25 4 | 7 47 | 1 10 | 9 23 | 3 18 | 1 11 | 2 10 | 26 15 | 6 70 | 1 87 | 425 | 86 |
| 4 | 7/8 | 27 2 | 7 99 | 1 12 | 9 75 | 3 39 | 1 11 | 2 12 | 27 73 | 7 15 | 1 86 | .421 | .86 |
| 4 | 15/16 | 28 9 | 8 50 | 1 14 | 10 26 | 3 59 | 1 10 | 2 14 | 29 26 | 7 59 | 1 86 | 418 | .86 |
| 4 | 1 | 30 6 | 9 00 | 1 17 | 10 75 | 3 79 | 1 09 | 2 17 | 30 75 | 8 02 | 1 85 | 414 | .86 |

## TABLE VII
### PROPERTIES OF Z BARS



| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Depth of Bar, Inches | Length of Legs, Inches | Thickness of Web and Legs, Inches | Weight per Foot, Pounds | Area of Section, Square Inches | Moment of Inertia, Axis 1-1, Inches$^4$ | Section Modulus, Axis 1-1, Inches$^3$ | Radius of Gyration, Axis 1-1, Inches | Moment of Inertia, Axis 3-3, Inches$^4$ | Section Modulus, Axis 3-3, Inches$^3$ | Radius of Gyration, Axis 3-3, Inches | Tangent of Angle | Least Radius of Gyration, Axis 3-3, Inches | Coefficient of Deflection, Uniform Load | Coefficient of Deflection, Center Load |
| $b$ | $a$ | $t$ | | $A$ | $I$ | $S$ | $r$ | $I'$ | $S'$ | $r'$ | $a$ | $r''$ | $N$ | $N'$ |
| 3 | 2⅛ | ¼ | 6.7 | 1.97 | 2.87 | 1.92 | 1.21 | 2.81 | 1.10 | 1.19 | .986 | .55 | .000270 | .000432 |
| 3¹⁄₁₆ | 2¼ | ¼ | 8.4 | 2.48 | 3.64 | 2.38 | 1.21 | 3.64 | 1.40 | 1.21 | 1.000 | .55 | .000213 | .000341 |
| 3 | 2⅛ | ⁵⁄₁₆ | 9.7 | 2.86 | 3.85 | 2.57 | 1.16 | 3.93 | 1.57 | 1.17 | .990 | .54 | .000201 | .000322 |
| 3¹⁄₁₆ | 2¼ | ⅜ | 11.4 | 3.36 | 4.57 | 2.98 | 1.17 | 4.75 | 1.88 | 1.19 | .975 | .55 | .000170 | .000272 |
| 3 | 2⅛ | ⁵⁄₁₆ | 12.5 | 3.69 | 4.59 | 3.06 | 1.12 | 4.85 | 1.99 | 1.15 | .965 | .53 | .000169 | .000271 |
| 3¹⁄₁₆ | 2¼ | ⅜ | 14.2 | 4.18 | 5.26 | 3.43 | 1.12 | 5.68 | 2.30 | 1.17 | .951 | .54 | .000148 | .000236 |

| | | | 12.4 | 3.00 | 9.03 | 4.07 | 1.02 | 6.71 | 4.20 | 1.50 | .190 | .09 | .00000 | .00000 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 4⅞ | 3 1/16 | ⅞ | 13.8 | 4.05 | 9.66 | 4.83 | 1.54 | 6.73 | 2.37 | 1.29 | .704 | .66 | .000080 | .000129 |
| 4 | 3 1/16 | ⅞ | 15 8 | 4 66 | 11 18 | 5 50 | 1.55 | 7 96 | 2 77 | 1 31 | 804 | 67 | 000069 | 000111 |
| 4⅛ | 3⅛ | 1/16 | 17 9 | 5 27 | 12 74 | 6 18 | 1 55 | 9 26 | 3 19 | 1 32 | 814 | 68 | 000061 | 000098 |
| 4⅜ | 3 1/16 | ½ | 18 9 | 5 55 | 12 11 | 6 05 | 1 48 | 8 73 | 3 18 | 1.25 | 808 | 65 | 000064 | 000103 |
| 4 | 3⅛ | 15/16 | 20 9 | 6 14 | 13 52 | 6 65 | 1 48 | 9 95 | 3 58 | 1 27 | 818 | 67 | 000057 | 000092 |
| 4¼ | 3⅜ | ⅝ | 23 0 | 6 75 | 14 97 | 7 26 | 1 49 | 11 24 | 4 00 | 1 29 | 828 | 68 | 000052 | 000083 |
| 4⅞ | 3⅛ | 13/16 | 11 6 | 3 40 | 13 36 | 5 34 | 1 98 | 6 18 | 2 00 | 1.35 | 611 | 75 | 000058 | 000093 |
| 5 | 3¼ | ¼ | 13.9 | 4.10 | 16.18 | 6 39 | 1 99 | 7 65 | 2 45 | 1 37 | 619 | 76 | 000048 | 000077 |
| 5⅛ | 3⅛ | 5/16 | 16.4 | 4 81 | 19 07 | 7 44 | 1 99 | 9 20 | 2 92 | 1 38 | 628 | 76 | 000041 | 000065 |
| 5⅜ | 3⅜ | ¾ | 17 9 | 5 25 | 19 19 | 7 68 | 1 91 | 10 51 | 3 02 | 1 31 | 616 | 74 | 000036 | 000065 |
| 5 | 3⅜ | 7/16 | 20 2 | 5 94 | 21 83 | 8 62 | 1 92 | 12 06 | 3 47 | 1 33 | 623 | 75 | 000032 | 000057 |
| 5⅜ | 3⅛ | ½ | 22.6 | 6 64 | 24 53 | 9 57 | 1 92 | 11 37 | 3 94 | 1 35 | 631 | 76 | 000033 | 000051 |
| 5⅛ | 3⅝ | ⅝ | 23 7 | 6 96 | 23.68 | 9 47 | 1 84 | 12 83 | 3 91 | 1 28 | 619 | 73 | 000030 | 000052 |
| 5 | 3½ | 13/16 | 26 0 | 7 64 | 26 16 | 10 34 | 1 85 | 14 37 | 4 37 | 1 30 | 626 | 74 | 000027 | 000048 |
| 5⅛ | 3 1/16 | ¾ | 28.3 | 8 33 | 28.70 | 11 20 | 1 86 | 9 11 | 4 84 | 1 31 | 633 | 76 | 000031 | 000043 |
| 5⅝ | 3⅜ | 7/16 | 15.6 | 4 59 | 25 32 | 8 44 | 2 35 | 10 94 | 2 75 | 1 41 | 519 | 83 | 000026 | 000049 |
| 6 | 3⅜ | ⅞ | 18 3 | 5 39 | 29 80 | 9 83 | 2.35 | 12 87 | 3 27 | 1 43 | 526 | 83 | 000023 | 000042 |
| 6⅛ | 3⅜ | ⅜ | 21 0 | 6 19 | 34 36 | 11 22 | 2 36 | 12 59 | 3 81 | 1 44 | 532 | 84 | 000022 | 000036 |
| 6⅜ | 3⅜ | 7/16 | 22.7 | 6 68 | 34.64 | 11 55 | 2 28 | 14 41 | 3 91 | 1 37 | 520 | 81 | 000020 | 000036 |
| 6 | 3⅜ | 15/16 | 25.4 | 7 46 | 38 87 | 12 82 | 2 28 | 16 34 | 4 44 | 1 39 | 526 | 82 | 000018 | 000032 |
| 6⅛ | 3⅜ | ⅝ | 28 1 | 8 25 | 43 18 | 14 10 | 2 29 | 15 44 | 4 98 | 1 41 | 532 | 84 | 000018 | 000029 |
| 6 | 3⅜ | 13/16 | 29.3 | 8 63 | 42 12 | 14 04 | 2 21 | 17 27 | 4 94 | 1 34 | 519 | 81 | 000017 | 000030 |
| 6¼ | 3⅜ | ⅞ | 31 9 | 9 39 | 46 13 | 15 22 | 2 22 | 19 18 | 5 47 | 1 36 | 525 | 82 | 000015 | 000027 |
| 6⅞ | 3⅜ | ⅞ | 34 6 | 10 17 | 50 22 | 16 40 | 2 22 | | 6 02 | 1 37 | 530 | 83 | | 000025 |
| 7¼ | 3 | ⅜ | 16 3 | 4 78 | 38 19 | 10 18 | 2 83 | 5 59 | 1 99 | 1 08 | 29 | 72 | 000020 | 000033 |
| 8 | 3 | ½ | 22 1 | 6 50 | 56.54 | 14 14 | 2 95 | 7 01 | 2 55 | 1.04 | 26 | 71 | 000014 | 000022 |

# TABLE VIII
## PROPERTIES OF T BARS



EQUAL LEGS

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 18 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Width of Flange, Inches, b | Depth of Bar, Inches, d | Dimensions — Thickness of Flange, Inches, s to s' | Thickness of Stem, Inches, t to t' | Weight per Foot, Pounds | Area of Section, Square Inches, A | Distance of Center of Gravity From Outside of Flange, Inches, x | Moment of Inertia, Axis 1–1, Inches⁴, I | Section Modulus, Axis 1–1, Inches³, S | Radius of Gyration, Axis 1–1, Inches, r | Moment of Inertia, Axis 2–2, Inches⁴, I' | Section Modulus, Axis 2–2, Inches³, S' | Radius of Gyration, Axis 2–2, Inches, r' |
| 1 | 1 | ⅛ to $\frac{5}{32}$ | ⅛ to $\frac{5}{32}$ | 1.0 | .27 | .29 | .02 | .03 | .30 | .01 | .02 | .21 |
| 1⅛ | 1⅛ | $\frac{3}{16}$ to $\frac{7}{32}$ | $\frac{3}{32}$ to $\frac{7}{32}$ | 1.4 | .41 | .33 | .04 | .05 | .32 | .02 | .04 | .25 |
| 1$\frac{3}{16}$ | 1$\frac{3}{16}$ | $\frac{3}{16}$ to ¼ | $\frac{3}{32}$ to $\frac{7}{32}$ | 1.6 | .45 | .34 | .05 | .06 | .33 | .03 | .05 | .26 |
| 1¼ | 1¼ | $\frac{3}{16}$ to ¼ | $\frac{3}{16}$ to ¼ | 1.7 | .48 | .36 | .06 | .07 | .35 | .03 | .05 | .27 |
| 1⅜ | 1⅜ | $\frac{3}{16}$ to ¼ | $\frac{3}{32}$ to ¼ | 1.9 | .55 | .39 | .08 | .08 | .39 | .05 | .07 | .29 |

| | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 2 | 2 | ¼ to 5⁄16 | ¼ to 5⁄16 | 3 7 | 1 07 | 59 | .37 | .26 | 59 | 18 | 18 | .18 | .42 |
| 2 | 2 | 5⁄16 to ⅜ | 5⁄16 to ⅜ | 4 4 | 1 28 | 61 | 43 | 31 | 59 | 23 | 23 | .23 | .42 |
| 2¼ | 2¼ | ¼ to 5⁄16 | ¼ to 5⁄16 | 4 2 | 1 21 | 68 | 51 | .32 | 65 | 24 | 21 | 45 | |
| 2¼ | 2¼ | 5⁄16 to ⅜ | 5⁄16 to ⅜ | 5 0 | 1 46 | 67 | .64 | .40 | 66 | 32 | 29 | 47 | |
| 2¼ | 2¾ | 5⁄16 to ⅜ | 5⁄16 to ⅜ | 5 6 | 1 63 | .73 | .87 | 49 | 74 | .44 | .35 | .52 | |
| 3 | 3 | ⅜ to 7⁄16 | 5⁄16 to ⅜ | 6.8 | 1 99 | .86 | 1 58 | .74 | 90 | .75 | .50 | .62 | |
| 3 | 3 | 7⁄16 to ½ | ⅜ to 7⁄16 | 7 9 | 2 31 | 88 | 1 82 | 86 | 90 | 92 | .61 | .64 | |
| 3 | 3 | ½ to 9⁄16 | 7⁄16 to ½ | 10 1 | 2 96 | 93 | 2 27 | 1 10 | 88 | 1.20 | 80 | .64 | |
| 3½ | 3½ | ⅜ to 7⁄16 | ⅜ to 7⁄16 | 9 3 | 2 74 | 99 | 3 10 | 1 23 | 1 08 | 1 42 | .81 | .73 | |
| 4 | 4 | ⅜ to 7⁄16 | ⅜ to 7⁄16 | 10 9 | 3 19 | 1 12 | 4 54 | 1 58 | 1 21 | 2 11 | 1 06 | .83 | |

UNEQUAL LEGS

| | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1½ | 1 1⁄16 | ⅜ to ¼ | 3⅝ to ¾ | 1 5 | 44 | 29 | .04 | .05 | 29 | .03 | 01 | .28 | |
| 2¼ | 1¾ | 5⁄16 to ⅜ | 1⁄16 to ⅛ | 3 0 | 86 | .30 | .08 | 09 | 31 | 28 | 22 | .58 | |
| 2¼ | 1⅞ | 11⁄16 to ⅞ | ¼ to ⅜ | 4 5 | 1.31 | 43 | .21 | 16 | 40 | 47 | 38 | .60 | |
| 2¼ | 3 | ⅜ to 9⁄16 | ⅜ to ⅞ | 7 2 | 2 10 | 95 | 1 72 | 84 | 91 | 53 | 42 | .50 | |
| 2¼ | 3 | 5⁄16 to ⅝ | ¾ | 7 5 | 2 21 | .74 | 83 | 66 | 62 | .64 | 46 | .54 | |
| 3 | 2¼ | ¾ to ⅞ | ⅜ to 7⁄16 | 7 2 | 2 11 | 71 | 1 08 | 60 | 64 | .90 | .60 | .66 | |
| 3½ | 4 | ⅜ to ⅝ | 7⁄16 to ⅞ | 9 3 | 2 74 | 1.27 | 4 12 | 1 51 | 1.24 | .90 | .60 | .58 | |
| 3½ | 4 | 7⁄16 to ⅝ | ⅜ to ⅞ | 10 0 | 2 94 | 1 20 | 4 33 | 1 54 | 1 23 | 1 42 | 81 | .70 | |
| 4½ | 2¼ | ⅜ to ⅞ | 5⁄16 to ⅞ | 8 0 | 2 29 | 57 | 1 04 | 54 | 68 | 2 51 | 1 12 | 1 05 | |
| 4½ | 3⅜ | 11⁄16 to ⅞ | 1⅜ to ¾ | 14 9 | 4 37 | 1 09 | 4 89 | 2 03 | 1 06 | 3 68 | 1 64 | 92 | |
| 4½ | 3¾ | 11⁄16 to ⅞ | 1⅝ to ⅞ | 15 9 | 4.65 | 1.11 | 5 08 | 2.13 | 1 05 | 3 73 | 1 66 | .90 | |
| 5 | 3 | ½ to 11⁄16 | 13⁄16 to ⅞ | 13.6 | 3 99 | .72 | 2 42 | 1 06 | 78 | 5 42 | 2 17 | 1.17 | |

## TABLE IX

### PROPERTIES AND PRINCIPAL DIMENSIONS OF STANDARD T RAILS
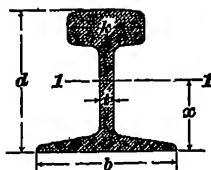


| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|
| | | | | | | | Axis $1-1$ | |
| Weight per Yard Pounds | Area Square Inches | $b$ Inches | $d$ Inches | $h$ Inches | $t$ Inches | $x$ Inches | Moment of Inertia | Section Modulus |
| | | | | | | | $I$ | $S$ |
| 8 | 78 | $1\tfrac12$ | $1\tfrac12$ | $\tfrac{13}{16}$ | $\tfrac{5}{32}$ | 75 | 23 | 31 |
| 12 | 1 18 | $1\tfrac78$ | $1\tfrac78$ | $1\tfrac{1}{10}$ | $\tfrac{3}{16}$ | 92 | 55 | 58 |
| 16 | 1 57 | $2\tfrac14$ | $2\tfrac14$ | $1\tfrac14$ | $\tfrac{15}{32}$ | 1 10 | 1 1 | 95 |
| 20 | 2 00 | $2\tfrac12$ | $2\tfrac12$ | $1\tfrac38$ | $\tfrac{17}{32}$ | 1 2 | 1 7 | 1 3 |
| 25 | 2 5 | $2\tfrac34$ | $2\tfrac34$ | $1\tfrac12$ | $\tfrac{19}{32}$ | 1 3 | 2 6 | 1 8 |
| 30 | 2 9 | 3 | 3 | $1\tfrac58$ | $\tfrac{21}{32}$ | 1 4 | 3 6 | 2 3 |
| 35 | 3 4 | $3\tfrac14$ | $3\tfrac14$ | $1\tfrac34$ | $\tfrac{23}{32}$ | 1 6 | 4 9 | 2 9 |
| 40 | 3 9 | $3\tfrac12$ | $3\tfrac12$ | $1\tfrac78$ | $\tfrac{25}{32}$ | 1 7 | 6 6 | 3 6 |
| 45 | 4 4 | $3\tfrac{11}{16}$ | $3\tfrac{11}{16}$ | 2 | $\tfrac{27}{32}$ | 1 8 | 8 1 | 4 2 |
| 50 | 4 9 | $3\tfrac78$ | $3\tfrac78$ | $2\tfrac18$ | $\tfrac{7}{16}$ | 1 9 | 9 8 | 4 9 |
| 55 | 5 4 | $4\tfrac{1}{16}$ | $4\tfrac{1}{16}$ | $2\tfrac14$ | $\tfrac{13}{32}$ | 2 0 | 12 2 | 5 9 |
| 60 | 5 9 | $4\tfrac14$ | $4\tfrac14$ | $2\tfrac38$ | $\tfrac{27}{32}$ | 2 1 | 14 7 | 6 7 |
| 65 | 6 4 | $4\tfrac{7}{16}$ | $4\tfrac{7}{16}$ | $2\tfrac{13}{32}$ | $\tfrac12$ | 2 2 | 17 0 | 7 4 |
| 70 | 6 9 | $4\tfrac58$ | $4\tfrac58$ | $2\tfrac{7}{16}$ | $\tfrac{33}{64}$ | 2 2 | 20 0 | 8 4 |
| 75 | 7 4 | $4\tfrac{13}{16}$ | $4\tfrac{13}{16}$ | $2\tfrac{15}{32}$ | $\tfrac{17}{32}$ | 2 3 | 23 0 | 9 1 |
| 80 | 7 8 | 5 | 5 | $2\tfrac12$ | $\tfrac{35}{64}$ | 2 4 | 26 7 | 10 1 |
| 85 | 8 3 | $5\tfrac{3}{16}$ | $5\tfrac{3}{16}$ | $2\tfrac{9}{16}$ | $\tfrac{9}{16}$ | 2 5 | 30 5 | 11 2 |
| 90 | 8 8 | $5\tfrac38$ | $5\tfrac38$ | $2\tfrac58$ | $\tfrac{9}{16}$ | 2 6 | 34 4 | 12 3 |
| 95 | 9 3 | $5\tfrac{9}{16}$ | $5\tfrac{9}{16}$ | $2\tfrac{11}{16}$ | $\tfrac{9}{16}$ | 2 7 | 38 6 | 13 3 |
| 100 | 9 8 | $5\tfrac34$ | $5\tfrac34$ | $2\tfrac34$ | $\tfrac{9}{16}$ | 2 8 | 43 4 | 14 7 |
| 150 | 14.7 | 6 | 6 | $4\tfrac14$ | 1 | 3 0 | 69 3 | 23 1 |

# TABLE X

## MOMENT OF INERTIA OF RECTANGULAR SECTIONS



| Depth Inches | Width of Rectangle   Inches | | | | | | |
|---|---|---|---|---|---|---|---|
| | ¼ | 5⁄16 | ⅜ | 7⁄16 | ½ | 9⁄16 | ⅝ |
| 2 | 17 | .21 | .25 | 29 | .33 | 38 | .42 |
| 3 | 56 | .70 | .84 | 98 | 1 13 | 1.27 | 1 41 |
| 4 | 1 33 | 1 67 | 2 00 | 2 33 | 2 67 | 3 00 | 3 33 |
| 5 | 2 60 | 3 26 | 3 91 | 4 56 | 5.21 | 5 86 | 6 51 |
| 6 | 4 50 | 5 63 | 6 75 | 7 88 | 9 00 | 10 13 | 11 25 |
| 7 | 7 15 | 8 93 | 10 72 | 12 51 | 14 29 | 16 08 | 17 86 |
| 8 | 10 67 | 13 33 | 16 00 | 18 67 | 21.33 | 24.00 | 26 67 |
| 9 | 15 19 | 18 98 | 22 78 | 26 58 | 30 38 | 34 17 | 37 97 |
| 10 | 20 83 | 26 04 | 31.25 | 36 46 | 41.67 | 46 87 | 52 08 |
| 11 | 27 73 | 34 66 | 41 59 | 48 53 | 55.46 | 62 39 | 69.32 |
| 12 | 36 00 | 45 00 | 54 00 | 63 00 | 72 00 | 81 00 | 90 00 |
| 13 | 45 77 | 57 21 | 68 66 | 80 10 | 91 54 | 102 98 | 114.43 |
| 14 | 57.17 | 71 46 | 85 75 | 100 04 | 114.33 | 128 63 | 142 92 |
| 15 | 70 31 | 87 89 | 105.47 | 123 05 | 140.63 | 158 20 | 175 78 |
| 16 | 85 33 | 106 67 | 128 00 | 149 33 | 170.67 | 192 00 | 213 33 |
| 17 | 102 35 | 127 94 | 153 53 | 179 12 | 204.71 | 230 30 | 255 89 |
| 18 | 121 50 | 151 88 | 182 25 | 212 63 | 243 00 | 273 38 | 303 75 |
| 19 | 142 90 | 178 62 | 214 34 | 250 07 | 285 79 | 321 52 | 357 24 |
| 20 | 166 67 | 208 33 | 250 00 | 291 67 | 333.33 | 375 00 | 416 67 |
| 21 | 192 94 | 241 17 | 289 41 | 337 64 | 385.88 | 434 11 | 482 34 |
| 22 | 221 83 | 277 29 | 332 75 | 388 21 | 443.67 | 499 13 | 554.58 |
| 23 | 253 48 | 316 85 | 380 22 | 443 59 | 506 96 | 570 33 | 633 70 |
| 24 | 288 00 | 360 00 | 432 00 | 504 00 | 576 00 | 648 00 | 720 00 |
| 25 | 325 52 | 406 90 | 488 28 | 569 66 | 651.04 | 732 42 | 813 80 |
| 26 | 366 17 | 457 71 | 549 25 | 640 79 | 732 33 | 823 88 | 915 42 |
| 27 | 410 06 | 512 58 | 615 09 | 717 61 | 820 13 | 922 64 | 1,025 16 |
| 28 | 457 33 | 571 67 | 686 00 | 800 33 | 914 67 | 1,029 00 | 1,143 33 |
| 29 | 508 10 | 635 13 | 762 16 | 889 18 | 1,016 21 | 1,143 23 | 1,270 26 |
| 30 | 562 50 | 703 13 | 843 75 | 984 38 | 1,125 00 | 1,265 63 | 1,406 25 |
| 32 | 682 67 | 853 35 | 1,024 00 | 1,194 67 | 1,365 33 | 1 536.00 | 1,706 67 |
| 34 | 818 83 | 1,023 54 | 1,228 25 | 1,432 96 | 1,637 67 | 1,842 38 | 2,047 08 |
| 36 | 972 00 | 1,215 00 | 1,458 00 | 1,701 00 | 1,944 00 | 2,187 00 | 2,430 00 |
| 38 | 1,143 17 | 1,428 96 | 1,714 75 | 2,000 54 | 2,286 33 | 2,572.13 | 2,857 92 |
| 40 | 1,333 33 | 1,666 67 | 2,000 00 | 2,333 33 | 2,666 67 | 3,000 00 | 3,333 33 |
| 42 | 1,543 50 | 1,929 38 | 2,315 25 | 2,701 13 | 3,087 00 | 3,472 88 | 3,858 75 |
| 44 | 1,774 67 | 2,218 33 | 2,662 00 | 3,105 67 | 3,549 33 | 3,993 00 | 4,436 67 |
| 46 | 2,027 83 | 2,534 79 | 3,041 75 | 3,548 71 | 4,055 67 | 4,562 63 | 5,069 58 |
| 48 | 2,304 00 | 2,880 00 | 3,456 00 | 4,032 00 | 4,608 00 | 5,184 00 | 5,760 00 |
| 50 | 2,604 17 | 3,255 21 | 3,906 25 | 4,557 29 | 5,208 33 | 5,859 38 | 6,510 42 |
| 52 | 2,929 33 | 3,661 67 | 4,394 00 | 5,126 33 | 5,858 67 | 6,591 00 | 7,323 33 |
| 54 | 3,280 50 | 4,100 63 | 4,920 75 | 5,740 88 | 6,561 00 | 7,381 13 | 8,201 25 |
| 56 | 3,658 67 | 4,573 33 | 5,488 00 | 6,402 67 | 7,317.33 | 8,232 00 | 9,146 67 |
| 58 | 4,064 83 | 5,081 04 | 6,097 25 | 7,113 46 | 8,129 67 | 9,145 87 | 10,162 08 |
| 60 | 4,500 00 | 5,625 00 | 6,750 00 | 7,875 00 | 9,000 00 | 10,125 00 | 11,250.00 |

**TABLE X**—(*Continued*)

$v$——$v$
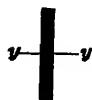
| Depth Inches | Width of Rectangle   Inches | | | | | |
|---|---|---|---|---|---|---|
| | $\frac{11}{16}$ | $\frac{3}{4}$ | $\frac{13}{16}$ | $\frac{7}{8}$ | $\frac{15}{16}$ | 1 |
| 2 | .46 | .50 | .54 | 58 | 63 | 67 |
| 3 | 1 55 | 1 69 | 1 83 | 1 97 | 2 11 | 2 25 |
| 4 | 3 67 | 4 00 | 4 33 | 4 67 | 5 00 | 5 33 |
| 5 | 7 16 | 7 81 | 8 46 | 9 11 | 9 77 | 10 42 |
| 6 | 12.38 | 13 50 | 14.63 | 15 75 | 16 88 | 18 00 |
| 7 | 19 65 | 21 44 | 23.22 | 25 01 | 26 80 | 28 58 |
| 8 | 29 33 | 32 00 | 34 67 | 37 33 | 40 00 | 42 67 |
| 9 | 41 77 | 45 56 | 49 36 | 53 16 | 56 95 | 60 75 |
| 10 | 57 29 | 62 50 | 67 71 | 72 92 | 78 13 | 83 33 |
| 11 | 76 26 | 83.19 | 90 12 | 97 05 | 103 98 | 110 92 |
| 12 | 99 00 | 108.00 | 117 00 | 126 00 | 135 00 | 144 00 |
| 13 | 125 87 | 137.31 | 148 75 | 160 20 | 171 64 | 183 08 |
| 14 | 157.21 | 171 50 | 185 79 | 200 08 | 214 38 | 228 67 |
| 15 | 193 36 | 210 94 | 228 52 | 246.09 | 263 67 | 281 25 |
| 16 | 234 67 | 256 00 | 277 33 | 298 67 | 320 00 | 341 33 |
| 17 | 281 47 | 307 06 | 332 65 | 358 24 | 383 83 | 409 42 |
| 18 | 334 13 | 364.50 | 394.88 | 425 25 | 455 63 | 486 00 |
| 19 | 392 96 | 428.69 | 464 41 | 500 14 | 535 86 | 571.58 |
| 20 | 458 33 | 500.00 | 541.67 | 583 33 | 625 00 | 666 67 |
| 21 | 530 58 | 578 81 | 627 05 | 675.28 | 723 52 | 771 75 |
| 22 | 610 04 | 655 50 | 720 96 | 776.42 | 831 87 | 887 33 |
| 23 | 697 07 | 760 44 | 823 81 | 887 18 | 950 55 | 1,013.92 |
| 24 | 792 00 | 864 00 | 936 00 | 1,008 00 | 1,800 00 | 1,152 00 |
| 25 | 895.18 | 976 56 | 1,057 94 | 1,139 32 | 1,220 70 | 1,302 08 |
| 26 | 1,006 96 | 1,098 50 | 1,190 04 | 1,281 58 | 1,373 13 | 1,464 67 |
| 27 | 1,127 67 | 1,230.19 | 1,332 70 | 1,435 22 | 1,537 73 | 1,640 25 |
| 28 | 1,257 67 | 1,372 00 | 1,486 33 | 1,600.67 | 1,715 00 | 1,829 33 |
| 29 | 1,397 29 | 1,524 31 | 1,651 34 | 1,778.36 | 1,905 39 | 2,032 42 |
| 30 | 1,546.88 | 1,687 50 | 1,828 13 | 1,968 75 | 2,109 38 | 2,250 00 |
| 32 | 1,877 33 | 2,048 00 | 2,218 67 | 2,380 33 | 2,560 00 | 2,730 67 |
| 34 | 2,251 79 | 2,456.50 | 2,661 21 | 2,865 92 | 3,070 63 | 3,275 33 |
| 36 | 2,673.00 | 2,916 00 | 3,159 00 | 3,402 00 | 3,645 00 | 3,888.00 |
| 38 | 3,143.71 | 3,429 50 | 3,715 29 | 4,001 08 | 4,286 88 | 4,572 67 |
| 40 | 3,666 67 | 4,000 00 | 4,333 33 | 4,666 67 | 5,000 00 | 5,333 33 |
| 42 | 4,244 63 | 4,630 50 | 5,016 38 | 5,402 25 | 5,788.13 | 6,174 00 |
| 44 | 4,880 33 | 5,324 00 | 5,767 67 | 6,211.33 | 6,655 00 | 7,098.67 |
| 46 | 5,576.54 | 6,083.50 | 6,590 46 | 7,097.42 | 7,604.38 | 8,111 33 |
| 48 | 6,336.00 | 6,912.00 | 7,488 00 | 8,064 00 | 8,640 00 | 9,216 00 |
| 50 | 7,161 46 | 7,812 50 | 8,463 54 | 9,114 58 | 9,765 63 | 10,416 67 |
| 52 | 8,055.67 | 8,788 00 | 9,520 33 | 10,252 67 | 10,985 00 | 11,717 33 |
| 54 | 9,021 38 | 9,841.50 | 10,661 63 | 11,481 75 | 12,301 88 | 13,122 00 |
| 56 | 10,061 33 | 10,976 00 | 11,890 67 | 12,805.33 | 13,720 00 | 14,634 67 |
| 58 | 11,178.29 | 12,194.50 | 13,210.71 | 14,226 92 | 15,243 12 | 16,259 33 |
| 60 | 12,375 00 | 13,500 00 | 14,625 00 | 15,750.00 | 16,875 00 | 18,000 00 |

## TABLE XI

### RADII OF GYRATION FOR TWO ANGLES, HAVING EQUAL LEGS, PLACED BACK TO BACK
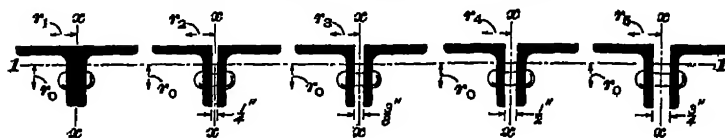


| Dimensions Inches | Thickness Inches | Area of Two Angles Square Inches | Radii of Gyration | | | | | |
|---|---|---|---|---|---|---|---|---|
| | | | $r_0$ | $r_1$ | $r_2$ | $r_3$ | $r_4$ | $r_5$ |
| 1½×1½ | 3/16 | 1 06 | 46 | .64 | .73 | 78 | 83 | .94 |
| 1½×1½ | ¼ | 1.98 | 44 | 67 | .77 | .82 | .88 | .99 |
| 1¾×1¾ | 3/16 | 1 26 | 54 | .74 | .83 | 88 | 93 | 1 03 |
| 1¾×1¾ | 7/16 | 2 68 | 51 | .78 | 88 | .93 | .98 | 1.09 |
| 2×2 | 3/16 | 1.44 | .62 | .84 | .93 | .98 | 1.03 | 1.13 |
| 2×2 | 5/16 | 2 32 | 60 | 86 | 95 | 1 00 | 1.05 | 1.16 |
| 2×2 | 7/16 | 3 12 | 59 | 88 | 98 | 1.03 | 1.08 | 1 19 |
| 2½×2½ | ¼ | 2.38 | .77 | 1 05 | 1.14 | 1 19 | 1 24 | 1.34 |
| 2½×2½ | ⅜ | 3 48 | .75 | 1 07 | 1 16 | 1.21 | 1.26 | 1 36 |
| 2½×2½ | ½ | 4 50 | 74 | 1.09 | 1.19 | 1 24 | 1 29 | 1 39 |
| 3×3 | ¼ | 2 88 | .93 | 1.26 | 1 34 | 1.39 | 1 43 | 1.53 |
| 3×3 | 7/16 | 4 88 | 91 | 1 28 | 1 37 | 1.42 | 1 47 | 1.57 |
| 3×3 | ⅝ | 6 72 | 88 | 1.32 | 1.41 | 1 46 | 1.51 | 1.61 |
| 3½×3½ | ⅜ | 4 98 | 1 07 | 1 48 | 1.56 | 1 61 | 1.66 | 1 75 |
| 3½×3½ | ⅝ | 7 98 | 1 04 | 1 52 | 1 61 | 1 66 | 1 71 | 1 81 |
| 3½×3½ | 13/16 | 10 06 | 1 02 | 1.55 | 1 65 | 1 70 | 1.75 | 1 85 |
| 4×4 | 5/16 | 4 82 | 1 24 | 1 67 | 1 76 | 1 80 | 1 85 | 1 94 |
| 4×4 | 9/16 | 8.38 | 1 21 | 1 71 | 1.80 | 1.85 | 1 89 | 1 99 |
| 4×4 | 13/16 | 11 68 | 1 18 | 1 75 | 1 85 | 1 89 | 1.94 | 2 04 |
| 6×6 | 7/16 | 10 12 | 1 87 | 2 50 | 2 58 | 2.63 | 2.67 | 2.76 |
| 6×6 | ⅝ | 14 22 | 1 84 | 2 53 | 2 62 | 2 66 | 2 71 | 2.80 |
| 6×6 | ⅞ | 19 48 | 1 81 | 2.57 | 2 66 | 2 70 | 2 75 | 2 85 |
| 8×8 | ½ | 15 50 | 2 51 | 3 32 | 3.41 | 3.45 | 3 49 | 3 58 |
| 8×8 | ⅝ | 19 22 | 2.49 | 3 34 | 3 43 | 3.47 | 3.51 | 3 60 |
| 8×8 | ¾ | 22 88 | 2 47 | 3 36 | 3 44 | 3 49 | 3 53 | 3 62 |
| 8×8 | ⅞ | 26 47 | 2 45 | 3 38 | 3 46 | 3 51 | 3.55 | 3 64 |
| 8×8 | 1 | 30 00 | 2 44 | 3.40 | 3 48 | 3 53 | 3 57 | 3 67 |
| 8×8 | 1⅛ | 33 47 | 2 42 | 3 42 | 3 51 | 3 55 | 3 60 | 3 69 |

# TABLE XII

## RADII OF GYRATION FOR TWO ANGLES, HAVING UNEQUAL LEGS, PLACED BACK TO BACK
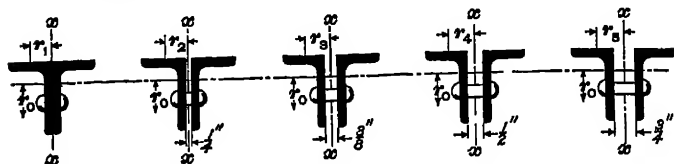


| Dimensions Inches | Thickness Inches | Area of Two Angles Square Inches | Radii of Gyration | | | | | |
|---|---|---|---|---|---|---|---|---|
| | | | $r_0$ | $r_1$ | $r_2$ | $r_3$ | $r_4$ | $r_5$ |
| 2½×2 | 3/16 | 1 62 | .79 | 79 | 88 | 92 | 97 | 1 07 |
| 2½×2 | 3/8 | 3 10 | .77 | .82 | 91 | 96 | 1 01 | 1 12 |
| 2½×2 | ½ | 4 00 | .75 | .84 | 94 | 99 | 1 04 | 1 15 |
| 3×2½ | ¼ | 2 64 | 95 | 1 00 | 1 09 | 1 13 | 1 18 | 1.28 |
| 3×2½ | ⅜ | 3 86 | 93 | 1 02 | 1 11 | 1 16 | 1 21 | 1 31 |
| 3×2½ | 9/16 | 5 56 | 91 | 1 05 | 1 15 | 1 20 | 1 25 | 1 35 |
| 3½×2½ | ¼ | 2 88 | 1 12 | .96 | 1 04 | 1 09 | 1 13 | 1 23 |
| 3½×2½ | ½ | 5 50 | 1.09 | 1 00 | 1 09 | 1 14 | 1 19 | 1 29 |
| 3½×2½ | 11/16 | 7 32 | 1 06 | 1 03 | 1 13 | 1 18 | 1 23 | 1 33 |
| 3½×3 | 5/16 | 3.88 | 1 10 | 1 21 | 1 30 | 1 35 | 1 39 | 1 49 |
| 3½×3 | 9/16 | 6 68 | 1.07 | 1 25 | 1 34 | 1 39 | 1 44 | 1 54 |
| 3½×3 | 13/16 | 9 26 | 1 04 | 1.30 | 1 40 | 1 45 | 1 50 | 1.60 |
| 4×3 | 5/16 | 4 18 | 1 27 | 1.17 | 1 25 | 1 30 | 1 34 | 1 44 |
| 4×3 | 9/16 | 7 26 | 1 24 | 1 21 | 1.30 | 1 34 | 1.39 | 1 49 |
| 4×3 | 13/16 | 10 06 | 1 21 | 1 25 | 1 35 | 1 40 | 1 45 | 1 55 |
| 5×3 | 5/16 | 4.82 | 1 61 | 1 09 | 1.17 | 1 22 | 1 26 | 1 36 |
| 5×3 | 9/16 | 8 38 | 1 58 | 1 13 | 1 22 | 1 26 | 1 31 | 1 41 |
| 5×3 | 13/16 | 11 68 | 1.55 | 1 17 | 1 27 | 1 32 | 1 37 | 1.47 |
| 5×3½ | ⅜ | 6 10 | 1 60 | 1 34 | 1 42 | 1.46 | 1 51 | 1 60 |
| 5×3½ | ⅝ | 9 86 | 1 56 | 1 37 | 1 46 | 1 51 | 1 56 | 1 66 |
| 5×3½ | ⅞ | 13 36 | 1 53 | 1 42 | 1 51 | 1 56 | 1 61 | 1 71 |
| 6×3½ | ⅜ | 6.86 | 1 94 | 1 26 | 1 34 | 1 39 | 1 43 | 1.53 |
| 6×3½ | ⅝ | 11 10 | 1 90 | 1 30 | 1 39 | 1 43 | 1 48 | 1 58 |
| 6×3½ | ⅞ | 15 10 | 1 87 | 1 34 | 1 44 | 1 49 | 1 53 | 1 64 |
| 6×4 | ⅜ | 7 22 | 1 93 | 1 50 | 1 58 | 1 62 | 1 67 | 1 76 |
| 6×4 | ⅝ | 11 72 | 1 90 | 1 53 | 1 62 | 1 67 | 1 71 | 1 81 |
| 6×4 | ⅞ | 15 98 | 1 86 | 1.58 | 1 67 | 1 71 | 1 76 | 1 86 |

## TABLE XIII

### RADII OF GYRATION FOR TWO ANGLES, HAVING UNEQUAL LEGS, PLACED BACK TO BACK
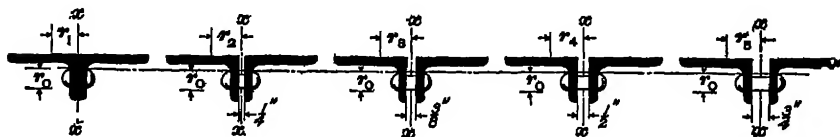


| Dimensions Inches | Thickness Inches | Area of Two Angles Square Inches | Radii of Gyration | | | | | |
|---|---|---|---|---|---|---|---|---|
| | | | $r_0$ | $r_1$ | $r_2$ | $r_3$ | $r_4$ | $r_5$ |
| 2½×2 | 3/16 | 1 62 | 60 | 1 10 | 1 19 | 1 24 | 1 29 | 1 39 |
| 2½×2 | ¼ | 3 10 | 58 | 1 13 | 1 23 | 1 28 | 1 33 | 1 43 |
| 2½×2 | ½ | 4 00 | 56 | 1 15 | 1.25 | 1 30 | 1 35 | 1.46 |
| 3×2½ | ¼ | 2 64 | 75 | 1 31 | 1 40 | 1 45 | 1 50 | 1.60 |
| 3×2½ | ⅜ | 3 86 | 74 | 1 33 | 1 42 | 1 47 | 1 52 | 1.63 |
| 3×2½ | 9/16 | 5 56 | 72 | 1 37 | 1 46 | 1 51 | 1 56 | 1 66 |
| 3½×2½ | ¼ | 2 88 | .74 | 1 58 | 1.67 | 1 72 | 1 76 | 1 86 |
| 3½×2½ | ½ | 5 50 | 70 | 1 62 | 1.72 | 1.77 | 1 81 | 1 92 |
| 3½×2½ | 11/16 | 7.32 | .69 | 1.66 | 1 75 | 1 80 | 1 86 | 1 96 |
| 3½×3 | 5/16 | 3 88 | 90 | 1 52 | 1 61 | 1.66 | 1.71 | 1.80 |
| 3½×3 | 9/16 | 6 68 | .87 | 1 57 | 1 66 | 1.71 | 1.76 | 1 86 |
| 3½×3 | 13/16 | 9 26 | 85 | 1 61 | 1 71 | 1 76 | 1 81 | 1 91 |
| 4×3 | 5/16 | 4 18 | .89 | 1 79 | 1 88 | 1 93 | 1 97 | 2 07 |
| 4×3 | 9/16 | 7 26 | 86 | 1 83 | 1 93 | 1 97 | 2 02 | 2 12 |
| 4×3 | 13/16 | 10 06 | 83 | 1 88 | 1 97 | 2 02 | 2.08 | 2 18 |
| 5×3 | 5/16 | 4 82 | 85 | 2.33 | 2 42 | 2 47 | 2 52 | 2 61 |
| 5×3 | 9/16 | 8 38 | 82 | 2 37 | 2 47 | 2 52 | 2 57 | 2 67 |
| 5×3 | 13/16 | 11 68 | .80 | 2.42 | 2 52 | 2 57 | 2 62 | 2.71 |
| 5×3½ | ⅜ | 6 10 | 1 02 | 2 27 | 2 36 | 2 41 | 2 45 | 2 55 |
| 5×3½ | 5/8 | 9 86 | 99 | 2 31 | 2 40 | 2 45 | 2 50 | 2.60 |
| 5×3½ | ⅞ | 13 36 | 96 | 2 36 | 2 45 | 2 50 | 2 55 | 2.65 |
| 6×3½ | ⅜ | 6 86 | 99 | 2 81 | 2 90 | 2 95 | 3 00 | 3 09 |
| 6×3½ | 5/8 | 11 10 | .96 | 2 86 | 2 95 | 3.00 | 3 05 | 3 15 |
| 6×3½ | ⅞ | 15 10 | 93 | 2 90 | 3 00 | 3 05 | 3 10 | 3.20 |
| 6×4 | ⅜ | 7.22 | 1.17 | 2 74 | 2 83 | 2 87 | 2 92 | 3.02 |
| 6×4 | 5/8 | 11 72 | 1 13 | 2 78 | 2 87 | 2 92 | 2 97 | 3 06 |
| 6×4 | ⅞ | 15 98 | 1 11 | 2 82 | 2 92 | 2.97 | 3 02 | 3.12 |

## TABLE XIV

### FORMULAS FOR DEFLECTION OF BEAMS
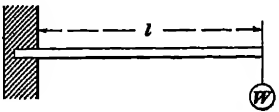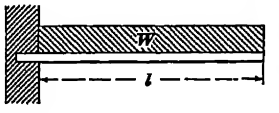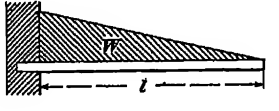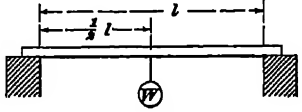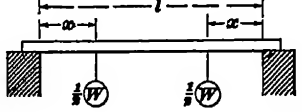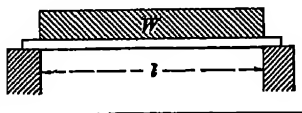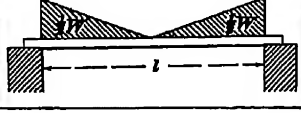
| Case | Method of Loading | Deflection Inches |
|------|-------------------|-------------------|
| I |  | $\dfrac{Wl^3}{3\,EI}$ |
| II |  | $\dfrac{Wl^3}{8\,EI}$ |
| III |  | $\dfrac{Wl^3}{15\,EI}$ |
| IV |  | $\dfrac{Wl^3}{48\,EI}$ |
| V |  | $\dfrac{Wxy\,(2\,l-x)\,\sqrt{3\,x\,(2\,l-x)}}{27\,l\,EI}$ |
| VI |  | $\dfrac{Wx}{48\,EI}(3\,l^2-4\,x^2)$ |
| VII |  | $\dfrac{5\,Wl^3}{384\,EI}$ |
| VIII |  | $\dfrac{3\,Wl^3}{320\,EI}$ |

**TABLE XIV—(*Continued*)**
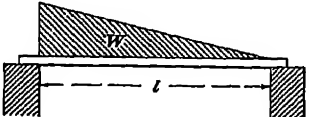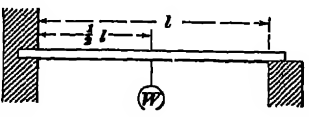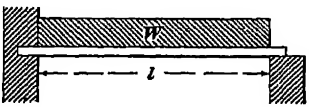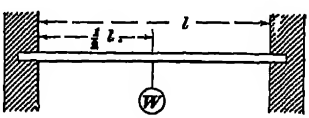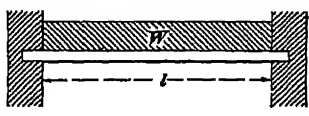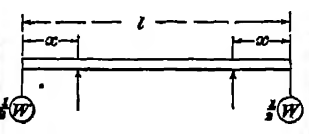
| Case | Method of Loading | Deflection Inches |
|------|-------------------|-------------------|
| IX | | $\dfrac{W l^3}{60 \, E I}$ |
| X | | $\dfrac{47 \, W l^3}{3,600 \, E I}$ |
| XI | | $\dfrac{3 \, W l^3}{322 \, E I}$ |
| XII | | $\dfrac{5 \, W l^3}{926 \, E I}$ |
| XIII | | $\dfrac{W l^3}{192 \, E I}$ |
| XIV | | $\dfrac{W l^3}{384 \, E I}$ |
| XV | | For overhang: $\dfrac{W x}{12 \, E I}(3 \, x \, l - 4 \, x^2)$ <br> For part between supports: $\dfrac{W x}{16 \, E I}(l - 2 \, x)^2$ |

## TABLE XV
### MODULI OF RUPTURE OF VARIOUS MATERIALS

| Material | Modulus of Rupture Pounds per Square Inch |
|---|---|
| Iron, cast . . . . . . . . . . . . . . . | 30,000 |
| Iron, wrought . . . . . . . . . . . . | 44,000 |
| Steel, structural, soft . . . . . . . . . | 54,000 |
| Steel, structural, medium . . . . . . . | 60,000 to 64,000 |
| White oak, air-dried . . . . . . . . . . | 7,000 |
| White pine, air-dried . . . . . . . . | 4,000 |
| Long-leaf, or Georgia, pine, air-dried . . | 7,000 |
| Douglas fir, air-dried . . . . . . . . | 5,000 |
| Hemlock, air-dried . . . . . . . . . . | 3,500 |
| California redwood, air-dried . . . . . | 5,000 |
| Spruce, air-dried . . . . . . . . . . . | 4,000 |